A NEW COMPLEAT and UNIVERSAL SYSTEM or BODY OF DECIMAL ARITHMETICK,

CONTAINING,

I. The Whole *Doctrine* of *Decimal Numbers*, not only the Plain and Terminate, but also such as *Repeat* or *Circulate ad Infinitum*; and a Plain but Perfect *Management* of both, laid down and explained in all the *Fundamental Rules* of *Plain Arithmetick*, and by *Logarithms*.

II. The Application and Use of *Decimal Arithmetick* in all the *Parts* or *Branches* of *Arithmetical Science*; viz. *Vulgar Arithmetick*, *Vulgar Fractions*, *Duodecimal*, and *Sexagesimal Arithmetick*; also in *Algebra* and *Logarithms*. In all which its *Excellency* and absolute *Necessity* is fully evinced.

III. Its Application and Use in all such *Parts* of the *Mathematicks* as absolutely require its *Assistance*; viz. *Plain Trigonometry*, and the Arts depending thereon; as, *Navigation*, *Fortification*, *Altimetry*, *and Longimetry*; Also the *Mensuration of* all Kinds of *Superficies and Solid Bodies*; and the Arts resulting therefrom; as, *Guaging*, *Surveying*, &c.

IV. A *New and compleat Sett of Decimal Tables* never before published, shewing by inspection the Value of all Kinds of Decimals (without the tedious Methods of Reductions hitherto used) to four or six Places of Figures; Also all the *Common Tables* very much *inlarged*, *corrected*, *and improved*; wherein all the *Circulating Numbers are marked*. With all other Tables of Interest, Annuities, Exchange, &c. *necessary* to render the Work *compleat*.

V. An exact and accurate *Canon of Logarithms*, for natural Numbers. And thro' the Whole, several Things new and useful, not here express'd.

By BENJAMIN MARTIN. K

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The PREFACE.

I Preſume 'tis entirely needleſs to write a Panegyric on the ſuperlative Excellency of the noble Science of Decimal Arithmetick, *ſince the World has already been ſufficiently apprized thereof in the extream Benefit and Service it has afforded the World of* Mathematical Literature, *even tho' in its* Embryo State ; *much leſs doth it need* Encomiums *to ſet forth its Nature, Worth, and valuable Properties, which have been diſcovered and illuſtrated by late Improvements ; of which the enſuing Work is but (as it were) an Inſtance.*

The Book I here preſent the World withal is a regular Syſtem of this valuable Art, according to all the lateſt Improvements of others, and many (in the ſeveral Parts thereof) of my own ; the two greateſt of which are, The Doctrine of infinite circulating Decimals *by the learned Mr.* Samuel Cunn ; *the other, A* New Sett of Tables *ſhewing the Value of any Decimal Part of any Integer, whether* Money, Weight, Meaſure, Motion, Time, *&c. by Inſpection only, to a ſufficient Exactneſs, without the* tedious Reductions *hitherto neceſſarily uſed ; which coſt me not a little Time and Pains to calculate, but was neceſſary to render this Art compleat.*

The Foundation on which I have built this Superſtructure is the abovementioned Gentleman's ſmall, but learned, Treatiſe of the Doctrine of Decimal circulating Numbers : But that great Maſter having laid the Foundation deep, and in a great Meaſure out of the Vulgar Ken ; I thought it might be of Service to young Students, *a little to diſcloſe and lay it more open to their View, and this was all I at firſt intended to do ; but having done that, Materials came in ſo faſt, that I went on and erected the* Syſtem *of* Decimal Science *thereupon, as you here ſee it ; of which take the following Account.*

In the Fundamental Rules of the Art, viz. Addition, Substraction, Multiplication, *and* Division, *I have been as plain as possibly I could without Prolixity, and shewn the compleat and perfect Management of both* Plain *and* Circulating Decimal Numbers *in each of the Rules, in so easy and obvious a Method as the meanest Capacity, with proper Attention, may comprehend; and have taken care, in its proper Place, to give the true* Reason, *or* Rationale, *of each particular and different Process, especially of those relating to* Circulating *or* Repeating Numbers *of all Kinds, as I went on, omitting nothing that I could communicate towards the perfecting this fundamental and important Part.*

In Reduction *I have perfected this Art beyond what it hath ever been; as I have not only largely exemplified all the common and vulgar Methods of Reducing to, and from Decimals, in all kinds of* Vulgar, Fractional, Duodecimal, *and* Sexagesimal Numbers, *by Arithmetical Operations; but have compleated the* Tabular Part, *which has been hitherto very deficient, both in the Tables already extant, and in the* Want *of others. The first of these Defects I have endeavoured to supply by correcting, inlarging, and compleating the common Tables for reducing the various Denominations of all Kinds of Quantities to Decimal Numbers; wherein I have taken care to* mark *all the* Circulating *or* Repeating Numbers, Single *or* Compound, *so far forth as they came within the Verge or Limits of the said Tables; which hath not till now been done by any. The other Defect, and that which renders this excellent Art most lame and imperfect,* viz. *The* Want *of* proper Tables *to express again the Value of a* Decimal *in the* Vulgar Denominations, *or known Parts of its* Integer, *without tedious and operose Arithmetical Reductions, I have also supplied by the Addition of a* compleat Sett of such Tables, *and shewn their Use in Examples of all Varieties. The Reader may have a farther Account of these Tables in the Place where they are inserted; of which I shall say no more, but that these are the* first Tables *of this Kind that were ever published.*

What I have hitherto said, relates purely to the Nature

and Substance of the Art it self; *what follows concerns its* Application *to* Arithmetical *and* Mathematical Sciences.

In Vulgar Arithmetick, *I have applied the Doctrine of* Decimal Numbers, *both* Plain *and* Circulating, *in every Part*; *and shewn its admirable Use, Service, and Expediency thro' the Whole. Particularly I have facilitated the* Business of Practice *by a* new Table *and Method of Working by* Decimals; *whereby the Difficulty and the Intricacy of this Rule by* common Arithmetick *is avoided.*

In Inchange *I have been very particular and explicit, having made this Affair (the* Basis *of* Merchandise) *more intelligible to meaner Capacities than I have met with it*; *explained the Nature and Meaning of the* Par *and* Course *of* Exchange, *and have exhibited* large *and* compleat Tables *of both*; *such as are very rare to be found in Books of this Nature, tho' they are an indispensable Part of Decimal Arithmetick. The Rules of* Interest, Simple *and* Compound, *being of so great and general Concernment and Importance, and yet so little truly understood, I have taken abundance of Pains and Care to let* young Students *see the* Theory, *or true Reason and Nature thereof, by a perspicuous Method of Resolving the most excellent Theorems of the late Mr.* Ward, *in* Decimal Numbers; *which in this Part of Arithmetick are* absolutely necessary. *And to facilitate and expedite* Calculations *of* Interest, *I have procured a compleat Sett of* Decimal Tables of Interest, *and shewn their* Construction *and* Use. *In the other Parts of common* Arithmetick, *I have been large and clear, and omitted Nothing that I could contribute to perfect them. In fine, in every Part I have shewn the* Necessity *and* Expediency, *and in the Whole, the* Preference *and* superior Utility *of* Decimal Computations.

In Vulgar Fractions, *I have shewn how all Questions are* most easily *and* commodiously *resolved by* Decimals; *and for that End have calculated a Table shewing* by Inspection *the* Decimal *equal to any* Vulgar Fraction *whose Denominator exceeds not* Twenty. *I have extended the Table no farther, because those small Fractions are most frequent and useful in Business.*

In Duodecimal *and* Sexagesimal *Arithmetick, I have shewn the Nature and Rules of the Arts*; *and, by many Examples, how Questions are most advantagiously wrought by* Decimal Numbers, *especially in* Duodecimals, *so much used in* Mensuration, *I have also inserted* compleat Tables *for turning* Duodecimal *and* Sexagesimal Numbers *into* Decimal *ones, and such as are not to be found every where*; *wherein (as in all my other Tables) I have noted the* Circulating Decimals, *which no one besides hath done.*

In the excellent Art of Logarithms, *I have not only explain'd the* Principles *and* Rules *of the Art it self, but largely shewn how all Kinds of* Decimals *are managed and ordered thereby*; *others have taught the World the Management of* plain *or* terminate Decimals *by* Logarithms; *But that of* circulating *or* repeating Decimals *of all Kinds, hath fallen to my Province only, since no one before has attempted it. I have here explain'd the Method of finding the* Logarithms *of any* Repetend, *whether* single *or* compound, pure *or* mixed *with absolute Numbers, with more Ease and Certainty than can be found for any* terminate Decimal *exceeding the Numbers in the* Canon. *I have shewn how to work all Sorts of* Decimals *in all the Rules of this noble Art*; *and to make this Part of the Work compleat, I have inserted a* Canon *of* Logarithms *for* Natural Numbers; *where by Rules you are taught to find the* Logarithms *of any Number not exceeding* 10000000, *and the contrary.*

In Algebra, *the Use and Necessity of Decimals in raising and resolving* Equations, *I have demonstrated in a* Select Number *of the most* curious *and* useful *Questions relating to the* Theory *of* Arithmetick, *to the* Philosophy *of* Motion, *&c. and shewn how lame and imperfect, even this most perfect and perfecting Art would be without the* Subservience *of* Decimal Numbers.

Hitherto of the Use *and* Application *of* Decimals *in the several Parts of* Arithmetical Science; *in the next Place take what concerns its* Application *to the Principal Parts of* Mathematical Knowledge.

In Plain Trigonometry, *I have convinced the Reader how* absolutely necessary Decimals *are in order to express the*

the Quantity of the Sides *of all* Right-lined Triangles, *in the Solution of all the Cases of* Right *and* Oblique-angled Trigonometry. *And as this Art is the* Foundation *and* Essence *of several others, as* Fortification, Navigation, Mensuration *of* Altitudes *and* Distances, *&c. I have likewise shewn the* Nature *and* Rudiments *of those Arts and Sciences, and the Manner of performing Conclusions by them in* Decimal *Numbers. So that any Person may here both learn the Art of* Trigonometrical Calculation, *and its* Application *to the aforesaid Arts, after the best Method, with the same Ease and to as good Purpose, as from many Books* wrote purposely *on the Subject.*

In the Mensuration of Superficies and Solids, *no one will pretend to dispute the* Superlative Use *of* Decimal Arithmetick; *whereof the small Tract I have here publish'd is a sufficient Instance. I have not only taught how to measure more* Superficies *and* Bodies *than any other one Book that I know of, but shew'd how this Art is the very* Basis *and* Substance *of* Gauging, Surveying, *and all other Kinds of* Measurements *used by* Artificers, *none of which can be obtain'd to any good Purpose without it; nor any Operations therein so well perform'd as by* Decimal Arithmetick; *and here I have provided the* Gauger *with a* Table *of* Multipliers *or* Divisors *for finding the Content of any* Superficies, *or the* Capacity *of any Vessel in* Wine Gallons, Ale Gallons, Corn Gallons *or* Corn Bushels, *whether the Dimensions be taken in* Inches, Feet, *or* Yards.

Thus I have given a general Account *of the Substance of the Book; it would be endless to descend to Particulars: Many Things of Importance in various Parts of the Book will offer themselves to the View of the Reader unexpectedly, and appear in the whole, with the Face of* Novelty. *I have spared no Pains in Consulting the best* modern Authors *on each particular Head as I went on, and extracted from them severally whatever I found of value and Worth and would contribute to perfect my Design; so that nothing of Consequence can be found in any other Piece of* Decimal Fractions *(as this* noble Art *has been* diminutively *called)*

led) but what may be here met with amidst a great Variety of other novel, but useful and curious Matters.

So that upon the Whole I hope this Book doth truly merit the Title it bears, viz. A New Compleat and Universal System or Body of Decimal Arithmetick.

If then any Person be desirous of a Good and thorough Knowledge of Decimal Numbers *of every Kind, and of their Compleat and perfect Management by the* Rules of Art, *they may be here satisfied. If they would learn its* Application, *or how to use it to the best Advantage in the various* Arts, Trades, *and* Business *of Life, they will here meet with plentiful Instructions, and Examples in every Sort, adapted to particular Cases. Would they learn the true Grounds, or* Rationale, *of all* Arithmetick *whether* Vulgar, Fractional, Duodecimal, Sexagesimal; *and of the* Mathematical Arts, Mensuration, Gauging, Surveying, Navigation, Fortification, Altimetry, Longimetry, *&c. let them please to spend a few of their* spare Hours *here. Are they disposed to learn the* Use *of* Logarithms, *or the Method of* Trigonometrical Calculations, *they are here with Ease informed. In short, they may here find in one* small Volume, *what I have been obliged to turn over* many *both* small *and* great *to procure; and therefore if* Variety, Utility, Novelty *and* Brevity *can please, I hope the* Publick *will candidly accept my Labours. I am not apprised of any Faults therein, and am very sure there are but few* material *Ones, having taken all the Care and Pains I was able, to prevent them.*

But if the well dispos'd *and* inquisitive *Reader, after having* perused *it* throughly, *shall then judge it* deficient; *I should be very glad if any thing better of the Kind should offer, that may afford him* greater Satisfaction; *and till* then *only, I entreat his* kind Acceptation *and* candid Perusal *of This.*

VALE.

THE

THE INTRODUCTION, CONCERNING

The Nature, Kinds, and Notation, of DECIMAL NUMBERS.

1. THE excellent Art of *Decimal Arithmetick* derives its Name from a Latin Word (*viz. Decem*, Ten) which denotes the Nature of its Numbers, which represent the Parts of any *Integral Quantity* divided in a *Decuple*, *Decimal*, or *tenfold Proportion*.

2. Any Integer, or whole Quantity, being divided into 10, 100, 1000, 10000, &c. Parts, because those Numbers are in a *Decimal* or tenfold Proportion; therefore such Numbers as represent any of those Parts, are call'd *Decimal Numbers*, or *Decimals*.

3. Thus, suppose I divide any whole Quantity into 10 Parts, and take 7 of them; those are call'd 7 decimal or tenth Parts of that Integer; and are thus vulgarly wrote $\frac{7}{10}$; suppose it divided into 100, 1000, &c. Parts; then 7 of them would be express'd thus $\frac{7}{100}$, $\frac{7}{1000}$, &c. and read as before.

4. Also many Integers and Parts of another, would be express'd thus, $8\frac{7}{10}$, $19\frac{67}{100}$, $475\frac{154}{1000}$, $2\frac{2946}{10000}$, &c. and read thus; 8, and 7 tenths; 19, and 67 Hundreths, or Parts of a 100; 475, and 154 Thousandths, or Parts of a Thousand; 2, and 2946 Parts of ten Thousand. Thus the *Denominator* of Decimal Parts or Numbers, is always an Unite with Cyphers annexed.

5. Therefore if the Places of Figures in the *Numerator* be equal in Number to the Places of Cyphers in the *Denominator*, (or be made equal thereto, by prefixing Cyphers;) the *Denominator* in ſuch a Caſe will be known, though it be not written; and therefore in the Notation of Decimal Numbers, is always omitted; and the *Numerator* (ordered as aforeſaid) alone is join'd to the *integral Quantity*, with a Comma, or Point, to diſtinguiſh it therefrom.

6. Hence $\frac{34}{100}$; $\frac{45}{1000}$; $\frac{257}{1000}$; $\frac{271}{100000}$; $\frac{4}{1000}$; &c. are thus written, .34; .045; .00271; .004. Alſo $27\frac{3}{10}$; $58\frac{4}{100}$ $129\frac{132}{10000}$; $1\frac{17}{10000}$: are wrote 27.3; 58,04; 129.0132; 1,0017. And on the contrary, by .12; 1.76; .006; 2,003,; .001; we underſtand $\frac{12}{100}$; $1\frac{76}{100}$; $\frac{6}{1000}$; $2\frac{35}{10000}$ $\frac{1}{1000}$.

7. Cyphers prefixed to decimal Numbers, decreaſe their Value in a decuple or tenfold Proportion; as affixed to Integers, they increaſe their Value in the ſame Proportion: thus .5; .05; .005; .0005; &c. are, as they proceed, each one ten times leſs then the preceding Decimal; as is eaſy to conceive.

8. When the *Denominator* is an Aliquot Part of the *Numerator* increaſed by Affixing Cyphers thereto, the Decimal equivalent to ſuch a Fraction, will be compleat and terminate; as, $\frac{5}{10} = .5$; $\frac{5}{20} = .25$; $\frac{15}{20} = .75$; $\frac{1}{20} = .05$; $\frac{6}{240} = .025$; $\frac{11}{80} = .1375$; $\frac{3}{960} = .003125$; $\frac{3}{16} = .1875$.

9. But if the *Denominator* be no aliquot Part of the *Numerator* thus increaſed; the Decimal equivalent to ſuch a Fraction will be interminate or endleſs; that is, it will conſtantly repeat one Digit only; as $\frac{1}{3} = .333333$, &c. *ad infinitum*; or $\frac{4}{6} = .666666$, &c. or $\frac{7}{12} = .5833333$, &c. or $\frac{5}{36} = .138888$, &c. or $\frac{31}{960} = .0322916666$, &c. *ſine fine*.

10. Or elſe a certain Number of Figures perpetually Circulate, or repeat in the Quotient. Thus $\frac{2}{11} = .18\ 18\ 18\ 18$, &c. *ad infinitum*; alſo $\frac{5}{27} = .185\ 185\ 185\ 185$, &c. And $\frac{20}{21} = .952380952380$, &c. And $\frac{3}{220} = .01\ 36\ 36\ 36$, &c. without End. And thoſe Numbers which thus infinitely circulate or repeat, are moſt fitly termed *Repetends*. Thoſe which circulate a Digit only, are called a *ſingle Repetend*;

petend; and thoſe in which ſeveral Figures circulate, are call'd a *Compound Repetend*, in the following Tract.

11. For the greater Elegance and Perſpicuity, in all the Operations of Circulating Numbers, I have daſhed the firſt and laſt Figure of the *Repetend*; thereby making one Place of the *Repetend* ſufficient. Thus the Examples above are thus wrote or expreſſed; $\cancel{3}$; $.\cancel{6}$; $.58\cancel{3}$; .138; $.032291\cancel{6}$. And the *Compound Repetends* thus; $.\cancel{1}\cancel{8}$; $.\cancel{1}8\cancel{5}$; $.\cancel{0}5238\cancel{0}$; and $.01\cancel{3}\cancel{6}$; herein following the Ingenious Mr. *Cunn*, the firſt Improver of this Part of *Decimal Arithmetick*.

12. In a *Compound Repetend*, any one of the circulating Figures may be made the firſt of the *Repetend*; for Inſtance, in the *Repetend* $8.6\cancel{3}2\cancel{5}$ 325 325, &c. it may be made $8.63\cancel{2}5\cancel{3}$; or $8.6325\cancel{3}\cancel{2}$. And by this Means any two or more *Repetends* may be made to begin and end in the ſame Place; and then they are ſaid to be conterminous.

13. Several other Things relating to the Nature and Properties of circulating Numbers, I have interſperſed in the following Treatiſe in their proper Places, where they may be underſtood, and which are not to be found in any other Book of *Decimal Arithmetick*.

14. In all *Decimal Numbers*, if the Point of Diſtinction be removed one Place towards the Right Hand, every Figure, and conſequently the Whole Expreſſion, will be increaſed in a tenfold Proportion; as in thoſe Decimal Expreſſions 3.756, 37.56, 375.6, 3756. which are each one 10 times greater than the preceding one. In which Proportion alſo, 'tis manifeſt, they decreaſe in Value, by removing the Decimal Point a Place to the Left Hand.

15. The Nature and Properties of *Decimal Numbers*, are the ſame with thoſe of *Integers* or *Whole Numbers*, and the Method of Working both the ſame (excepting Repetends). Hence ariſeth the Excellency and ſuperior Uſefulneſs of *Decimal Arithmetick*, above all other kinds of Computation.

16. To make the preceding Propoſition evident, ſuppoſe 'twere required to expreſs the Time ſince our Saviour's Incarnation to the Year preſent, in Centuries and Decimal Parts of a Century; it would be thus 17.33; where you obſerve one half of the Number conſiſt of *Intgers*, and the other half of *Decimals*. But ſuppoſe the Time ex-

preſs'd in Years, the Number conſiſts of the ſame Figures, 1733; and is whole or integral.

17. Hence 'tis plain the ſame Number may be either *Integral* or *Decimal*, and that either in Whole or in Part, according to what is made the Integer; for in the foregoing Caſe, if a Myriad be the Integer, the Time will be expreſſed by a pure Decimal 0.1733; if a Century be the Integer, by a mix'd Decimal 17.33; if a Year be the Integer, by the integral Number 1733; as before.

18. All the different Species, or Parts of different *Kinds* and *Denominations*, of *Money*, *Weights*, and *Meaſures*, and all other Quantities, are to be reduced to *Decimals*, or may be expreſſed in *Decimal Parts* of their reſpective Integers, by proper Tables calculated for that Purpoſe; alſo any *Decimal* may very nearly by Inſpection only (without the tedious Reductions hitherto uſed) be read in the vulgar Parts or Denominations of its reſpective Integer, by a Set of new Tables, which I have compoſed for the Eaſe of thoſe who are converſant in this excellent Science.

19. Since then it has been ſhewn that *Decimals* are the ſame with *whole Numbers*, as to their Nature and the Manner of Operation; and that all mix'd Numbers, or ſuch as conſiſt of divers and different Denominations, are reducible thereto, and *vice verſa*; it follows that all the Arithmetick of mix'd and heterogeneous Numbers is to be perform'd by *Decimals*, with the ſame Eaſe, Expedition, and Pleaſure as that of *whole Numbers*.

20. And by Conſequence, That *Vulgar Arithmetick*, *Vulgar Fractions*, *Duodecimal* and *Sexageſimal Arithmetick*, (thoſe Parts of the Science of Computation hitherto deem'd ſo hard and intricate, and therefore but little ſtudied or known) are all by this noble Art of *Decimal Arithmetick* perform'd with the utmoſt Eaſe and Pleaſure, that any Arithmetick is capable of, and which I have abundantly evinced in the Sequel of the enſuing Work.

21. The Figures of a *Decimal Number* are to be numerated as thoſe of whole Numbers, *viz.* from the Right Hand to the Left; but they muſt be denominated of the Number of Parts the Integer is divided into. The following Table will make the Numeration and Denomination of Decimals very eaſy.

A TABLE of the Numeration and Denomination of DECIMALS.

Integer	Tens	Hundreds	Thousands	Tens of Thousands	Hundreds of Thousands	Millions	Tens of Millions	Hundreds of Millions	Thousands of Millions	
1.	0	0	0	0	0	0	0	0	0	= Parts of the Integer.
0.	1	.	.	.	.	.	.	.	.	= Parts of Ten.
0.	1	2	.	.	.	.	.	.	.	= Parts of an Hundred.
0.	1	2	3	.	.	.	.	.	.	= Parts of a Thousand.
0.	1	2	3	4	.	.	.	.	.	= Parts of Ten Thousand.
0.	1	2	3	4	5	.	.	.	.	= Parts of an Hundred Thousand.
0.	1	2	3	4	5	6	.	.	.	= Parts of a Million.
0.	1	2	3	4	5	6	7	.	.	= Parts of Ten Millions.
0.	1	2	3	4	5	6	7	8	.	= Parts of an Hundred Millions.
0.	1	2	3	4	5	6	7	8	9	= Parts of a Thousand Millions.

0.	0	0	0	0	0	0	0	0	9
0.	0	0	0	0	0	0	0	8	9
0.	0	0	0	0	0	0	7	8	9
0.	0	0	0	0	0	6	7	8	9
0.	0	0	0	0	5	6	7	8	9
0.	0	0	0	4	5	6	7	8	9
0.	0	0	3	4	5	6	7	8	9
0.	0	2	3	4	5	6	7	8	9
0.	1	2	3	4	5	6	7	8	9

These Numbers are all of them the reſpective Parts of a Thouſand Millions, into which the Integer is divided as above.

An

An Explanation of the Characters and Abbreviatures used in the following Book.

It has been of late an Expedient to avoid Prolixity in Writing, to make uſe of ſome convenient and ſignificant Characters to expreſs thoſe Words which moſt often occur, and occaſion Tediouſneſs and Tautology in the Work, the moſt irkſome Vices that can attend it; and accordingly I have here uſed them; which, with their Significations, are thus to be underſtood.

Figure.	*Names.*	*Significations.*
$+$	Plus, or more.	As $a+b$, is a more b; in Addition.
$-$	Minus, or leſs.	As $a-b$, is a leſs b; in Subtraction.
$\times$	Multiplied into.	As $a\times b$, is a multipl. into b; in Mult.
$\div$	Divided by.	As $a\div b$, is a divided by b; in Diviſ.
$=$	Equal to.	As $a=b$, is a equal to b; in Equat.
$:$	Is to.	As $a:b::c:d$; as a is to b, ſo is c to d; in Proportions.
$::$	So is.	
⊖	Involved.	As 2 ⊖, is the 2d involved.
ω	Evolved.	As 2 ω, is the 2d evolved, or extracted.
$\surd$	Surd Root.	As $\sqrt[2]{ab}$, $\sqrt[3]{ab}$, $\sqrt[4]{ab}$, &c. is the Root Square, cub'd, biquadrate, &c. of ab a Surd.

CHAP. I.

Addition of DECIMALS.

ADdition of Decimal Parts admits of various Caſes, according to their different Kinds; either as they are *terminate and compleat*, or *interminate*, and continually *repeat* either *one* or *more* Figures. I ſhall illuſtrate all the ſeveral Varieties by ſuitable Examples of *Money*, *Weight*, *Meaſure*, &c.

Caſe

Case I. If your Decimals be terminate, place Units under Units, Tens under Tens, *&c.* in whole Numbers, and annex the Decimals in order towards the Right Hand; then add them, and cut off from the Sum to the Right Hand so many Places for Decimals as are equal to the greatest Number of Decimal Places in any of the given added Numbers.

EXAMPLE I. Add together

	l.		Then *l.*	*s.*	*d.*	*qr.*
	56,7375					
	24,025					
	20,0125					
	19,4875		25, =	. —	. —	2,4
	12,05		31, =	6 —	2 —	1,6
The Sum	132,3125	=	132 —	6 —	3 —	0

Example II. Add together

	l. *Troy.*		*l.*	*oz.*	*pwt.*	*gr.*
	457,825					
	570,065625					
	24,5375					
	806,253125					
	0,003125		43 =	. —	1 —	0,768
	695,05		73 =	8 —	15 —	4,8
The Sum	2553,73.43.75	=	2553 —	8 —	16 —	6

Case 2. If you have a great many several Sums to add, and their Decimals run to a great Number of Places, it will not be necessary to add them all, but only so many Places, *as are sufficient to give the Value* in the aggregated Sum, which will require but 4 or 5 Places, or 6 at most; for so far only the largest Tables go.

Observe to make that Figure (at which you break off) more by a Unit, if the next rejected Figure be more than 5; but if the next Figure be less than 5, reject the Figures only.

Then add the several Sums, and the certain Places of the Decimal are generally fewer by one than the Decimal Places retained in any of the given Sums. I shall subjoin an Example at large, and the same thus contracted.

EXAMPLE

EXAMPLE.

47.982774354	Thus contracted	47.98277
175.6732956		175.6733
2.43752432		2.43752
97.702006764		97.70201
276.92301762		276.92302
30.00420999		30.00421
630.722828648		630.72283

In this Example there are 4 Places of Decimals certain, or the ſame, in both Operations; and the Rule will ſcarce ever err *above an Unit* in the laſt Place. Now whatever you ſuppoſe the Integral Quantities to be, the *Tables* will ſhew you the Value of the Decimal Parts.

Caſe 3. Suppoſe the Numbers you are to add have *repeating Decimals*; if they are *ſingle Repetends*, make them all *conterminous*, that is, end together; and then add as before, only to the laſt, or Right Hand Place of Decimals, add as many *Units* as there are *Nines* in it; and that *laſt Digit* ſhall be one of the *Repetends*.

N. B. It may be proper to give the firſt and laſt Places of all Repetends *a Daſh* with the Pen, for Diſtinction.

EXAMPLE I.

	l.		*l.*	*s.*	*d.*	*qr.*
	124.233					
	64.516					
	0.333					
	59.800					
	3.833	,30 =	.	— .	—	2,88
	45.016	,73 =	14	— 7	—	08
Sum =	297.733	= 297	— 14	— 7	—	3,68

EXAMPLE

EXAMPLE II.

Feet.		*F.*	*In.*	*qrs.*
5,91$\dot{6}$66				
0.0208$\dot{3}$				
2.5625$\dot{0}$				
4.8$\dot{3}$333				
6.041$\dot{6}$6	,16	= .	—	.17
2.$\dot{6}$6666	,04	= .	—	1.92
Sum = 22,041$\dot{6}$6	=	22	— 0	— 2

EXAMPLE III.

Oz. Troy Wt.		*oz.*	*pwt.*	*gr.*
4,72708$\dot{3}$				
2.58$\dot{3}$333				
0.00208$\dot{3}$				
9.0291$\dot{6}$6				
4.03125$\dot{0}$				
7.03541$\dot{6}$				
10.758$\dot{3}$33	,33	= .	—	1,58
6.91$\dot{6}$666	,08	= 1	—	14,4
45.08$\dot{3}$333	=	45	— 1	— 16

EXAMPLE IV.

Days.		*D.*	*H.*
275.252777			
47.87$\dot{1}$111			
436.02708$\dot{3}$			
10.6$\dot{7}$7777			
101.2$\dot{5}$5555			
127.769444	,20	= .	— 2,8
243.958$\dot{3}$33	,81	= 19	— 26,4
Sum = 1242.81208$\dot{3}$	= 1242	— 19	— 29,2

Case 4. If your Decimal be a *compound Repetend*, that is, consists of several Places of Figures which *continually repeat or return*; the Sum or Aggregate of any given Number of such Decimals will also *repeat*; and the Number of Places, or of the Figures, in each repetend, will be equal to the least *common Multiple* of those several Numbers which represent the Places of Figures in the Repretends added. Hence (tho' it be scarce ever necessary to have above five or six Places of Decimals, yet) if any one be minded to see the *Repetend compleat*, he must observe this *Rule*;

From the Place where all the Repetends *begin together*, continue each Decimal to a Number of Places equal to the *Multiple aforesaid*; then add, and to the last Place add as many Units as there are 10's in the Place where the Repetends all begin together, and the Figures in those two Places are the *first* and the *last* of the *Repetend*. The Examples following will make all plain.

	l.		*l.*	*s.*	*d.*	*qr.*
EXAM. I.	13 . 0462					
	2 . 8043	,60 =	.	1		1,78
	5 . 7234	,37 =	7	4		3,2
Sum	21 . 3760	=	21	7	6	1

	C. Wt. Av.		*C.*	*Q.*	*lb.*	*oz.*	*dr.*
EXAM. II.	14 . 472956						
	12 . 307243	,38 =	.	.		.	1,09
	9 . 020763	,33 =	.	.		5	14,61
	11 . 912378	,71 =	2	23		8	5,12
Sum =	47 . 713338	=	47	2	23	14	4,82

	Rods.		*R.*	*Yd.*	*F.*	*In.*
EXAM. III.	121 . 47237					
	80 . 27555					
	64 . 90634	,87 =	.	.		1,73
	89 . 07444	,72 =	3	2		10,56
Sum =	355 . 72871	=	355	4	0	0,3

EXAM-

Years.

			Y.	M.	W.	D.	H.
	175.3724						
	84.2634						
EXAM. IV.	126.4526						
	72.3279						
	105.7105	,46	= .	— .	— 1	— 16,3	
	28.4378	,56	= 7	— 1	— 1	— 9,6	
Sum =	599.5646	= 599	— 7	— 1	— 3	— 2	

Signs.

		Sig.	°	'	''
	4.212213				
	2.524756				
EXAM. V.	6.030303				
	5.215215				
	7.070707	,68	= .	— 12	— 14
	0.732612	,78	= 23	— 24	— 0
Sum =	1.786807	= 1	— 23	— 36	— 14

Feet Square.

			Fq.	Iq.	Qq.
	2.954395439543				
	1.041041041041				
EXAM. VI.	3.737373737373				
	4.065826065826	,01	= .	— 0,23	
	4.731473147314	,53	= 76	— 5,12	
Sum =	16.530109431099	= 16	= 76	— 5,35	

These Six Examples, I imagine, are sufficient to illustrate this last Case of *compound Repetends*; but if it chances to happen that *a compleat* or *terminate* Decimal be to be added with them, you must affix *Cyphers* thereto, to esteem and deal with them as a *Repetend*.

CHAP. II.

SUBSTRACTION.

Case I. IF your Decimals be *terminate* and *compleat*, place as directed in this Case of *Addition*, and Substract as in Whole Numbers; imagining all the *vacant Places* fill'd with Cyphers.

EXAMPLE I.

	L.		*l.*	*s.*	*d.*	*qr.*
From	729.4726	{ ,56=	.	—	1	—1,38
Substract	634.927	{ ,54=	10	—	9	—2,4
Remains	94.5456	=	94 :	10 :	10 :	3,78

EXAMPLE II.

	C.		*C.*	*Q.*	*lb.*	*oz.*	*dr.*
From	472,07	{ ,46=	.	— .	—8	—	3,88
Substract	392,4354	{ ,63=	2	—14	—8	—	5,36
Remains	79,6346	=	79 :	2 :	15 :	0 :	9,24

EXAMPLE III.

	lb.		*lb.*	*oz.*	*pwt.*	*gr.*
From	147,	{ ,52=	.	—1—		5,95
Subtract	94,7248	{ ,27 =	3	—4—		19,2
Remains	52,2752	=	52	—3—	6 —	1,15

Case 2. If your Decimals run to *many Places* of Figures, do as directed in the Case of *Addition*; and *substract* as in the *last Case*; and the last place of the Decimal Remainder will never *Err more than an Unit*.

EXAM-

EXAMPLE I.

From 2,752804624 take 1,476937679.

	Miles,		*M.*	*F.*	*P.*	*Y.*	*F.*	*In.*
		,67 =	.	.	.	.		4,23
Thus,	2,752805	,58 =	.	1	4	2		1,14
	1,476938	,27 =	2	6	2	0		7,2
Remains	1,275867 =		1	2	8	0	1	6,57

In this Example the last Figure 7 is a Unit too much, but that is not to be regarded; for in this Case the Value of a Unit in that Place is but ,063 of an *Inch.*

Case 3. If your Decimal *repeat Single Figures*, proceed (as in this Case of *Addition)* to place them, and *substract* as usual; except that when the Subtrahend is the greater Number, you must increase the upper Figure by 9 only, and in every such Case carry one to the next Place.

EXAMPLE I.

	L.		*l.*	*s.*	*d.*	*qr.*
From	54,7333$\dot{3}$	,91 =	.	—	2	0,73
Substract	17,9541$\dot{6}$	,77 =	15	—	4	3,2
Remains	36,7791$\dot{6}$ =		36	15	6	3,93

EXAMPLE II.

	L.		*l.*	*s.*	*d.*	*qr.*
From	57,5289	,56 =	.	—	1	1,38
Substract	49,583$\dot{3}$	,94 =	18	—	9	2,4
Remains	7,945$\dot{6}$ =		7	18	10	3,78

EXAM-

EXAMPLE III.

	Hogsheads.		*H.* *G.* *P.*
From	1672,4516	{	,16 = . — . 8
Substract	879,3000	{	,15 = 9 — 3,6
Remains	793.1516	=	793 — 9 — 4,4

EXAMPLE IV.

	Loads.		*L.* *Q.* *B.* *G.*
From	472,222	{	,60 = . — . — 1,92
Substract	346,175	{	,04 = . — 1 — 4,8
Remains	126,046	=	126 — 0 — 1 — 6,72

EXAMPLE V.

	Years.		*Y.* *M.* *W.* *D.* *H.*
From	47,957200	{	,88 = . — . — 3 — 5,08
Substract	,008316	{	,94 = 12 — 1 — 0 — 2,4
Remains	47,948883	=	47 — 12 — 1 — 3 — 7,48

Case 4. If the Decimals be *compound Repetends*, order them as directed in the Case of *Addition*; then *substract*; and look if you must borrow one in the Place where *both Repetends* begin together; if so, you must add one to the Right-Hand place of the *Subtrahend*; and the Remainder either Whole or in Part, will shew the *Repetend*.

EXAMPLE I.

	L.		*l.* *s.* *d.* *qr.*
From	47,4178178	{	,21 = . — . — 2,01
Substract	15,5656565	{	,85 = 17 — 0 — 0
Remaius	31,8521612	=	31 — 17 — 0 — 2,01

EXAM-

EXAMLE II.

	Ounces.		*Oz. pw. gr.*
From	153,92749	{	,49 = . — 2,35
Substract	142,85353	{	,07 = 1 — 9,6
Remains	11,07495	=	11 — 1 — 11,95

EXAMPLE III.

	Rods Sq.		*Rq. Yq. Fq.*
From	75.5333	{	,35 = . — . 94
Substract	42.7597	{	,77 = 23 — 2,44
Remains	32.7735	=	32 — 23 — 3,38

EXAMPLE IV.

	Yards.		*Y. F. In. Q.*
From	47.8540260	{	,81 = . — . — 1,16
Substract	40.9259259	{	,92 = 2 — 9 — 0,48
Remains	6,9281001	=	6 — 2 — 9 — 1,64

EXAMPLE V.

	Days.		*D. H. M.*
From	75,2758000	{	,94 = . — 13,53
Substract	47,3563563	{	,91 = 21 — 50,4
Remains	27,9194436	=	27 — 22 — 3,93

EXAMPLE VI.

	Degrees.		*D. ′ ″*
From	49,5285285	{	,49 = . — 17,64
Substract	38,4736000	{	,05 = 3 — 0
Remains	11,0549285	=	11 — 3 — 17,64

CHAP.

CHAP. III.

MULTIPLICATION.

Case I. IF your Decimals be *compleat* and *terminate*, whether they be *pure* or join'd with *Integers*, Multiply them as if they were all *whole Numbers*; and cut off (to the Right-Hand) so many Places for Decimal Parts in the Product as there were in both the *Multiplier* and *Multiplicand* counted together. But if it so happen that there are not so many Places in the Product, supply the Defect by prefixing Cyphers.

EXAMPLE I.

Multiply	32.12	} *Feet.*	
by	24.3		
	9636		*Fq. Inq. Qr.*
	12848	{	,60= . —13,8
	6424		,51=73— 7,04
Product	780,516	=	780—74— 4,84

EXAMPLE II.

Multiply	42,51	} *Yards.*	
by	,241		
	4251		*Yq. Fq. Inq.*
	17004	{	,99= .—12,84
	8502		,24=2—23,04
Product	10,24991	=	10—2—35,88

EXAM-

EXAMPLE III.

Multiply	78,546 *Miles,*		
by	436		
	471276		*M. Fq. R. Y. F. In.*
	235638		,60 = .— 1—5—0—1,8
	314184		,05 = .—16—0—0—0
Product	34246,056	=	34246—0—17—5—0—1,8

EXAMPLE IV.

Multiply	,02365		of a *Mile.*
by	,0435		
	11825		*Mq. Aq. Rq. Pq.*
	7095		,28 = .— .— 2,85
	9460		,10 = 0—2—22,4
Product	,001028775	=	0—0—2—25,25

Case 2. When it happens that the Places of Decimals run far in both Factors, and consequently would make a very large Decimal in the Product, you may contract your Work, in such a Case, to as few Places of Decimals in the Product as you please, or is suitable to your Design, by the following *Rule*, *viz*. set the Units Place of the *Multiplier* directly under that Figure of the Decimal Part of the *Multiplicand* whose Place you would preserve in the Product.

Then *invert*, or place all the other Figures of the *Multiplier* in a contrary order to the common way.

Lastly, in Multiplying always begin at the Figure of the Multiplicand which stands over the Figure wherewith you are then a Multiplying, setting down the *first Figure* of each particular Product *directly under one another*. But withal take care to see what *Increase* would arise from the Multiplying of the *two* next Right-Hand Figures of the Multiplicand, which you must constantly *add* to the *first Figure* in every *Product*.

EXAMPLE I.

Suppose I would multiply 92.412031 Yards by 47,29195 Yards, and to have only *four Places* of Decimals in the Product.

Place them as before directed, and they will stand

Thus	92,412031	The Multiplicand as usual.
	59192,74	The Multiplier inversed.
	36964812	
	6468842	
	184824	
	83171	
	924	
	832	
	46	
	4370,3451	

The *Reason* of, and how *great a part* of the Work is saved by, this *Contraction*, will appear from the *Operation at large.*

Thus	92,412031	
	59192,74	
46	2050155	
831	708279	
924	12031	
83170	8279	
184824	062	
6468842	17	
36964812	4	
4370,3451	4935045	

Hence it appears, that *half* the Work is useless, *viz.* all those Figures included in the *Square*, whose Sum make indeed 7 places of Decimals, but are of *no value*, and therefore superfluous.

EXAM-

EXAMPLE II.

Multiply 14,794, by 12,123; and to have thereby reſerv'd *two Places* of Decimals in the Product, place them

Thus	
	14,794
	321,21
	14794
	2959
	148
	30
	4
	179,35

The common way at large.		
	14,794	
	12,123	
	4	4382
	29	588
	147	94
	2958	8
	14794	
	179,34	7662

EXAMPLE III.

Multiply 257,356 with 76,48, and for an *Intire Product of Integers*, place them as by the *Rule*

Thus	
	257,356
	84,67
	18015
	1544
	103
	20
	19682

The ſame at large.		
	257,356	
	7648	
	20	58848
	102	9424
	1544	136
	18014	92
	19682,	58688

From *theſe Examples*, tis manifeſt how advantagous theſe *compendious Contractions* are to *facilitate* and *ſhorten* the Work of thoſe *long* and *operous* Calculations and Computations, which the *experienced* Practitioner finds occur but too often in *Arithmetick*, *Algebra*, and *Geometry*.

Caſe 3. If the *Multiplicand* be a *Repetend only*, and the *Multiplier* a *ſingle Digit*, Multiply as uſual; only obſerve to *add* in the *laſt place* of the Product *as many Units* as it contains *Nines*, and that place is a *Repetend*.

EXAMPLE I.

			Signs of the Zodiack. ° ′ ″
Multiply	10,7016	,83	= · — 15 — 0
by	5	,50	= 15 — 0 — 0
Product	5,5083	=	5 — 15 — 15 — 0

EXAMPLE II.

				Fq. *In. q.*
Multiply	9,305	*Yards.*	,80	= · — 10,33
by	,7		,13	= 1 — 24,48
Product	65,138	=		65 — 1 — 34,81

EXAMPLE III.

				Yq. *Fq.*
Multiply	476,05	*Poles.*	,44	= · — 1,18
by	,08		,08	= 2 — 3,76
Product	38,0844	=		38 — 2 — 4,94

But if the *Multiplier* consist of *several* Digits or Figures, then make each particular Product *conterminous*, by continuing the *single Repetend* of each towards the Right-Hand.

EXAMPLE I.

Multiply	748,64
by	,634
	299457
	2245933
	4491866
Product	474,63057

If

If the *Multiplier* be a *Repetend*, multiply as usual; but in the *Product*, cut off *one place less* for Decimals than usual (which is all one as *multiplying by Ten)* and divide by Nine; continue the Quotient till it becomes a *Single* or *Compound Repetend*; and this shall be the *true Result* or Answer.

EXAMPLE I.

Multiply	724,35
by	,04
	9)289,740
True Product	32,193

EXAMPLE II.

Multiply	26,54
by	,03
	9)7,963
True Product =	,8847

EXAMPLE III.

Multiply	251,43
by	8,74
	9)100,473
	111636
	176001
	201144
Product	2198,6048

EXAM.

EXAMPLE IV.

Multiply	48,754
by	2,13
	9)146263
	1625148
	4875444
	97508888
Product	104,009481

Case 4. If the *Multiplicand* be a *Compound Repetend*, and the *Multiplier* but a *single Digit*, *Multiply* as in *Case the First*; but observe to add to the Right-Hand Place of the Product *so many* Units as there are *Tens* in the Product of the Left-Hand Place of the *Repetend*. And the *Product* shall contain a *Repetend* whose Places are *equal* to those in the *Multiplicand*.

EAAMPLE I.

Multiply	582,347
by	8
Product	4658,778

EXAMPLE II.

Multiply	5924,378
by	,03
Product	177,73138

EXAMPLE III.

Multiply	3749,23
by	,007
Product	26,24464

If the *Multiplier* consists of Places *more than one*, make all the several Products *conterminous* towards the *Right-Hand*, as taught in the *last Case*.

EXAMPLE I.

Multiply	73,2586
by	43,7
	5128106
	21977597
	2931734634
Product	3202,40538

EXAMPLE II.

Multiply	4027,3012
by	4370,2
	80546025
	28191108911
	120819038190
	1610920509205
Product	17600112,02332

But if not *only* the *Multiplicand*, but the *Multiplier* also be a *Compound Repetend*, Multiply (as has been before taught,) each Figure of the *Repetend*, and add the several Products together ; Then add the Result to it self in this Manner, set the first Left-Hand Figure so many Places forward as *exceeds* the Number of Places in the Repetend by one ; and the Rest of the Figures in order after it ; and thus proceed till the Result *last added* be carried *beyond the first* ; Lastly, add these several Results together, beginning under the Right-Hand Place of the first, and from thence *dash as many* Figures for a *Repetend*, as *the Repetend* of the Multiplier does *consist of*.

EXAM-

EXAMPLE I.

Multiply 235,01
by 3,28

151006
67002
70503

First Product 7871326
7871326
7871326

True Product 787,9205

EXAMPLE II.

Multiply 432067
by ,02436

2592402
1296201
1728268
884134

First Product 1052515212
1052515 &c.
105 &c.

True Product 10526,20474

EXAMPLE III.

Multiply 42710,36
by ,20403

12813108
17084144
8542072

First Product 8714,1957508
87141957 &c.
871 &c.

True Product 8714,2828936

If

If the *Multiplier* has any *terminate* Places join'd with the *Repetend*, and if the Repetend *be small* and these *many*, the best way will be to multiply and add the Products of the Repetend first; then after multiply by the *terminate Figures*, and add their Products to the *Sum* of the Product of the *Repetend*; and to this *last Result* add the said *Sum* of the *Repetend Products*, as in the last Examples.

EXAMPLE.

Multiply 432,43
by 23,414

172972
43243

The Sum 605402 of the Product of the Repetends.

172972
129729
86486

1012491602
605402
6054 &c.
60 &c.

10124,97717

But if the *terminate Figures* are *few*, and the Places of the Repetend are *many*; the shortest way will be to *substract* the *terminate Figures* from those of the *Repetend*, and multiply by the *Remainder* as a *Repetend*.

EXAMPLE.

Multiply 1243,2701
by 423,436
From which substract 42 The terminate Figures.

Remains a new Multiplier } 423394

49730804
111894309
37298103
37298103
24865402
49730804

First Product = 5263931007194
5263931007 &c.
526393 &c.
52 &c.

True Product = 526445,7452939

But if *both Factors* are *interminate*, or have *compound Repetends*, the Places of the Repetend in the Product will be *uncertain* as to their *Number*, and can only be determined (in any Manner fit for Practice) by *continuing* and *repeating* the *first Product*, which will contain a *certain Repetend*, being equal in Places, to that of the Multiplicand.

EXAMPLE I.

Multiply 3,145
by 4,297
Substract 4 The *terminate* Part.

Remains 4293 a *new* Multiplier.

9436
283090
629090
1258181818

First Product 1350343636 &c.
135034363 &c.
135034 &c.
135 &c.

True Product 13,5169533

If

If the Product runs *far* 'ere it begins to repeat, and your *Curiosity* leads you to investigate the Repetend, or, at least, the true Product to any given Number of Places, you may do it thus; continue the Repetends of the first Product at pleasure; then divide it into *Periods*, each Period to consist of *as many Places as the Repetend of the Multiplier did*; then add the first Period to the second, and the Result to the third Period, and this Result to the fourth; and so on till you arrive at the Repetend, or as many places in the *true Product* as you please; Lastly, in adding the last place to the Left-hand of every Period, if it increase to *Ten* or *more*, place *One* under the Right-hand Place of the former *Period*, and what is thus placed in one Period, must constantly be added to the Unit thus placed in the next, and so on. An Example will make the Rule *perspicuous.*

EXAMPLE.

Multiply	121,0308			
by	28,0321			
	1210308			
	24206106			
	363091591			
	9682444424			
	24206106061061			
First Product =	3392,72	993488	488488	
		339273	332761	
	1	332761	821249	
		1	2	
True Product =	3392,74	332762	821251	+
First Product continued =	488488	488488	488488	
	821249	309737	868225	
	309737	868225	356713	
	2	3	3	
Remainder of the true Product	309739	868228	356716	&c.

Thus you have thirty six Places of the *true Product*, which is indeed twenty six more than are *necessary*.

 CHAP.

CHAP. IV.

DIVISION of DECIMALS.

DIVISION of Decimals is perform'd in the same manner as *Division of Integers*, both in regard of *placing* the Numbers, and the *Work* it self.

The *chiefest Difficulty*, in general, is to discover the *true Value* of the *Quotient* Figures, that is, how to separate *justly* the *Integers* and *Decimals* it contains. However the Business of *Valuing* the Quotient is render'd very plain and obvious, by a due Observation of either of the following *Rules*, viz.

Rule I. The *Quotient Figure* is always of the same Value with *that Figure* of the *Dividend*, under which the *Units Place* of its Product stands. Or thus,

Rule II. The *Decimal Parts* in the Divisor and Quotient must be always *equal in Number* to *those* of the Dividend.

Some Authors give *one* of these Rules, and some the other; but I have supplied you with both, that nothing may be wanting to render this *necessary* and *frequent* Part of the *Art* as *easy* and *ready* as possible.

From the second general Rule may be deduced these four *particular* and very *useful* Directions, *viz.*

1 *Direct.* When the Decimal Places in the Divisor and Dividend are equal, the Quotient will be whole Numbers.

2 *Direct.* When the Places of Decimals in the Dividend exceed those of the Divisor, the Decimal Parts in the Quotient must be equal to that Excess.

3 *Direct.* If the Divisor exceed the Dividend in Decimal Places, annex Cyphers to make them equal, then will the Quotient be Integers, by *Direct* 1.

4 *Direct.* If after you have finished Division and find not so many Figures in the Quotient as there ought to be places of Decimal Parts by the general Rule, supply that Defect by prefixing Cyphers to the Quotient.

The Learner being thus fraught with general Rules and particular Directions; cannot, I think, without Impeachment of his Ingenuity, require any thing farther to be said or done

to

to make Division of Decimals evident and easy, except the Operations themselves in all the various Cases; to which I now proceed.

Case 1. When your Decimals are *compleat* and *soon terminate*, place them and work as in *Whole Numbers*, having a strict regard to the Rules and Directions before given for *Valuing the Quotient*.

In *Division of Decimals* there may happen *Nine Varieties*, with respect to the Nature of the Numbers, which may be of *three* Sorts; *viz.*

First, *Integers*; or Whole Numbers.

Secondly, *Mixt*; consisting of Integers and Decimals.

Thirdly, *Pure Decimals*; without any Whole Numbers. Now the Dividend being it self of *three Kinds*, and capable of a Divisor of three kinds, there follows of consequence, these *nine Diversities*, viz.

Any *Whole Number* may be divided by a { *Whole Number.* / *Mixt Number.* / *Decimal.* }

A *Mixt Number* may be divided by a { *Whole Number.* / *Mixt Number.* / *Decimal.* }

A *Pure Decimal* may be divided by a { *Whole Number.* / *Mixt Number.* / *Decimal.* }

I shall explain and exemplify this by one Example, *viz.* by 1722 divided by 12 thus, at large.

```
12) 1722.0 (143.5
    12
    ----
    · 52
      48
    ----
     ·42
      36
    ----
     · 60
       60
    ----
       ··
```

Here

Here you see the *Divisor* and *Dividend* are both *whole Numbers*; and because there was a Remainder of 6, I bor-row a Cypher in order to divide it off clean, which give (by *Direct* 2.) one Place, to wit, 5 in the Quotient for Decimal. I shall subjoin this one Example varied according to all the Varieties aforesaid.

Variety 1— —	12)1722.0	(143,5	by *Direct*. 2.
2— —	12)172,20	(14,5	by *Direct*. 2.
3— —	12),17220	(,01435	by *Direct*. 4.
4— —	1,2)1722.0	(1435	by *Direct*. 1.
5— —	1,2)172,20	(143,5	by *Direct*. 2.
6— —	1,2),17220	(,1435	by *Direct*. 2.
7— —	,12)1722.00	(14350	by *Direct*. 3.
8— —	,12)17,220	(143,5	by *Direct*. 2.
9— —	,12),17220	(1,435	by *Direct*. 2.

But notwithstanding I have given a *Specimen* of *all the Varieties* in the last Example, and pointed to the *Direction*, by which each Quotient was form'd; yet 'twill be necessary to illustrate the *general Rule* by Examples wrought at *large*, wherein the *immediate Use* of the particular Directions will more *obviously* appear.

Example 1. Wherein the *Places of Decimal Parts* in the Divisor and Dividend *are equal.*

```
8,45) 295,75 (35          ,0074) ,4884 (66
      2535                        444
      ----                        ---
       4225                       444
       4225                       444
       ----                       ---
       ....                       ...
```

Here because the *Decimals* in Divisor and Dividend are *equal in Number*, therefore the Quotients in both Instances are *whole Numbers*, by *Direct*. 1.

Example

Example 2. When the *Decimal Parts* of the Dividend *exceed those* of the Divisor.

24,3) 780,516 (32,12
729
515
486
291
243
486
486
...

,0067) ,3953 (,59
335
603
603
...

In this Case the *Excess* is cut off in both the Quotients for Decimal Parts; by *Direct.* 2.

Example 3. When there are not so many Places of Parts in the Dividend as in the Divisor.

7,684) 192,100 (25
15368
38420
38420
.....

,7875) 441,0000 (560
39375
47250
47250
.....

Here *Cyphers are annexed* to the Dividend, to answer the Decimal Places of the Divisor, that the Quote might be whole Numbers; as in *Direct* 1. by *Direct.* 3.

Example 4. When, after *Division is finished*, there are not *so many* Figures in the Quotient as there should be *Decimal Parts* by the *General Rule.*

957) 7,25406 (,00758
6699
5550
4785
7656
7656
....

,575),0007475(,0013
575
1725
1725
....

In

In both these Instances, by *Direction* 4. I prefix Cypher to the Quotients, that together with those in the Divisor they might be equal to the Decimal Places of the Dividend.

If any *Whole*, *Mixt*, or *Decimal Number* is given to b divided by 10, 100, 1000, *&c.* you only remove the separa ting Point towards the Left-hand *so many Places* as there ar *Cyphers in the Divisor*; as on the contrary in *Multiplicatio* the *separating Point* is moved to the Right-hand so man Places as there are Cyphers in the Multiplier.

EXAMPLES *in*

Multiplication.	*Division.*
1,523 × 10 = 15,23	10)1523(152,3
1,523 × 100 = 152,3	100)1523(15,23
1,523 × 1000 = 1523	1000)1523(1,523
,0072 × 10 = ,072	10)72 (7,2
,0072 × 100 = ,72	100)72 (,72
,0072 × 1000 = 7,2	1000)72 (,072
,0072 × 10000 = 72	10000)72 (,0072

I shall next give a *Method* whereby you may work any Cas of *Division* by *Multiplication*, and *vice versa*, any Case of *Multiplication* by *Division*; and this, in many Instances, will be found very excellent and useful.

PROBLEM I.

Suppose I have any Number, 7315, to *multiply* by any other Number 125; but yet have a mind to *divide* the said Number, and to have a Quotient equal to the Product of those two Numbers; *Quere* the Divisor?

Rule. Divide a *Unit* with *Cyphers annexed* by the *given Multiplier*, and the *Quotient* is the *Divisor sought*.

EXAMPLE.

Given Multiplier 125) 1.000 (,008 = the Divisor sought.
1000

...

Then

Then { Multiply 7315
by 125

$$\begin{array}{r} 7315 \\ 125 \\ \hline 36575 \\ 14630 \\ 7315 \\ \hline 914375 \end{array}$$

,008) 7315,000 (914375

$$\begin{array}{l} 72 \\ \hline 11 \\ 8 \\ \hline 35 \\ 32 \\ \hline 30 \\ 24 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline \cdot\cdot \end{array}$$

Thus I have obtain'd a Quotient *the same* as the Product. *Q. E. I.*

PROBLEM II.

Suppose I have any Number 7315 to be *divided* by any other Number ,008; but would *multiply* the said Number, and have a *Product* equal to the *Quotient* of the same Number divided by ,008; *Quere the Multiplier?*

Rule. Divide an *Unit*, with *Cyphers annexed*, by the *given Divisor*, and the *Quotient* will be the *Multiplier sought.*

,008) 1000 (125 = the Multiplier sought.

$$\begin{array}{l} 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline \cdot\cdot \end{array}$$

Thus you see this, and the Remainder of the Work, is only the Reverse of the former; and therefore need not be repeated.

Case 2. If your *Divisor* consist of *many Places* of Decimal Parts, the Work may be very much *contracted*, and yet a *just Quotient* obtained by the following *Rule*. Having determin'd the Value of the *Quotient Figures*, proceed in *multiplying the Divisor* with the first Quotient Figure as usual; but for *every Figure after*, in multiplying, *omit* or *prick off one* in the *Divisor*; still having a due regard to the *Increase*, which would arise from the *Figure* and *Figures* so *omitted*.

EXAMPLES.

Contracted.	*At large.*	
7,9863) 70,2300 (8,7938	7,9863) 70,2300 (8,7938	
···· 638904	638904	
·63396	63396	0
55904	55904	1
·7492	7491	90
7187	7187	67
·305	304	230
239	239	589
·66	64	6410
63	63	8904
·3	0	7506

Tho' much Labour be this way saved, yet it is not proper to use it unless the Decimals in the Quotient be sure to four or six Places; since 'tis obvious, the next Place, or all the Remainder of the Quotient in the *contracted Work* would be three times more or greater than the same in the Work at large.

If the *Dividend* contain *many Places of Decimals*, there's no occasion for using but a *very few of the first*, as appears by this second Example.

EXAM-

EXAMPLE II.

3,141592) 165,6995 | 001296 (52,7438
...... 1570796 | .
86199 | . .
62831 | . .
23368 | . . .
21991 | . . .
1377 | . . . .
1256 | . . . .
121 | . . . . .
94 | . . . . .
27 | . . . . . .
25 | . . . . . .
2 |

Here you may obſerve, that of *Ten Decimal Places* in the Dividend, I have uſed only *Four*; and yet have a Quotient *to four Places* of Decimals true: Hence all the Figures which would have fill'd the *dotted Space*, had it been work'd at large, are *ſuperfluous*; and thoſe, tis evident, make *half* the Work.

Caſe 3. If your *Dividend* contain a *ſingle Repetend*, and your Diviſor be a *ſingle terminate Digit*, divide as uſual; and when you take down your *Repetend*, the Quotient will *begin to repeat*.

EXAM. I.

4) 195,0$\dot{2}$ (48,7$\dot{5}$
16
35
32
30
28
22 } *Ad Infinitum.*
20
2

EXAM. II.

,6) 3176,$\dot{0}$ (529$\dot{3}$
30
17
12
56
54
20 } *Ad Infinitum.*
18
2

 If

If the *Divisor* be any Number of *terminate Digits*, the *Quotient* will *repeat a single Digit*; but not always *begin*, when the *Repetend* is taken down.

EXAM. III.

```
,48)79,26(165,138
    48
    ---
    312
    288
    ---
     246
     240
     ---
      66
      48
      ---
      186
      144
      ---
      426 } ad infinitum.
      384 }
      ---
       42
```

EXAM. IV.

```
487,65)106036,783(217,4
       97530
       -----
        85067
        48765
        -----
        363028
        341355
        ------
        216733 } ad infinitum.
        195060 }
        ------
         21673
```

If your *Divisor* be only a *single Repetend*, and the *Dividend* a *terminate Number*, multiply the Dividend by *9*, cutting off *one* more Right-hand Figure in the *Product*, which is now your *new* Dividend; then *divide* as usual, and the Quotient will be *just*.

EXAMPLE V. Divide 572,4 by ,8.

```
The Dividend = 572,4
Multiply by        9
              ------
Divisor = ,8) 515,16 ( = the new Dividend.
              48       643,95 = the true Quotient.
              --
               35
               32
               --
                31
                24
                --
                 76
                 72
                 --
                  40
                  40
                  --
```

Other-

Otherwise thus; place the Dividend *under it ſelf*, but *one Place forward* to the Right-hand; and then ſubſtract, the Remainder will be the new Dividend, the ſame as before.

The Dividend	572,40	as before.
The ſame placed	5724	one Figure forwards.
Remains a new	515,16	Dividend as before.

From hence alſo appears the *Reaſon* of cutting off *one more Figure* in the new Dividend for *Decimals*.

That *either* of theſe Ways will give the ſame Quotient you have ſeen, and that the Quotient this way produced is the *only true one* will appear from the Work of the laſt Example at large.

8,)	572,4	0000 &c.	(643,95
	5333	3333 &c.	
	·390	6666 &c.	
	385	5555 &c.	
	·35	1111 &c.	Ad Infinitum.
	28	6666 &c.	
	·8	4444 &c.	
	8	0000 &c.	
	·	4444 &c.	

....

In this Operation, tis manifeſt though the *Repetends* in every particular Step would proceed to *Infinity*, yet in the laſt Place you ſee there is an *infinite Product* equal to an *infinite Remainder*; and conſequently the Work muſt there ceaſe, and the Quotient nevertheleſs be *true*.

If the *Diviſor* conſiſts of *terminate Numbers* join'd to the *Repetend*, and the *Dividend* be *compleat*; proceed thus: Subſtract the *terminate* Numbers of the *Diviſor* from the *Diviſor it ſelf*, and the *Remainder* ſhall be a *new Diviſor*; and deal with the *Dividend* as in the laſt Example, for a *new Dividend*.

EXAM-

EXAMPLE VI.

Suppose it required to divide 8569,88 by 4,86; Work a
follows;

```
4,86) 8569,88 (1760,9 the Quotient if work'd at larg
 48   856988
-----------------
4,38) 7712,892 (1760,9 the same.
      438
      ----
      3332
      3066
      ----
      ·2668
       2628
       ----
       ··4092
         3942
         ----
           50
```

If the *Divisor* and *Dividend* do *each* contain a *Repetend*, order them as before directed; and the *Quotient* will be either *terminate*, *repeat* a *single Digit*, or else a *compound Repetend*.

EXAMPLE VII.

```
Divide  134,26  by ,6
        13426
        ------
  ,6) 120,84 (201,4 the true Quotient.
      12
      ---
      ··08
         6
       ---
        24
        24
       ---
        ··
```

EXAM-

EXAMPLE VIII.

Divide 450,95 by ,06

```
          45095
,06)405,860(6764,3
    36
     45
     42
      38
      36
       26
       24
        20 } Ad Infinitum.
        18 }
         2
```

EXAMPLE IX.

Divide 23,46 by 7.

```
Thus { 23,46
     {  2 346
 7) 21,120(3,0171428 5
    21
    ..12
       7
       50
       49
        10
         7
         30
         28   } Ad Infinitum.
          20
          14
          60
          56
           40
           35
            5
```

Case. 4. If *Compound Repetends* are found in your *Divisor*, or *Dividend*, or *both*; then observe to set the Divisor and Dividend *under themselves* so many Places *forwards* to the *Right-hand*, as there are Places in the *Repetend* of the Divisor *exclusively*; next, *substract* them, and the *Remainders* will be respectively a *new* Divisor and Dividend.

EXAMPLE I.

Divide 243306, by 11198.

```
111,98 ) 243,306
    11 )     243
111,87 ) 243,063 (2,172
          22374
         ·19323
          11187
         ·81360  }
          78309  }
         ·30510  } Ad Infinitum.
          22374  }
          ·8136
```

The Truth of the Work will appear as well by the *common* Rule of multiplying the Divisor and Quotient, as by the Work at large.

If there be no *terminate* Part of the Divisor, you substract nothing from it.

EXAM-

EXAMPLE II.

Divide 395,273814 by ,317
Then 395273

,317) 394,878341 (1245,673
317

·778
634

·1447
1268

·1798
1585

·2133
1902

:2314
2219

:·951
951

···

EXAMPLE III.

Divide 70005 by 1,48.
70005
70005

1,48)69934,995(47253,375
592

1073
1036

:·374
296

789
740

499

499
444

·555
444

1110
1036

:·740
740

:·:

If there be no *Repetend* in the *Divisor*, whatever the Dividend may be, there's *no Substraction* to be made of either Divisor or Dividend.

EXAMPLE IV.

Divide 1761,3840x, by 417,64.

417,64) 1761,3840x (,4217
167056

90804
83528

72760
41764 } *Ad Infinitum.*

309961
292348

17613

EXAMPLE V.

Divide 31928,007x12, by 76,45

76,45) 31928,007x12 (417,6325
30580

13480
7645

58350
53515

48357
45870

24871
22935

19361
15290 } *Ad Infinitum.*

40712
38225

2487

In *Division* it may often happen that the Quotient may not *repeat* so soon as is desired; in such Case the Value of the Quotient may be expressed compleatly by a *Vulgar Fraction.*

But

But in order to underſtand this, 'twill be neceſſary to premiſe the following *Lemma's*.

Lemma 1.

A *Series* of Nines *infinitely* continued, is equal to *Unity*, or One, in the next Left-hand Place; thus 0,999, *&c.* is equal to 1; and ,0999 *&c.* = ,1; and ,00999 *&c.* = ,01; and 549,99 *&c.* = 55.

Demonſtration. 'Tis evident that ,9 = $\frac{9}{10}$ wants, only $\frac{1}{10}$ of Unity; and ,99 wants $\frac{1}{100}$; ,999 wants $\frac{1}{1000}$ of Unity: ſo that if the Series were continued to *Infinity*, the *Difference* between that *Series of Nines* and *an Unit*, would be equal to *Unity divided by Infinity*, that is, *Nothing* at all. *Q. E. D.*

Lemma 2.

Any *ſingle Repetend multiplied* by 10, and then *ſubſtracted* from that *Product*; the *Remainder* will be the *ſame Number compleat or terminate*, in the next ſuperior Left-hand Place.

Demonſtration. Let the given Repetend be ,6666, *&c.* this multiplied by 10 is 6,666 *&c.*

From which Subſtract ,666 *&c.*

There will remain 6, . . . a whole Number.

Thus 47,77 *&c.* will become 430. and ,0333 *&c.* will be ,3. *Q. E. D.*

Corollary 1.

Hence it follows that if any *Compound Repetend* be multiplied by an Unit with ſo many Cyphers annexed as are equal to the Places of the Repetend, and then ſubſtracted from the Product, there will be left to the Left-hand the ſame Numbers *terminate* and *compleat*, that conſtituted the *Repetend*; thus, $\dot{3}2\dot{5}$ multiplied by 1000, will be $\dot{3}2\dot{5},325$ from which if you ſubſtract $,\dot{3}2\dot{5}$ there will remain the *terminate* Number 325; Thus $12,\dot{7}4\dot{3}$ will be 12731; and $,000\dot{7}4\dot{3}$ will be ,743, and $52\dot{7}5,\dot{3}$ will become 5270,1.

Corollary. 2.

Hence alſo if any *Repetend* be multiplied by ſo many *Nines* as it contains Places, the Reſult will be the ſame as before;

fore; that is, the ***Repetend terminate*** and ***compleat.*** For any thing multiplied by *Ten*, and *once* ſubſtracted, is the ſame as ***multiplied by Nine***;

Thus; ,666 &c. $\times$ 9 = 5,999 &c. = 6. by *Lemma* 1.
And $,\dot{5}2\dot{7} \times 999 = 526,999 = 527$. *per idem.*

Corollary 3.

Hence it muſt follow that, *vice verſa*, any Number divided by as many *Nines* as it contains Figures is equal to the ſame Number *perpetually circulating*; Thus $\frac{6}{9}$ = ,666 &c.

And $\frac{527}{999} = ,\dot{5}2\dot{7}$. And $12\,\frac{743}{999} = 12,\dot{7}4\dot{3}$.

Corollary 4.

Hence, laſtly, appears the *Reaſon* of all the different Methods and peculiar Proceſſes uſed in the *Arithmetick of Circulating Numbers*, call'd *Repetends.*

The preceeding *Lemma's* and *Corollaries* being well underſtood, it will then be very eaſy to *value* any kind of *Decimals* in the manner of *Vulgar Fractions*. For the Quotient in Diviſion, take this Example from Mr. *Cunn.*

EXAMPLE VI.

Divide $2\dot{1}347\dot{6}$, by $4\dot{1}7\dot{6}$.

$4\dot{1}7,\dot{6})\ 2\dot{1}3,47\dot{6}134\ ($
$\quad 4 \qquad 2\dot{1}347\dot{6}$

$417,2)\ 21326\,2658\ (511176 \qquad 314\,\frac{26586}{99999} \Big/ 417,2$

20860
··4662
4172
·4906
4172
·7345
4172
31738
29204
: 25346
25032
··314

The

The Reason why the Quotient is thus expressed will be evident if we consider.

First, That 314 is not the *entire* Remainder because the Dividend is a *Repetend*, perpetually supplying a *circulating Remainder*, which expressed in its proper Terms would be, where we leave off, wrote thus $314\dot{2}658\dot{6}$. But this *infinite Series* of Figures is truly expressed thus, $314\frac{36586}{99999}$ by *Corol.* 3.

Secondly, It being plain that $314\frac{36586}{99999}$ is the true Remainder, and 417,2 the Divisor, 'tis necessary they should be expressed in the Quotient as here you see them by the Rules of *common Division.*

If instead of $314\frac{26586}{99999}$ you write its *Equivalent* $314\dot{2}658\dot{6}$, and from it substract the *terminate* part 314, there will remain 31426272 a new Numerator. And if to 417,2 you *add* as many *Cyphers* as the Repetend consists of Places, thus 417,200000; and again substract it, as a *terminate* part, there will remain 417195828 for a *new Denominator*; and then this new and more *simple Fraction* $\frac{31426272}{417195828}$ will be equal to *that* in the *Quotient*.

The *Reason* of reducing the *Fraction* of the Quotient in this Manner is obvious from *Corol.* 1. of *Lem.* 2.

$$\begin{array}{lr}
\text{For } 314\dot{2}658\dot{6} \times 100000 = & 314\dot{2}658\dot{6}26586 \text{ \&c.} \\
\text{From which substract it self} & 314\dot{2}658\dot{6} \text{ \&c.} \\
\hline
 & \ldots\ldots
\end{array}$$

And there will remain the new Numerator 31426271 as before.

$$\begin{array}{lr}
\text{Then } 417{,}2 \times 100000 = & 417{,}200000 \\
\text{From which substract it self} & 4172 \\
\hline
\text{There remains the Denominator} & 417195828 \text{ as before.}
\end{array}$$

Thus I have supplied you with Rules for managing the *whole Doctrine of Circulating Numbers*; and given the *Theory* and *Reasons* for the same; which you may search for in vain in any other Book (that I know of) so fully as here laid down.

CHAP.

CHAP. V.

REDUCTION *of* DECIMALS.

IT being sufficiently experienced that all *Arithmetica[l]* *Operations* are with the greatest *Facility* and *Expe[-]dition* work'd in *whole Numbers*; that *Vulgar Fracti[-]ons*, and Numbers of *diverse Denominations* in their Manage[-]ment require *great Art*, and are attended with *much Per[-]plexity*; and that the *noble Art of Decimal Arithmetick* alone is susceptible of all the various kinds of Numbers, and at the same time hath all its Operations perform'd by the same *eas[y]* and *common Rule*, and in the very *same* manner of *Integra[l]* *Quantities*, or *whole Numbers*; This, I say, being well known to all versed in the *Science of Numbers*, hath justly rendered *Decimal Arithmetick* in the *greatest Esteem* among those who *understand* it; and is most *generally used* by them in almost *all kinds of Numerical Calculations*.

The Part we now treat of is *absolutely necessary* to the *true Understanding* and *Use* of this *excellent* Art; and teaches,

First, To *reduce* or express any *Vulgar Fraction* in *Deci[-]mal Parts* of the *Integral Quantity*.

Secondly, To *reduce* such Numbers as consist of *various* Parts and Denominations, as those of *Money*, *Weight*, *Mea[-]sure*, &c. into *Decimals* for more *easy* Operation.

Thirdly, To *reduce Decimal Parts* into the *common* and *known* Parts of *Money*, *Measure*, &c.

Case 1. To reduce *Vulgar Fractions* into *Decimals*, the common Rule is, to divide the *Numerator* by the *Denomina[-]tor*, and the *Quotient* will be the *Decimal required*; that is, equivalent to the Vulgar Fraction given.

EXAMPLE I.

What is the Decimal *equivalent* to the Fraction $\frac{3}{4}$?

4) 3,00 (,75 The Decimal required.
28
——
20
20
——
..

EXAMPLE II.

Reduce $\frac{3}{8}$ of a Pound into *Decimal Parts* of a Pound.

```
                  L.   s.  d.
8) 3,000 ( ,375 = 7 : 6 the Anſwer.
   24
   ---
   ·60
    56
   ---
    40
    40
   ---
    ··
```

EXAMPLE III.

Reduce $\frac{3}{16}$ of a Pound *Troy* into *Decimal Parts.*

```
                    oz. pwt.
16) 3,0000 (,1875 = 2 : 5 the Anſwer.
    16
    ---
    140
    128
    ---
     120
     112
     ---
     ··80
       80
     ---
       ··
```

EXAMPLE IV.

Reduce $\frac{4}{27}$ of a *Rod* into *Decimals.*

```
                  Rod. F. In. Qr.
27) 4,000 (,148 = 2 : 5 : 1 Anſwer.
    27    ⎫
    ---   ⎪
    130   ⎪
    108   ⎪
    ---   ⎬ Ad Infinitum.
    220   ⎪
    216   ⎪
    ---   ⎪
    ··4   ⎭
```

EXAM-

EXAMPLE V.

Reduce $5\frac{11}{27}$ of a *Hogshead* into *Decimals*.

$$\begin{array}{l} \qquad\qquad\qquad\qquad\qquad\quad Gal.\ Pts. \\ 27)\ 11,000\ (,407 = 25 : 5 \\ \quad\ \ 108 \\ \quad\ \ \overline{\ \ \ 200} \\ \quad\ \ \ \ \ 189 \\ \quad\ \ \overline{\ \ \ \ \ 11} \end{array} \Bigg\} \textit{Ad Infinitum.}$$

$\qquad\qquad\qquad\qquad\qquad$ *H. Gal. Pints.*

Hence the Answer is 5,407 = 5 : 25 : 5.

Reduction of *Vulgar Fractions* into *Decimals* is also commodiously perform'd by *Logarithms*, especially when the *Fraction is large*; thus, substract the *Logarithm* of the *Denominator* from the *Logarithm* of the *Numerator*, the *Remainder* is the *Logarithm* of the *Decimal Parts*.

EXAMPLE I. By *Logarithms*.

Reduce the Fraction $\frac{127}{4123}$ into *Decimal* Parts.

Thus, the Logarithm of 127 is	— —	2.1038037
the Logarithm of 4123 substract,	—	3.6152133
Remains the Logarithm or the Decim. ,0308 =		8.4885904

EXAMPLE II.

What is the Decimal of the mix'd Fraction $5\frac{91}{3479}$

From the Logarithm of the Numerator 91	1.9590414
Substract the Logarithm of the Denominator	3.5414544
There Remains the Log. of the Dec. ,026157 —	8.4175870

To which prefix the *integral Quantity* 5, and the Answer will be compleat, thus 5.026157.

The same thing might as well have been done by reducing the *mixt* Fraction into an *improper* one, *viz.* $\frac{17486}{3479}$

Then,

Then, as in the firſt Example.

From the Log. of the Numerator 17486 — 4.2426904
Subſtract the Log. of the Denominator 3479 — 3.5414544

Remains the Log. of the Anſwer 5,026157 — 0.7012360

Note, In the firſt and ſecond Examples, and in all ſuch Caſes where you ſubſtract a *greater* Index from a *leſs*, you borrow *Tens* and as *many Digits* as the *remaining Index* want; of *Nine*, *ſo many Cyphers* prefix to the *Decimal*.

Caſe 2. To *reduce* Numbers which expreſs Quantities of *various Kinds and Denominations*, as *Money*, *Meaſure*, &c. into *Decimals*, there are *three* Ways or Methods, which are as follows.

Method 1. Reduce the *different* Species to *one*; that is, to the loweſt Denomination they conſiſt of: then reduce the *Integer* to the *ſame* Denomination; the *firſt* will be the *Numerator*, the *latter* the *Denominator* of a *Vulgar Fraction*; which Fraction reduced to a *Decimal* (by *Caſe* 1.) will be *that* required.

EXAMPLE I.

What Decimal Part of a Pound is 5*s*. 7*d*. $\frac{3}{4}$?

	s.		*l.*	
Multiply	5	Then reduce	1	the Integer.
By 12*d*.	12		20	
	60		20	
Add the 7*d*.	7		12	
	67 *Pence.*		240	
Mul. by 4*q*	4		4	
	268		960	*Farthings* in a *Pound.*
Add the 3*q*	3			
	271 *Farthings.*			

That is $\left\{ \begin{array}{l} 271 \text{ the Numerator} \\ 960 \text{ the Denominator} \end{array} \right\}$ of the Vulgar Fraction.

Then 960) 271,0000 (,28229 &c. The Anſwer.

So that ,28229 is the *Decimal* Part of a *Pound* in one Denomination, equal to 5 *s.* 7 *d.* $\frac{3}{4}$, the Part of a Pound in *diverse* Denominations.

EXAMPLE II.

What *Decimal* Part of a *Hundred Weight* is 2 *q.* 21 *lb.* 12 *oz.* ?

Q. *lb.* *oz.*	*C.*
Reduce 2 : 21 : 12 to Ounces.	And 1 reduce to Ounces.
28	4
77	4
16	28
464	112
78	16
1244 *Ounces*	1792 *Ounces* in *C. W.*

Then $\left\{\frac{1244}{1792}\right\}$ is the Vulgar Fraction.

And 1792) 1244,0 (=,694196 the Decimal Part of an *Hundred Weight* (answering to 2 *q.* : 21 *lb.* : 12 *oz.* required.

EXAMPLE III.

What *Decimal* Part of a *Rod* or *Pole*, is 4 *Y.* : 2 *F.* : 8 *In*?

Y. *F.* *In.*	
4 : 2 : 8	1 *Pole.*
3	5,5
14	5,5
12	3
176 *Inches.*	16,5
	12
	198,0 *Inches* in a *Pole.*

But $\frac{176}{198} = \frac{88}{99} =$,8 the (repeating) Decimal of a *Pole*, equal to the 4 *Yards*, 2 *Feet*, and 8 *Inches*.

In the *same Manner* proceed with any other given *Species*.

Method

Method 2. Find what *Decimal* Part the *least* Denomination of the given *Species* is of the next *superior*, to which prefix the given Part of the next superior Denomination; then see what *Decimal* Part this *mixt* Number is of the next *superior* Denomination, to which again prefix what is *given of it*; and thus proceed till you ascend to the *Integer* it self, and find what *Decimal* Part of it the last *mixed Number* is, which will be ***that*** sought.

EXAMPLE I.

What *Decimal* Part of a *Pound* is 12 *s.* 6 *d.* $\frac{1}{2}$?
First 4) 2,0 (,5 the Decimal of one Penny for $\frac{1}{2}$.
Secondly 12) 6,5 (,5416 the Dec. Part of a Shill. for 6*d.* $\frac{1}{2}$;
Thirdly 20) 12,5416 (,627083 the Decimal Part of a Pound, as was required, for 12 *s.* 6 *d.* $\frac{1}{2}$.

EXAMPLE II.

What *Decimal* of a Pound *Troy* is 2 *oz.* 18 *pwt.* 20 *gr.*?
First 24) 20,000 (.875 the Decimal for 20 *gr.*
Secondly 20) 18,875 (.94375 the Decimal for 18 *pwt.* 20 *gr.*
Thirdly, 12) 2,94375 (.2453125 the Decimal Part of a Pound *Troy* for 2 *oz.* 18 *pwt.* 20 *gr.* as was required.

EXAMPLE III.

What *Decimal* Part of a *Year* is 6 *m.* 3 *w.* 5 *d.* 6 *h.* 40′ 50″?
First 60) 50,00 (,83 Decimals for 50″ of 1′.
Secondly 60) 40,83 (,6805 Dec. for 40′ : 50″ of an *Hour.*
Thirdly, 24) 6,6805 (,2783148 = 6 *H.* 40′ : 50″ of a *Day.*
Fourthly, 7) 5,2783148 (,75404 *&c.* Decimals of a *Week.*
Fifthly, 4) 3,75404 *&c.* (,93851 Decimals of a *Month.*
Sixthly, 13) 6,93851 (,53373 Decimals of a *Year.*

So that we see the *six* different Parts of Time above specified are reduced to this small *Decimal* ,53373; which expresseth the *same* Part of a *Year* as they do; which, by the way, may be an Instance of the great *Simplicity*, *Ease*, and *Excellency* of this *admirable Art*.

In these three Examples I have omitted the Work at large, setting down only the *Divisors*, *Dividends*, and *Quotients* as

sufficient to give the *Learner* as good a Notion of the Metho as the Operations at length, which he may make his *Exer-cise at pleasure* to good advantage.

Method 3. The *third Method* for finding the *Decima* of any *given Part* of Quantity consisting of *diverse Denomi-nations*, is by *Tables* ready calculated for that purpose.

This is not only the most *easy*, but the most *expeditiou* Manner of working the Processes of *all Kind* of Computation in general; and is of particular service in this Case of *pre-paring* Numbers for *Decimal Operations.*

For that reason I have here inserted a *Set of Tables*, which tho' some are *common* of this kind, are the most *compleat* an *universal* of any I have seen extant; and in order to ren-der them so, I have not only very much *enlarged* and *new vamped* the old ones, but also *added* other very useful ones as those who are read in these Matters, will soon perceive.

By the *following Tables*, all the *Species* of *Money*, *Weigh Measure*, &c. consisting of *what ever* Denomination, an be the *Integer* what you please, are immediately turn'd int *Decimal Parts*; and are then work'd with the known *Faci-lity* and *Pleasure* of Whole Numbers.

As to the *manner of using* those Tables, that is so *obviou and natural*, even by a *bare Inspection*, that I presume i needless to say any thing to a Person of *Genius*, though *Learner*, about that. The Scheme of *Examples* followin being sufficient to testify the great *Use* and *Excellency* of suc Tables, and are both *Precedents* and *Precepts* themselves.

EXAMPLE I.

What is the *Decimal* Part of a *Pound* for 13 *s*. 7 *d*. $\frac{3}{4}$?

In *Table* I. you find answering to	13 Shillings	— —	,65
	7 Pence, 3 Farthings	— —	,032291
The Answer is		— — — —	,682291

EXAMPLE II.

What *Decimal* Part of a *Mark* is 11 *s*. 2 *d*. $\frac{1}{2}$?

In *Table* I. un-der a Mark, against	11 Shillings	— —	,65
	2 Pence	— —	,0125
	2 Farthings	— —	,003125
The Answer		— — —	,840625

EXAM-

EXAMPLE III.

What *Decimal* Part of a *Moidore* is 9 *s.* 3 *d.* ¼?

In *Table* I. under Moidore you find against { 9 Shillings — ,333333; 3 Pence — ,009259; 1 Farthing — ,000771 }

The Answer is this *compound* Repetend ,343363592

EXAMPLE IV.

What *Decimal* of a Pound *Troy* answers to 5 *oz.* 17 *pwts.* 22 *gr.*?

In *Table* II. you find against { 5 Ounces — — ,416666; 17 Penny Weight — — ,070833; 22 Grains — — ,003819 }

The *Sum* of which is the Answer — — ,491319

EXAMPLE V.

What *Decimal* Part of an *Hundred Weight* is 21 *lb.* 14 *oz.*?

In *Table* III. you find against { 21 Pounds — — ,1875; 14 Ounces — — ,007812 }

The Answer is — — ,195312

EXAMPLE VI.

What *Decimal* Part of a *Tun* is 3 *qr.* 6 *bush.* 7 *gall.*?

In *Table* V. *Dry Measure*, against { 3 Quarters — — ,6; 6 Bushels — — ,15; 7 Gallons — — ,021875 }

The Answer is — — — ,771875

EXAMPLE VII.

What *Decimal* Part of a *Hogshead* of Wine, is 2½ *Rund.* 14 *Gall.*?

In *Table* VI. *Liquid Measure*, you find against { 2½ Rundlets ,71425; 14 Gallons ,22222 &c. }

The Answer (*repeating* a *single* Figure is ,936472 &c.

EXAM.

EXAMPLE VIII.

How is 27 *Miles*, 7 *Furlongs*, 25 *Rod*, and 4 *Yards* expressed in *Decimals*?

In *Table* VIII. *Long Measure* (one Mile the Integer) you find against	7 Furlongs —	,875
	35 Rod, or Poles, —	,109375
	4 Yards —	,002272
The 27 *Miles* prefixed, the Answer will be		27,986647

EXAMPLE IX.

What *Decimal* Part of a *Year* is 7 *Months*, 3 *Weeks*, and 2 Days?

In *Table* IX. you observe against	7 Months — —	,538461
	3 Weeks — —	,057630
	2 Days — —	,005494
The Answer is		,601585

EXAMPLE X.

What *Decimal* Part of a *Sign of the Zodiac* is 25° 46′ 8″?

In *Table* X. you find against	25 Degrees —	,8333333
	46 Minutes —	,0255555
	8 Seconds —	,000073
The Answer is a *single Repetend* —		,8589618

EXAMPLE XI.

What *Decimal* Part of a *Degree*, is 49′ 57″?

In the *same Table* you see against	49 Minutes — —	,816666
	57 Seconds — —	,015833
The Answer is — —		,8325

Having thus so largely *exemplified* the *Use* of the *Tables*, the *Tables themselves* follow; wherein observe, 1. I have *dash'd* the *first* Figure of all *single Repetends*, and the *first* and *last* of the *compound Repetends* that come within the Table. 2. I have nevertheless *continued* each to six Places for *their sakes* who wou'd be *exact*, but *know not well* how to manage *Repetends*.

Common Tables *of* Money, Weights, Measures, *and* Time.

Table I. *Money.*

Farth.
4 = 1 *Penny.*
48 = 12 = 1 *Shilling.*
960 = 240 = 20 = 1 Pound.

Table II. *Apothecaries Weight.*

Grains.
20 = 1 *Scruple.*
60 = 3 = 1 *Dram.*
480 = 24 = 8 = 1 *Ounce.*
5760 = 288 = 96 = 12 = 1 *lb.*

Table III. *Troy Weight.*

Grains.
24 = 1 *Penny Weight.*
480 = 20 = 1 *Ounce.*
7560 = 240 = 12 = 1 *lb.*

Table IV. *Moneyers Weight.*

Blanks.
24 = 1 *Periot.*
480 = 20 = 1 *Droite.*
11520 = 480 = 24 = 1 *Mite.*
230400 = 9600 = 480 = 20 = 1 *Grain.*

Table V. *Averdupois Weight.*

Drams.
16 = 1 *Ounce.*
256 = 16 = 1 *Pound.*
28672 = 1792 = 112 = 1 *Hundred.*
573440 = 35840 = 2240 = 20 = 1 *Tun.*

Table VI. *Wine Measure.*

Cubic In.
231 = 1 *Gallon.*
9702 = 42 = 1 *Terce.*
14553 = 63 = $1\frac{1}{2}$ = 1 *Hogshead.*
19404 = 84 = 2 = $1\frac{1}{3}$ = 1 *Punch.*
29106 = 126 = 3 = 2 = $1\frac{1}{2}$ = 1 *Butt.*
58212 = 252 = 6 = 4 = 3 = 2 = 1 *Tun.*

Table VII, *Ale Measure.*

Cubic In.
282 = 1 *Gallon.*
2256 = 8 = 1 *Firkin.*
4512 = 16 = 2 = 1 *Kilderkin.*
9024 = 32 = 4 = 2 = 1 *Barrel.*
13536 = 48 = 6 = 3 = $1\frac{1}{2}$ = 1 *Hogshead.*

Table VIII. *Beer Measure.*

Cubic In.
282 = 1 *Gallon.*
2583 = 9 = 1 *Firkin.*
5076 = 18 = 2 = 1 *Kilderkin.*
10152 = 36 = 4 = 2 = 1 *Barrel.*
15228 = 54 = 6 = 3 = $1\frac{1}{2}$ = 1 *Hogshead.*

Table IX. *Dry Measure.*

Cubic In.
268.8 = 1 *Gallon.*
537.6 = 2 = 1 *Peck.*
2150.4 = 8 = 4 = 1 *Bushel.*
8601.6 = 32 = 16 = 4 = 1 = *Coomb.*
17203.2 = 64 = 32 = 8 = 2 = 1 *Quarter.*
68812.8 = 256 = 128 = 32 = 8 = 4 = 1 *Chalder.*
86016.0 = 350 = 160 = 40 = 10 = 5 = 1 *Wey*, or *Load.*
172032.0 = 640 = 320 = 80 = 20 = 10 = 2 = 1 *Last.*

Table

Table X. Of *Time.*

Seconds.
60 = 1 *Minute.*
3600 = 60 = 1 *Hour.*
86400 = 1440 = 24 = 1 *Day.*
604800 = 10080 = 168 = 7 = 1 *Week.*
2419200 = 40320 = 672 = 28 = 4 = 1 *Month.*
31556937 = 525949 = 8765 = 365 = 52 = 13 + 1 *Day,* + 5 *Hours,* + 48′ + 57″ = *Year.*

Table XI. *Long Measure.*

Barly Corns.
3 = 1 *Inch.*
36 = 12 = 1 = *Foot.*
108 = 36 = 3 = 1 *Yard.*
594 = 198 = $16\frac{1}{2}$ = $5\frac{1}{2}$ = 1 *Pole.*
23760 = 7920 = 660 = 220 = 40 = 1 *Furlong.*
190080 = 63360 = 5280 = 1760 = 320 = 8 = 1 *Mile.*

Table XII. *Square Measure.*

Square In.
144 = 1 *Feet sq.*
1296 = 9 = 1 *Yards sq.*
3600 = 25 = $2\frac{7}{10}$ = 1 *Paces sq.*
39204 = $372\frac{1}{4}$ = $30\frac{1}{4}$ = $10\frac{9}{10}$ = 1 *Poles sq.*
1568160 = 10890 = 1210 = $435\frac{6}{10}$ = 40 = 1 *Rood sq.*
6272640 = 43560 = 4840 = $1742\frac{2}{5}$ = 160 = 4 = 1 *Acres sq.*

Table XIII. *Scripture Measure.*

Digits.
4 = 1 *Palm.*
12 = 3 = 1 *Span.*
24 = 6 = 2 = 1 *Cubit.*
96 = 24 = 8 = 4 = 1 *Fathom.*
144 = 36 = 12 = 6 = $1\frac{1}{2}$ = 1 *Ezekiels Reed.*
192 = 48 = 16 = 8 = 2 = $1\frac{1}{3}$ = 1 *Arabian Pole.*
1920 = 480 = 160 = 80 = 20 = $13\frac{1}{3}$ = 10 = 1 *Schœnus* or *measuring Line.*

Table IV. *Eastern Measure.*

Cubits.

400 = 1 *Stadium.*

2000 = 5 = 1 *Sabbath Days Journey.*

4000 = 10 = 2 = 1 *Eastern Miles.*

12000 = 30 = 6 = 3 = 1 *Parasang.*

96000 = 240 = 48 = 24 = 8 = 1 *Days Journey*

Table XV. *Hebrew Measure.*

Gachal.

20 = 1 *Cab.*

36 = $1\frac{4}{5}$ = 1 *Omer.*

120 = 6 = $3\frac{1}{3}$ = 1 *Seah.*

360 = 18 = 10 = 3 = 1 *Epha.*

1800 = 90 = 50 = 15 = 5 = 1 *Lethech.*

3600 = 180 = 100 = 30 = 10 = 2 = 1 *Homer*, or *Coron.*

Table XVI. *Hebrew Measure.*

Eaph.

$1\frac{1}{3}$ = 1 *Log.*

$5\frac{1}{3}$ = 4 = 1 *Cab.*

16 = 12 = 3 = 1 *Hin.*

32 = 24 = 6 = 2 = 1 *Seah.*

96 = 72 = 18 = 6 = 3 = 1 *Bath Epha.*

960 = 720 = 180 = 60 = 30 = 10 = 1 *Coron Chomer.*

Table XVII. *Hebrew Money.*

Gerahs.

10 = 1 *Bekah.*

20 = 2 = 1 *Shekel.*

1200 = 120 = 60 = 1 *Maneh.*

60000 = 6000 = 3000 = 50 = 1 *Talent.*

Decima

Decimal Tables *of* Money, Weight, &c.

TABLE I. *Of* Money; *one* Pound *the Integer.*

S.	Dec. Par.
1	,05
2	,1
3	,15
4	,2
5	,25
6	,3
7	,35
8	,4
9	,45
10	,5
11	,55
12	,6
13	,65
14	,7
15	,75
16	,8
17	,85
18	,9
19	,95

P.	q.	Dec. Par.
0	$\frac{1}{8}$	,0005208
0	$\frac{1}{4}$	,0010416
0	$\frac{1}{2}$	,002083
0	$\frac{3}{4}$	,003125
1	,0	,00416
1	$\frac{1}{4}$	,0052
1	$\frac{1}{2}$	,00625
1	$\frac{3}{4}$	,00729
2	,0	,00833
2	$\frac{1}{4}$	,009375
2	$\frac{1}{2}$	,010416
2	$\frac{3}{4}$	,0114583
3	,0	,0125
3	$\frac{1}{4}$	,0135416
3	$\frac{1}{2}$	,014583
3	$\frac{3}{4}$	,015625
4	,0	,016666
4	$\frac{4}{4}$	,0177083
4	$\frac{1}{2}$	,01875
4	$\frac{3}{4}$	,0197916
5	,0	,02083
5	$\frac{1}{4}$	,021875
5	$\frac{1}{2}$	,022916
5	$\frac{3}{4}$	,0239583
6	,0	,025
6	$\frac{1}{4}$	,0260416
6	$\frac{1}{2}$	,0270083
6	$\frac{3}{4}$	,028125
7	,0	,029166
7	$\frac{1}{4}$	,0302083
7	$\frac{1}{2}$	,03125
7	$\frac{3}{4}$	,0322916

P.	q.	Dec. Par.
8	,0	,033333
8	$\frac{1}{4}$	,034375
8	$\frac{1}{2}$	,035416
8	$\frac{3}{4}$	,0364583
9	,0	,0375
9	$\frac{1}{4}$	,0385416
9	$\frac{1}{2}$	,039583
9	$\frac{3}{4}$	,040625
10	,0	,041666
10	$\frac{1}{4}$	,0427083
10	$\frac{1}{2}$	,04375
10	$\frac{3}{4}$	,0447916
11	,0	,045833
11	$\frac{1}{4}$	,046875
11	$\frac{1}{2}$	,047916
11	$\frac{3}{4}$	,0489583

One Shilling *the Integer.*

P.	q.	D. Par.
0	$\frac{1}{8}$	,010416
0	$\frac{1}{4}$	,020833
0	$\frac{1}{2}$	,041666
0	$\frac{3}{4}$	,0625
1	,0	,083333
1	$\frac{1}{4}$	,104166
1	$\frac{1}{2}$	,125
1	$\frac{3}{4}$	,145833
2	,0	,16666
2	$\frac{1}{4}$	,1875
2	$\frac{1}{2}$	,208333
2	$\frac{3}{4}$	,229166
3	,0	,25
3	$\frac{1}{4}$	,270833
3	$\frac{1}{2}$	,291666
3	$\frac{3}{4}$	,3125
4	,0	,333333
4	$\frac{1}{4}$	,354166
4	$\frac{1}{2}$	,375
4	$\frac{3}{4}$	,395833
5	,0	,416666
5	$\frac{1}{4}$	,4375
5	$\frac{1}{2}$	,458333
5	$\frac{3}{4}$	,479166
6	,0	,5
6	$\frac{1}{4}$	,520833
6	$\frac{1}{2}$	,541666
6	$\frac{3}{4}$	,5625
7	,0	,583333
7	$\frac{1}{4}$	,604166
7	$\frac{1}{2}$	,625
7	$\frac{3}{4}$	,645833
8	,0	,666666
8	$\frac{1}{4}$	,6875
8	$\frac{1}{2}$	,708333
8	$\frac{3}{4}$	,729166
9	,0	,75
9	$\frac{1}{4}$	,770833
9	$\frac{1}{2}$	,791666
9	$\frac{3}{4}$	,8125
10	,0	,833333
10	$\frac{1}{4}$	,854166
10	$\frac{1}{2}$	,875
10	$\frac{3}{4}$	,895833
11	,0	,916666
11	$\frac{1}{4}$	,9375
11	$\frac{1}{2}$	,958333
11	$\frac{3}{4}$	,979166

1 Crown *the Integer.*

S.	D.Par.
1	,2
2	,4
3	,6
4	,8

P.	D.Par.
1	,01666
2	,03333
3	,05
4	,06666
5	,08333
6	,1
7	,11666
8	,13333
9	,15
10	,16666
11	,18333

qr.	D.Par.
1	,00416
2	,00833
3	,0125

A Noble *the Integer.*

S.	D.Par.
1	,15
2	,3
3	,45
4	,6
5	,75
6	,9

P.	D. Part.
1	,0125
2	,025
3	,0375
4	,05
5	,0625
6	,075
7	,0875
8	,1
9	,1125
10	,125
11	,1375

qr.	D. Part.
1	,003125
2	,00625
3	,009375

A Mark *the Integer.*

S.	D. Part.
1	,075
2	,15
3	,225
4	,3
5	,375
6	,45
7	,525
8	,6
9	,675
10	,75
11	,825
12	,9
13	,975

P.	D. Part.
1	,00625
2	,0125
3	,01875
4	,025
5	,03125
6	,0375
7	,04375
8	,05
9	,05625
10	,0625
11	,06875

qr.	D. Parts.
1	,0015625
2	,003125
3	,0046875

A Guinea *the Integer.*

S.	D. Parts.
1	,047619
2	,095238
3	,142857
4	,190476
5	,238095
6	,285714
7	,333333
8	,380952
9	,428571
10	,476190
11	,523809
12	,571428
13	,619047
14	,666666
15	,714285
16	,761904
17	,809523
18	,857142
19	,904761
20	,952380

P.	Dec. Part.
1	,0039682
2	,007936
3	,011904
4	,015873
5	,019841
6	,023809
7	,027777
8	,031746
9	,035714
10	,039682
11	,043650

qr.	De. Part.
1	,000992
2	,001984
3	,002976

A Corol *the Integer.*

S.	,043478
1	,043478
2	,086957
3	,130436
4	,173914
5	,217393
6	,260872
7	,304350
8	,347829
9	,391308
10	,434786
11	,478265
12	,521744
13	,565222
14	,608701
15	,652180
16	,695659
17	,739137

S.	D. Part.
18	,782616
19	,826095
20	,869573
21	,913052
22	,956531

P.	D. Part.
1	,003623
2	,007246
3	,010869
4	,014492
5	,018215
6	,021938
7	,025561
8	,029184
9	,032707
10	,036232
11	,039855

qr.	D. Part.
1	,000905
2	,001811
3	,002717

A Jacobus *the Integer.*

S.	D. Part.
1	,04
2	,08
3	,12
4	,16
5	,2
6	,24
7	,28
8	,32

S.	D. Part.
9	,36
10	,4
11	,44
12	,48
13	,52
14	,56
15	,6
16	,64
17	,68
18	,72
19	,76
20	,8
21	,84
22	,86
23	,92
24	,96

P.	D. Part.
1	,003333
2	,006666
3	,01
4	,013333
5	,016666
6	,02
7	,023333
8	,026666
9	,03
10	,033333
11	,036666

qr.	D. Part.
1	,000833
2	,001666
3	,0025

A Moidore *the Integer.*

S.	D. Parts.
1	,037037
2	,074074
3	,111111
4	,148148
5	,185185
6	,222222
7	,259259
8	,296296
9	,333333
10	,370370
11	,407407
12	,44444
13	,481481
14	,518518
15	,555555
16	,592592
17	,629629
18	,666666
19	,703703
20	,740740

S.	D. Parts.
21	,777777
22	,814814
23	,851851
24	,888888
25	,925925
26	,962962

P.	D. Parts.
1	,003086
2	,006172
3	,009259
4	,012345
5	,015432
6	,018185
7	,021604
8	,024691
9	,027777
10	,030864
11	,033950

qr.	D. Parts.
1	,000771
2	,001543
3	,002314

TABLE II.

Troy Weight; *one Pound the Integer.*

Oz.	D. Par.	Oz.	D. Par.
1	,08333	7	,58333
2	,16666	8	,66666
3	,25	9	,75
4	,33333	10	,83333
5	,41666	11	,91666
6	,5		

Pw.	*D. Part.*
1	,004166
2	,008833
3	,0125
4	,016666
5	,020833
6	,025
7	,029166
8	,033333
9	,0375
10	,041666
11	,045833
12	,05
13	,054166
14	,058333
15	,0625
16	,066666
17	,070833
18	,075
19	,079166

Gr.	*D. Part.*
1	,000173
2	,000347
3	,000521
4	,000694
5	,000868
6	,001042
7	,001215
8	,001389
9	,001562
10	,001736
11	,00191
12	,002083
13	,002257
14	,602431
15	,002604
16	,002778
17	,002951
18	,003125

Pw.	*D. Part.*
19	,003298
20	,003472
21	,003646
22	,003819
23	,003993

One Ounce *the Integer.*

Pw.	*D. Part.*
1	,05
2	,1
3	,15
4	,2
5	,25
6	,3
7	,35
8	,4
9	,45
10	,5
11	,55
12	,6
13	,65
14	,7
15	,75
16	,8
17	,85
18	,9
19	,95

Gr.	*D. Part.*
1	,002083
2	,004166
3	,00625
4	,008333
5	,010416
6	,0125
7	,014583

Gr.	*D. Part.*	*Gr.*	*D. Part.*
8	,016666	16	,033333
9	,01875	17	,035416
10	,020833	18	,0375
11	,022916	19	,039583
12	,025	20	,041666
13	,027083	21	,04375
14	,029166	22	,045833
15	,03125	23	,047916

TABLE III.

Averdupois Weight, *one Hundred Weight the Integer.*

Qr.	*D. Part.*
1	.25
2	,5
3	,75

lb.	*D. Part.*
1	,008928
2	,017857
3	,026786
4	,035714
5	,044643
6	,053571
7	,0625
8	,071428
9	,080357
10	,089286
11	,098214
12	,107143
13	116071
14	,125
15	,133928
16	,142857
17	,151785
18	,160714
19	,169643
20	,178571
21	,1875
22	,196428
23	,205357
24	,214286
25	,223214
26	,232143
27	,241071

Oz.	*D. Part.*
1	,000558
2	,001116
3	,001674
4	,002232
5	,00279
6	,003348

Oz.	D. Part.
7	,003906
8	,004464
9	,005022
10	,00558
11	,006138
12	,006696
13	,007254
14	,007812
15	,00837

One Pound *the Integer.*

Oz.	D. Part.
1	,0625
2	,125
3	,1875
4	,25
5	,3125
6	,375
7	,4375
8	,5
9	,5625
10	,625
11	,6875
12	,75
13	,8125
14	,875
15	,9375

Dr.	D. Part.
1	,003906
2	,007812
3	,011718
4	,015625
5	,019531
6	,023437
7	,027343
8	,03125
9	,035156
10	,039062
11	,042968
12	,046875
13	,050781
14	,054687
15	,058593

TABLE IV.

Apothecary's Weight, *one* Pound *the Integer.*

Oz.	D. Part.	Oz.	D. Part.
1	,08333	6	,5
2	,16666	7	,58333
3	,25	8	,66666
4	,33333	9	,75
5	,41666	10	,83333

Oz.	D. Part.
11	,91666

Dr.	D. Part.
1	,010416
2	,020833
3	,031250
4	,041666
5	,052083
6	,0625
7	,072916

Sc.	D. Part.
1	,003472
2	,006944

One Ounce *the Integer.*

Dr.	D. Part.
1	,125
2	,25
3	,375
4	,5
5	,625
6	,75
7	,875

Sc.	D. Part.
1	,041666
2	,083333

Gr.	D. Part.
1	,002083
2	,004166
3	,00625
4	,008333
5	,010416
6	,0125
7	,014583
8	,016666
9	,01875
10	,020833
11	,022916
12	,025
13	,027083
14	,029166
15	,03125
16	,033333
17	,035416
18	,375
19	,039583

TABLE V.

Dry Measure; *one* Tun *or* Load *the Integer.*

Qr.	D. Part.	Qr.	D. Part.
1	,2	3	,6
2	,4	4	,8

B.	D. Part.
1	,025
2	,05
3	,075
4	,1
5	,125
6	,15
7	,175

G.	D. Part.
1	,003125
2	,00625
3	,009375
4	,0125
5	,015625
6	,01875
7	,021875

One Quarter *the Integer.*

B.	D. Part.
1	,125
2	,25
3	,375
4	,5
5	,625
6	,75
7	,875

G.	D. Part.
1	,015625
2	,03125
3	,046875
4	,0625
5	,078125
6	,09375
7	,109375

One Bushel *the Integer.*

G.	D. Part.
1	,125
2	,25
3	,375
4	,5
5	,625
6	,75
7	,875

P.	D. Part.
1	,015625
2	,03125
3	,046875
4	,0625
5	,078125
6	,09375
7	,109375

TABLE VI.

Liquid Measure. For Wine, &c. *One* Tun *the Integer.*

H.	D. Part.
1	,25
2	,5
3	,75

R.	D. Part.
½	,035714
1	,071428
1½	,107143
2	,142857
2½	,178572
3	,214285

G.	D. Part.
1	,003968
2	,007936
3	,011904
4	,015872
5	,019841
6	,023808
7	,027776
8	,031744
9	,035714
10	,039682
11	,04365
12	,047618
13	,051586
14	,055555
15	,059523
16	,063491
17	,06746

One Hogsh. *the Integer.*

R.	D. Part.
½	,14285
1	,28571
1½	,42856
2	,57142
2½	,71425
3	,85713

G.	D. Part.
1	,015873
2	,031746
3	,047619
4	,063492
5	,079365
6	,095239
7	,111111
8	,126984
9	,142857
10	,15873
11	,174603
12	,190476
13	,206349
14	,222222
15	,238095
16	,253968
17	,269841

One

One Rundlet *the Integer.*

G.	D. Part.
1	,055555
2	,111111
3	,166666
4	,222222
5	,277777
6	,333333
7	,388888
8	,444444
9	,5
10	,555555
11	,611111
12	,666666
13	,722222
14	,777777
15	,833333
16	,888888
17	,944444

P.	D. Part.
1	,006944
2	,013888
3	,020833
4	,027777
5	,034722
6	,041666
7	,048611

One Gallon *the Integer.*

P.	D. Part
1	,125
2	,25
3	,375
4	,5
5	,625
6	,75
7	,875

TABLE VII.

Of Ale *and* Beer Meaſure ; *one* Hogſhead *the Integer.*

Fr.	D. Part.
1	,16666
2	,33333
3	,5
4	,66666
5	,83333

G.	D. Part.
1	,020833
2	,041666
3	,0625
4	,083333
5	,104166
6	,125
7	,145833

G.	D. Part.
1	,018518
2	,037037
3	,055555
4	,074074
5	,092592
6	,111111
7	,129629
8	,148148

TABLE VIII.

Long Meaſure ; *one* Mile *the Integer.*

F.	D. Part.
1	,125
2	,25
3	,375
4	,5
5	,625
6	,75
7	,875

P.	D. Part.
1	,003125
2	,00625
3	,009375
4	,0125
5	,015625
6	,01875
7	,021875
8	,025
9	,028125
10	,03125
11	,034375
12	,0375
13	,040625
14	,04375
15	,046875
16	,05
17	,053125
18	,05625
19	,059375
20	,0625
21	,065625
22	,06875
23	,071875
24	,075
25	,078125
26	,08125
27	,084375
28	,0875
29	,090625
30	,09375
31	,096875
32	,1
33	,103125
34	,10625
35	,109375
36	,1125
37	,115625
38	,11875
39	,121875

Y.	D. Part.
½	,000284
1	,000568
2	,001136
3	,001704
4	,002272
5	,002841

One Furlong *the Integer.*

P.	D. Part.
1	,025
2	,05
3	,075
4	,1
5	,125
6	,15
7	,175
8	,2
9	,225
10	,25
11	,275
12	,3
13	,325
14	,35
15	,375
16	,4
17	,425
18	,45
19	,475
20	,5
21	,525
22	,55
23	,575
24	,6
25	,625
26	,65
27	,675
28	,7
29	,725
30	,75
31	,775
32	,8
33	,825
34	,85
35	,875
36	,9
37	,925
38	,95
39	,975

Y.	D. Part
1	,004545
2	,009090
3	,013636
4	,018181
5	,022727

F.	D. Part.
1	,001515
2	,003030

One Pole *the Integer.*

Y.	D. Part.
1	,181818
2	,363636
3	,545454
4	,727272
5	,909090

F.	D. Part.
1	,060606
2	,121212

In.	D. Part.
1	,005050
2	,010101
3	,01515
4	,020202
5	,02525
6	,030303
7	,035353
8	,040404
9	,045454
10	,050505
11	,055555

One Yard *the Integer.*

F.	D. Part.
1	,3333
2	,6666

In.	D. Part.
1	,0277
2	,0555
3	,0833
4	,1111
5	,1388
6	,1666
7	,1944
8	,2222
9	,25
10	,2777
11	,3055

TABLE IX.

Of Time; *one* Year *the Integer.*

M.	D. Part.
1	,076923
2	,153846
3	,230769
4	,307692
5	,384615
6	,461538
7	,538461
8	,615384
9	,692307
10	,76923
11	,84615
12	,92307

W.	D. Part.
1	,01923
2	,03846
3	,0576

D.	*D. Part*
1	,002747
2	,005494
3	,008241
4	,010988
5	,013735
6	,016482

One Month *the Integer.*

W.	*D. Part*
1	,25
2	,5
3	,75

D.	*D. Part*
1	,035714
2	,071428
3	,107142
4	,142856
5	,17857
6	,214284

One Week *the Integer.*

D.	*D. Part*
1	,142857
2	,285714
3	,428571
4	,571428
5	,714285
6	,857142

H.	*D. Part.*
1	,005952
2	,011904
3	,017856
4	,023808
5	,02976
6	,035712
7	,041664
8	,047616
9	,053568
10	,05952
11	,065472
12	,071424
13	,077376
14	,083328
15	,08928
16	,095232
17	,101184
18	,107136
19	,113088
20	,11904
21	,124992
22	,130944
23	,136896

TABLE X.

Of Motion, *A Sign of the* Zodiac *the* Integer.

D°.	*D. Part.*
1	,033333
2	,066666
3	,1
4	,133333
5	,166666
6	,2
7	,233333
8	,266666
9	,3
10	,333333
11	,366666
12	,4
13	,433333
14	,466666
15	,5
16	,533333
17	,566666
18	,6
19	,633333
20	,666666
21	,7
22	,733333
23	,766666
24	,8
25	,833333
26	,866666
27	,9
28	,933333
29	,966696

M'.	*D. Part.*
1	,000555
2	,001111
3	,001666
4	,002222
5	,002777
6	,003333
7	,003888
8	,004444
9	,005
10	,005555
11	,006111
12	,006666
13	,007222
14	,007777
15	,008333
16	,008888
17	,009444
18	,01
19	,010555
20	,011111
21	,011666
22	,012222
23	,012777
24	,013333
25	,013888
26	,014444
27	,015
28	,015555
29	,016111
30	,016666
31	,017222
32	,017777
33	,018333
34	,018888
35	,019444
36	,02

M'.	D. Part.
37	,020555
38	,021111
39	,021666
40	,022222
41	,022777
42	,023333
43	,023888
44	,024444
45	,025
46	,025555
47	,026111
48	,026666
49	,027222
50	,027777
51	,028333
52	,028888
53	,029444
54	,03
55	,030555
56	,031111
57	,031666
58	,032222
59	,032777

S''.	D. Part.
1	,000009
2	,000018
3	,000027
4	,000036
5	,000046
6	,000055
7	,000064
8	,000073
9	,000083
10	,000092
11	,000101
12	,00011
13	,000119
14	,000128

S''.	D. Part.
15	,000137
16	,000146
17	,000155
18	,000164
19	,000173
20	,000184
21	,000193
22	,000202
23	,000211
24	,00022
25	,000229
26	,000238
27	,000247
28	,000256
29	,000265
30	,000276
31	,000285
32	,000294
33	,000303
34	,000312
35	,000322
36	,000331
37	,00034
38	,000349
39	,000358
40	,000368
41	,000377
42	,000386
43	,000395
44	,000404
45	,000414
46	,000423
47	,000432
48	,000441
49	,00045
50	,00046
51	,000469
52	,000478
53	,000487
54	,000496
55	,000506
56	,000515
57	,000524
58	,000533
59	,000542

A Degree *the Integer.*

M'.	D. Part.
1	,016666
2	,033333
3	,05
4	,066666
5	,083333
6	,1
7	,116666
8	,133333
9	,15
10	,166666
11	,183333
12	,2
13	,216666
14	,233333
15	,25
16	,266666
17	,283333
18	,3
19	,316666
20	,333333
21	,35
22	,366666
23	,383333
24	,4
25	,416666
26	,433333
27	,45
28	,466666
29	,48333
30	,5
31	,51666
32	,53333
33	,55
34	,56666
35	,58333
36	,6
37	,61666
38	,63333
39	,65
40	,66666
41	,68333
42	,7
43	,71666
44	,73333
45	,75
46	,76666
47	,78333
48	,8
49	,81666
50	,83333
51	,85
52	,86666
53	,88333
54	,9
55	,91666
56	,93333
57	,95
58	,96666
59	,98333

S''.	D. Part.
1	,000277
2	,000555
3	,000833
4	,001111
5	,001388

S″.	D. Part.	S″.	D. Part.	S″.	D. Part.	S″.	D. Part.
6	,001666	20	,005555	34	,009444	48	,013333
7	,001944	21	,005833	35	,009722	49	,013611
8	,002222	22	,006111	36	,01	50	,013888
9	,0025	23	,006388	37	,010277	51	,014166
10	,002777	24	,006666	38	,010555	52	,014444
11	,003055	25	,006944	39	,010833	53	,014722
12	,003333	26	,007222	40	,011111	54	,015
13	,003611	27	,0075	41	,011388	55	,015277
14	,003888	28	,007777	42	,011666	56	,015555
15	,004166	29	,008055	43	,011944	57	,015833
16	,004444	30	,008333	44	,012222	58	,016111
17	,004722	31	,008611	45	,0125	59	,016388
18	,005	32	,008888	46	,012777		
19	,005277	33	,009166	47	,013055		

A General Decimal Table for all Divisions of Parts under Twenty.

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	,5	,3	,25	,2	,16	,1482	,125	,1	,1	,09	,083	,0769	,0714	,6	,0625	,0588	,05	,0526	,05
2		,6	,5	,4	,3	,2875	,25	,2	,2	,18	,16	,1538	,1428	,13	,125	,1176	,1	,1052	,1
3			,75	,6	,5	,4285	,375	,3	,3	,27	,25	,2307	,2142	,2	,1875	,1765	,16	,1579	,15
4				,8	,6	,5714	,5	,4	,4	,36	,3	,3076	,2856	,26	,25	,2352	,2	,2105	,2
5					,83	,7142	,625	,5	,5	,45	,416	,3846	,3571	,3	,3125	,2941	,27	,2631	,25
6						,8571	,75	,6	,6	,54	,5	,4615	,4285	,4	,375	,3529	,3	,3157	,3
7							,875	,7	,7	,63	,583	,5384	,5	,46	,4375	,4117	,38	,3683	,35
8								,8	,8	,72	,6	,6153	,5714	,53	,5	,4705	,4	,421	,4
9									,9	,81	,75	,6922	,6428	,6	,5625	,5293	,5	,4736	,45
10										,90	,83	,7692	,7142	,6	,625	,5882	,5	,5262	,5
11											,916	,8461	,7856	,73	,6875	,647	,61	,5788	,55
12												,923	,8571	,8	,75	,7058	,6	,6314	,6
13													,9285	,86	,8125	,7646	,72	,684	65
14														,93	,875	,8234	,7	,7366	,7
15															,9375	,8822	,83	,7892	,75
16																,941	,8	,8418	,8
17																	,94	,8944	,85
18																		,9473	,9
19																			,95

The Explanation and Use of the foregoing general *Decimal Table.*

1. The Figures at *Top*, which run to 20, ſhew the *Number* of *Parts* any *Quantity* or *Integer* is *divided* into; and the Figures in the *ſide Column* are thoſe Parts themſelves; The Figures forming the *triangular* Space, and diſpoſed into proper Columns, are the *Decimal Parts* of the *Integral* Quantity anſwering thereto. Hence any Quantity divided into any Number of Parts under 20, the *Decimals* anſwering to each of thoſe Parts are ſeen in *one view* in their proper Column.

Example. Suppoſe a Quantity divided into *Eight equal Parts*, and you would know the *Decimal Part* equivalent to *each*: Look at top for 8, *under* which are diſpoſed the *Decimals*, viz. ,125 ,25 ,375 *&c.* anſwering in order to the *Parts* in the *ſide Column.*

2. The Figures in the *ſide Column* may be taken for the *Numerator*, and thoſe at top, for the *Denominator* of a Vulgar Fraction.

Then the *Decimal correſponding* to thoſe *two Numbers* reſpectively, is equal to the foreſaid *Fraction:* Thus the *Decimal* 5714 anſwering to 4 in the ſide, and 7 at *top*, is *equal* to the *Fraction* $\frac{4}{7}$. So $\frac{9}{13}=,6922$. $\frac{5}{6}=,83$. $\frac{6}{11},=54$. $\frac{13}{16}=,8125$. $\frac{16}{19}=,8418$. $\frac{17}{20}=,85$ *&c.*

Alſo any *larger Fraction* whoſe Parts are an *Equimultiple* of any of theſe *tabular Fractions*, or may be reduced to them, are equally anſwer'd in *Decimals* by this *Table*; ſee this *Example* $\frac{588}{1512}=\frac{84}{210}=\frac{21}{54}=\frac{7}{18}=,38$ Decimals in the Table.

Caſe 3. To *reduce* any *Decimal* into the *equivalent known Parts* of *Coin*, *Weights*, *Meaſure*, *Motion*, &c. obſerve this

Rule. Multiply the given *Decimal* by the *Number of Units* contain'd in the next *lower* Denomination of that *Species* of Quantity, which your *Decimal* is of; and thus proceed, till

till you have *converted your Decimals*, or come to the *lowest* Part; and the *several Products* will be the *several Parts* of the *Quantity* required. See the following Examples.

EXAMPLE I.

What common Parts of a Pound (*viz.* Shillings, Pence, *&c.*) are contain'd in 0,73825 *Decimal* Parts of a Pound?

First, Multiply by	20	Shillings, the next lower Dem.
	14,76500	Shillings.
Then Multiply by	12	the next lower De. to the last.
	9,18000	Pence.
Lastly, Multiply by	4	the lowest Denomin. of all.
	0,72000	Farthings.

Hence the Answer is 14 Shillings, 9 Pence, and 7 Tenths, or 72 Hundredths of a Farthing.

EXAMPLE II.

Reduce 0,72083 to the known Parts of a Crown.

```
0,72083
      5
-------
3,60416
     12
-------
7,24999
      4      Answer 3 s. 7 d. 1/4.
-------
0,99999
```

EXAMPLE III.

Reduce 5,890625 into known Parts of a Mark.

```
   5,890625
       13,3
-----------
9) 2,671875
-----------
    ,2968750
    2671875
    890625
-----------
  11,8750000      Thus the Answer is exact with-
          12        out any Remainder, viz.
-----------       Marks   s.    d.
  10,5000000        5  :  11  :  10  :  1/2.
           4
-----------
   2,00000000
```

EXAM-

EXAMPLE IV.

Reduce ,727564 into the known Parts of a Pound *Troy*.

```
    ,727564
         12
   --------
   8,730768
         20
  ---------
  14,615360
         24
  ---------
    2461440
   1230720
  ---------
  14,768640
```

The Answer is 8 *oz.* 14 *penny wt.* 14 *gr.* $\frac{3}{4}$.

EXAMPLE V.

Reduce ,49723 into the known Parts of an *C. Weight*.

```
  ,49723
       4
  ------
 1,98892
      28
  ------
  791136
 197784
 -------
27,68976
      16
 -------
  413856
  68976
 -------
11,03616
      16
 -------
   21696
   3616
 -------
 0,57856
```

The Answer will stand thus, 1 *qr.* 27 *lb.* 11 *oz.* 0$\frac{1}{2}$ *dr.*

EXAMPLE VI.

Reduce ,578 of a *Rod* into its known Parts.

$$
\begin{array}{r}
,578 \\
5,5 \\
\hline
2892 \\
28928 \\
\hline
3,1821 \\
3 \\
\hline
0,5465 \\
12 \\
\hline
6,5585
\end{array}
$$

The Answer is 3 *yds.* 0 *Ft.* 6½ *In.*; and this repeating Decimal ,0585 over.

These six Examples I imagine sufficient to shew the *common* Method of *reducing Decimal Parts* into the common and known Parts of any Species of Quantity.

But as those *Operations of Reduction* are for the most part very *laborious*, *tedious*, and require *abundance of Figures*, I have sometimes wondred that a *Set of Tables* have not been composed to *facilitate* this Part of *Decimal Arithmetick*, as well as for the *contrary* Operations of *reducing different Species* into Decimals; especially since *one* is as *necessary* as the *other*. *Tables* for *that* purpose have long since been contrived, but none for the *reverse*; to turn *into*, but not to turn *out* of *Decimals*.

'Tis true, some *Decimals* (as those of *Money*) have the *first* and sometimes the *second* Figure pretty easily *valued* by a small Application of thought; but even this is for the *Skilful* to do, not for any that are but *Tyro's*, or *rude* in the Art. Yet how much *Tables* for *Reduction* of *Decimals* to *vulgar* Parts are wanting, may appear from the great Industry many have used to lay down *Rules* for that purpose. which being *so prolix*, *verbose*, *obscure*, and consequently *impertinent*, that a Person wou'd sooner and with more ease and pleasure work out his Answer by the *ordinary* Method than by those *unintelligible* and *insignificant* Rules; and according to the *old Saw*, wou'd find the farthest way about, the nearest way home.

But

But having for Reasons already render'd determined to write a *compleat Treatise of Decimal Arithmetick*, I thought it could by no means be *worthy* of, or *answer* that *Title*, unless with many other *Improvements*, I could make one more to render this Part of the Art *most easy* and *expeditious*; and having imploy'd my Thoughts a little on this *Topic*, I soon perceived an *Expedient* that wou'd do the business, which was this.

Viz. To *divide* the Figures of *Decimals* into *Pairs*, from the left Hand to the right; then to *tabulate* the *Digits* of every *Pair* from *Units* to an *Hundred*, in proper Columns; and lastly, to *affix the true Value* of every Place of Figures in the Column *appositely* answering thereto.

So that by this means the *Value* of any *two Places* of Figures (so far as they are valuable) in any *Decimal*, is seen by *Inspection* only; and the Value of four or six Places are as it were placed in one view, and with the greatest *ease and readiness* are obtain'd in any common Species of *Coins*, *Measures*, and *Weight*. But it being a Contrivance of *my own*, I shall not on that Account say any thing as to their Merit; but only give a short *Description* of the *Tables* and the *Manner* of *using* them by an Example of each.

A Description of the following new Decimal Tables.

For every *different Kind of Decimals* in *common Use*, I have compos'd a proper Set of *Tables*, and according as the *Decimal* is *more* or *less* valuable, it is divided into two or three Pairs; and to each of these Pairs is a *Table* exhibiting the *true value* of the *Figures* in *each Pair* from *Unit* to 100. The *Tables* of every sort are seen at the *top* of the Page; thus represented *Table* 1. *Table* 2. &c. *Each Table* consists of *several* Columns; the *first* Column has at *Top* *N°.* to signifie the *Numbers* of the *Decimal* Pairs. The other Columns (all but the last) represent the *Value of the Numbers* in the *first* Column in the *various Denominations* of the Parts the *Integer* are *vulgarly* known by. The *last* Column contains the *Decimal Parts* of the *last* Denomination, and in every Table is marked with *P.ts*.

Having given a *brief Explanation* of the Tables in general, I shall now shew their *Use* in discovering the *Value* of

any *Decimal* given, which is thus. Suppose you would know the *Value* of the *Decimal* ,689457 of a Tun *Averdupois Weight*, proceed thus; first divide those Numbers into *Pairs* as here, 68,94,57; then take the *first* right Hand Pair, *viz.* 57, and seek in *Table* 1. *Averdupois Weight*, and you will find against $\left\{\begin{matrix} 7 - \overset{oz.}{0} - ,25 \\ 50 - 1 - ,79 \end{matrix}\right\}$ that is, $\overset{oz.}{2,04}$; then for the second Pair, 94, look in *Table* 2. and against 94 you see 21 *lb.* 0 *oz.* 52 *pwts.*; and for the third Pair, 68, seek in *Table* 3. and against 68 you observe 13 *C.* 2 *qr.* 11 *lb.* 3,2 *oz.* So that these several Numbers added in a proper manner will stand thus,

			C.	*qr.*	*lb.*	*oz.*
Against the	First Pair	,57 —	00 :	0 :	00 :	02,04
	Second Pair	,94 —	00 :	0 :	21 :	00,52
	Third Pair	,68 —	13 :	2 :	11 :	03,2
Hence the value of ,689457 is			13 :	3 :	04 :	05,76

Now that the Reader may see at once both the *Advantage* and *Exactness* of the *Tables*; I shall shew the Work of finding the value of the said *Decimal*, at large in the *common* way of *Reduction*, which is as follows.

$$
\begin{array}{rl}
,689457 & \text{of a } \textit{Tun.} \\
20 & \\
\hline
13,789140 & = \textit{Hundred.} \\
4 & \\
\hline
3,156560 & = \textit{Quarter} \text{ of } C. \\
28 & \\
\hline
1252480 & \\
313120 & \\
\hline
4,383680 & = \textit{Pounds.} \\
16 & \\
\hline
2302080 & \\
383680 & \\
\hline
6,138880 & = \textit{Ounces.}
\end{array}
$$

The

	C.	qr.	lb.	oz.
The Value this way is	13	3	04	06,1388 &c.
The Value by the Table is	13	3	04	05,76
The Difference only	00	0	00	00,3788

Hence it appears how *exact*, and yet how easy and *expeditious* these *Tables* are in the business of *Reducing Decimals* to their *proper Value* in the *common* and *vulgar* Parts of their proper *Integer*. I shall next give Examples of all kinds of *Decimal Parts*, in order to render the *Use* and *Emoluments* of these *new invented Tables*, as *plain* and *obvious* as may be; tho' they are of themselves as easy to be understood as any *Arithmetical Tables* whatsoever.

EXAMPLE I.

What is the Value of ,4725 of a *Pound Sterling*?

				s.	d.
In *Table* 1.	against	,25 is	—	00	0 : $\frac{1}{2}$,4
In *Table* 2.	against	,47 is	—	9	4 : $\frac{3}{4}$,2
			Answer	9	5 : $\frac{1}{4}$,6

EXAMPLE II.

What is the Value of ,147 of a *Shilling* or *Foot*?

				d.	qr.
In *Table* 1.	against	,70 is	—	00	0,336
In *Table* 2.	against	,14 is	—	01	2,72
The Answer in *d*, and *f*, or *In.* and *qr.* is				01	3,056

EXAMPLE III.

What is the Value of ,7347 of a *Pound Troy*?

				oz.	pwt.	gr.
In *Table* 1.	against	,47 is	—	00	01	3,07
In *Table* 2.	against	,73 is	—	08	15	4,8
			Answer	08	16	7,87

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EXAMPLE IV.

What is the Value of ,91249 of a *C. Weight Averdupois?*

					Qr.	*lb.*	*oz.*	*dr.*
In	*Table* 1.	against	,90 is	—	0	00	00	02,58
	Table 2.		,24 is	—	0	00	04	04,8
	Table 3.		,91 is	—	3	17	14	11,52
			Answer		3	18	03	02,9

EXAMPLE V.

What is the Value of ,7777 of a Pound *Apothecaries Weight?*

					℥	ʒ	℈	*gr.*
In	*Table* 1.	against	,77 is	—	00	0	2	04,88
	Table 2.		,77 is	—	09	1	2	15,2
	—		Answer		09	2	2	00,08

EXAMPLE VI.

What is the Value of ,8754 of a *Tun*, Wine Measure.

					P.	*Hg.*	*T.*	*G.*
In	*Table* 1.	against	,54 is	—	0	0	0	01,36
	Table 2.		,87 is	—	1	1	0	30,24
			Answer		1	1	0	31 6

EXAMPLE VII.

What is the Value of ,7509 of a Load of *Corn?*

					2	*B.*	*G.*
In	*Table* 1.	against	,09 is	—	0	0	0,28
	Table 2.		,75 is	—	3	6	0
			Answer		3	6	0,28

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EXAMPLE VIII.

What is the Value of ,8495 of a *Year?*

				M.	W.	D.	H.
In	*Table* 1.	against	,95 is —	00	0	3	11,22
	Table 2.		,84 is —	10	3	5	14,4
			Anſwer	11	0	2	01,62

EXAMPLE IX.

What is the Value of ,889 of an *Hour* or *Degree?*

				′	″
In	*Table* 1.	against	,90 is — —	00	32,4
	Table 2.		,88 is — —	52	48,
			Anſwer	53	20,4

EXAMPLE X.

What is the Value of ,0596 of a *Sign* of the *Zodiac?*

				°	′	″
In	*Table* 1.	against	,96 is —	00	17	16
	Table 2.		,05 is —	01	30	00
			Anſwer	01	47	16

EXAMPLE XI.

What is the Value of ,976305 of a *Mile?*

				Fr.	R.	Yd.	Ft.	In.
In	*Table* 1.	against	,05 is	0	00	0	0	00,31
	Table 2.		,63 is	0	02	0	0	02,79
	Table 3.		,97 is	7	30	2	0	07,2
			Anſwer	7	32	2	0	10,3

EXAM-

EXAMPLE XII.

What is the Value of ,278 of a *Yard square?*

				F.q.	In.q.
In	*Table* 1.	against	,80 is	—	0 : 10,33
	Table 2.		,27 is	—	2 : 61,92
			Answer		2 : 72,25

EXAMPLE XIII.

What is the Value of ,795007 of a *Mile square?*

				A.	R.	P.
In	*Table* 1.	against	,07 is —	0	0	00,71
	Table 2.		,50 is —	3	0	32
	Table 3.		,79 is —	505	2	16
			Answer	508	3	08,71

EXAMPLE XIV.

What is the Value of ,974 of an *Acre?*

				R.	P.	Y.
In	*Table* 1.	against	,40 —	0	00	19,36
	Table 2.		,97 —	3	35	06,05
			Answer	3	35	25,41

EXAMPLE XV.

What is the Value of ,629 of a *Yard solid?*

				F.	In.
In	*Table* 1.	against	,90 is —	00	419,94
	Table 2.		,62 is —	16	1278,72
			Answer	16	1698,66

TABLE

A Set of New Decimal Tables *expressing the Value of any* Decimal *in the Known or Vulgar Denominations of the Integral Quantity, whether* Money, Weight, Time, Motion, *or* Measure *of every Kind.*

Table I. *Of* Money, *One Pound the Integer.*

N°.	d.	s.	Pt.	N°.	d.	s.	Pt.	N°.	d.	s.	Pt.
1	—	—	,09	29	—	2	,78	57	1	1	,48
2	—	—	,19	30	—	2	,88	58	1	1	,58
3	—	—	,28	31	—	2	,98	59	1	1	,68
4	—	—	,38	32	—	3	,08	60	1	1	,78
5	—	—	,48	33	—	3	,17	61	1	1	,87
6	—	—	,57	34	—	3	,27	62	1	1	,97
7		—	,67	35	—	3	,37	63	1	2	,06
8	—	—	,76	36	—	3	,46	64	1	2	,16
9	—	—	,86	37	—	3	,56	65	1	2	,26
10	—	—	,96	38	—	3	,65	66	1	2	,35
11	—	1	,05	39	—	3	,75	67	1	2	,45
12	—	1	,15	40	—	3	,84	68	1	2	,54
13	—	1	,24	41	—	3	,94	69	1	2	,64
14	—	1	,34	42	1	0	,03	70	1	2	,74
15	—	1	,44	43	1	0	,12	71	1	2	,83
16	—	1	,53	44	1	0	,22	72	1	2	,93
17	-	1	,63	45	1	0	,32	73	1	3	,02
18	—	1	,72	46	1	0	,41	74	1	3	,12
19	—	1	,82	47	1	0	,51	75	1	3	,22
20	—	1	,92	48	1	0	,6	76	1	3	,31
21	—	2	,01	49	1	0	,7	77	1	3	,41
22	—	2	,11	50	1	0	,8	78	1	3	,5
23	—	2	,20	51	1	0	,89	79	1	3	,6
24	—	2	,3	52	1	0	,99	80	1	3	,7
25	—	2	,4	53	1	1	,09	81	1	3	,79
26	—	2	,49	54	1	1	,19	82	1	3	,89
27	—	2	,59	55	1	1	,29	83	1	3	,98
28	—	2	,68	56	1	1	,38	84	2	0	,06

Nº.	d.	f.	Pt.	Nº.	d.	f.	Pt.	N8.	d.	f.	Pt.
85	2	0	,16	90	2	0	,64	95	2	1	,12
86	2	0	,25	91	2	0	,73	96	2	1	,21
87	2	0	,35	92	2	0	,83	97	2	1	,31
88	2	0	,44	93	2	0	,92	98	2	1	,4
89	2	0	,54	94	2	1	,02	99	2	1	,5

Table II. *Of Money, one Pound the Integer.*

Nº.	f.	d.	f.	Pt.	Nº.	f.	d.	f.	Pt.	Nº.	f.	d.	f.	Pt.
1	—	2	1	,6	29	5	9	2	,4	57	11	4	3	,2
2	—	4	3	,2	30	6	0	0	,—	58	11	7	0	,8
3	—	7	0	,8	31	6	2	1	,6	59	11	9	2	,4
4	—	9	2	,4	32	6	4	3	,2	60	12	0	0	,—
5	1	0	0	,—	33	6	7	0	,8	61	12	2	1	,6
6	1	2	1	,6	34	6	9	2	,4	62	12	4	3	,2
7	1	4	3	,2	35	7	0	0	,—	63	12	7	0	,8
8	1	7	0	,8	36	7	2	1	,6	64	12	9	2	,4
9	1	9	2	,4	37	7	4	3	,2	65	13	0	0	,—
10	2	0	0	-	38	7	7	0	,8	66	13	2	1	,6
11	2	2	1	,6	39	7	9	2	,4	67	13	4	3	,2
12	2	4	3	,2	40	8	0	0	,—	68	13	7	0	,8
13	2	7	0	,8	41	8	2	1	,6	69	13	9	2	,4
14	2	9	2	,4	42	8	4	3	,2	70	14	0	0	,—
15	3	0	0	,—	43	8	7	0	,8	71	14	2	1	,6
16	3	2	1	,6	44	8	9	2	,4	72	14	4	3	,2
17	3	4	3	,2	45	9	0	0	,—	73	14	7	0	,8
18	3	7	0	,8	46	9	2	1	,6	74	14	9	2	,4
19	3	9	2	,4	47	9	4	3	,2	75	15	0	0	,—
20	4	0	0	,—	48	9	7	0	,8	76	15	2	1	,6
21	4	2	1	,6	49	9	9	2	,4	77	15	4	3	,2
22	4	4	3	,2	50	10	0	0	,—	78	15	7	0	,8
23	4	7	0	,8	51	10	2	1	,6	79	15	9	2	,4
24	4	9	2	,4	52	10	4	3	,2	80	16	0	0	,—
25	5	0	0	,—	53	10	7	0	,8	81	16	2	1	,6
26	5	2	1	,6	54	10	9	2	,4	82	16	4	3	,2
27	5	4	3	,2	55	11	0	0	,—	83	16	7	0	,8
28	5	7	0	,8	56	11	2	1	,6	84	16	9	2	,4

N°.	f.	d.	f.	Pt.	N°.	f.	d.	f.	Pt.	N°.	f.	d.	f.	Pt.
85	17	0	0	,—	90	18	0	0	,—	95	19	0	0	,—
86	17	2	1	,6	91	18	2	1	,6	96	19	2	1	,6
87	17	4	3	,2	92	18	4	3	,2	97	19	4	3	,2
88	17	7	0	,8	93	18	7	0	,8	98	19	7	0	,8
89	17	9	2	,4	94	18	9	2	,4	99	19	9	2	,4

Table I. Troy Weight, *one Pound the Integer.*

N°.	Pwt.	gr.	Pt.	N°.	Pwt	gr.	Pt.	N°.	Pwt.	gr.	Pt.
1	—	—	,57	29	—	16	,7	57	1	8	,84
2	—	1	,15	30	—	17	,28	58	1	9	,41
3	—	1	,72	31	—	17	,85	59	1	9	,99
4	—	2	,3	32	—	18	,43	60	1	10	,57
5	—	2	,88	33	—	19	,—	61	1	11	,14
6	—	3	,45	34	—	19	,58	62	1	11	,72
7	—	4	,03	35	—	20	,16	63	1	12	,29
8	—	4	,6	36	—	20	,73	64	1	12	,87
9	—	5	,19	37	—	21	,31	65	1	13	,45
10	—	5	,76	38	—	21	,88	66	1	14	,02
11	—	6	,34	39	—	22	,46	67	1	14	,6
12	—	6	,91	40	—	23	,04	68	1	15	,17
13	—	7	,48	41	—	23	,61	69	1	15	,75
14	—	8	,06	42	1	0	,19	70	1	16	,33
15	—	8	,64	43	1	0	,76	71	1	16	,9
16	—	9	,21	44	1	1	,34	72	1	17	,48
17	—	9	,79	45	1	1	,92	73	1	18	,05
18	—	10	,36	46	1	2	,49	74	1	18	,63
19	—	10	,94	47	1	3	,07	75	1	19	,2
20	—	11	,52	48	1	3	,64	76	1	19	,77
21	—	12	,09	49	1	4	,22	77	1	20	,35
22	—	12	,67	50	1	4	,8	78	1	20	,92
23	—	13	,24	51	1	5	,37	79	1	21	,5
24	—	13	,82	52	1	5	,95	80	1	22	,08
25	—	14	,4	53	1	6	,52	81	1	22	,65
26	—	14	,97	54	1	7	,1	82	1	23	,23
27	—	15	,55	55	1	7	,69	83	1	23	,8
28	—	16	,12	56	1	8	,26	84	2	0	,38

N°.	Pwt.	gr.	Pt.	N°.	Pwt.	gr.	Pt.	N°.	Pwt.	gr.	Pt.
85	2	0	,96	90	2	3	,84	95	2	6	,72
86	2	1	,53	91	2	4	,41	96	2	7	,29
87	2	2	,11	92	2	4	,99	97	2	7	,87
88	2	2	,68	93	2	5	,56	98	2	8	,44
89	2	3	,26	94	2	6	,14	99	2	9	,02

Table II. Troy Weight, *one Pound the Integer.*

N°.	oz.	pw.	gr.	pt.	N°.	oz.	pw.	gr.	pt.	N°.	oz.	pw.	gr.	pt.
1	—	2	9	,6	29	3	9	14	,4	57	6	16	19	,2
2	—	4	19	,2	30	3	12	0	,—	58	6	19	4	,8
3	—	7	4	,8	31	3	14	9	,6	59	7	1	14	,4
4	—	9	14	,4	32	3	16	19	,2	60	7	4	0	,—
5	—	12	0	,—	33	3	19	4	,8	61	7	6	9	,6
6	—	14	9	,6	34	4	1	14	,4	62	7	8	19	,2
7	—	16	19	,2	35	4	4	0	,—	63	7	11	4	,8
8	—	19	4	,8	36	4	6	9	,6	64	7	13	14	,4
9	1	1	14	,4	37	4	8	19	,2	65	7	16	0	,—
10	1	4	0	,—	38	4	11	4	,8	66	7	18	9	,6
11	1	6	9	,6	39	4	13	14	,4	67	8	0	19	,2
12	1	8	19	,2	40	4	16	0	,—	68	8	3	4	,8
13	1	11	4	,8	41	4	18	9	,6	69	8	5	14	,4
14	1	13	14	,4	42	5	0	19	,2	70	8	8	0	,—
15	1	16	0	,—	43	5	3	4	,8	71	8	10	9	,6
16	1	18	9	,6	44	5	5	14	,4	72	8	12	19	,2
17	2	0	19	,2	45	5	8	0	,—	73	8	15	4	,8
18	2	3	4	,8	46	5	10	9	,6	74	8	17	14	,4
19	2	5	14	,4	47	5	12	19	,2	75	9	0	0	,—
20	2	8	0	,—	48	5	15	4	,8	76	9	2	9	,6
21	2	10	9	,6	49	5	17	14	,4	77	9	4	19	,2
22	2	12	19	,2	50	6	0	0	,—	78	9	7	4	,8
23	2	15	4	,8	51	6	2	9	,6	79	9	9	14	,4
24	2	17	14	,4	52	6	4	19	,2	80	9	12	0	,—
25	3	0	0	,—	53	6	7	4	,8	81	9	14	9	,6
26	3	2	9	,6	54	6	9	14	,4	82	9	16	19	,2
27	3	4	19	,2	55	6	12	0	,—	83	9	19	4	,8
28	3	7	4	,8	56	6	14	2	,6	84	10	1	14	,4

N°.

Nº.	oz.	pw.	gr.	pt.	Nº.	oz.	pw.	gr.	pt.	Nº.	oz.	pw.	gr.	pt.
85	10	4	0	,—	90	10	16	0	,—	95	11	8	0	,—
86	10	6	9	,6	91	10	18	9	,6	96	11	10	9	,6
87	10	8	19	,2	92	11	0	19	,2	97	11	12	19	,2
88	10	11	4	,8	93	11	3	4	,8	98	11	15	4	,8
89	10	13	14	,4	94	11	5	14	,4	99	11	17	14	,4

Table I.

Nº.	Pwt.	gr.	Pts.
1	--	--	,048
2	--	--	,096
3	--	--	,144
4	--	--	,192
5	--	--	,24
6	--	--	,288
7	--	--	,336
8	--	--	,384
9	--	--	,432
10	--	--	,48
20	--	--	,96
30	--	1	,44
40	--	1	,92
50	--	2	,4
60	--	2	,88
70	--	3	,36
80	--	3	,84
90	--	4	,32

Table II. { Troy Weight, *one Ounce the Integer.*

Nº.	Pwt.	gr.	Pt.	Nº.	Pwt.	gr.	Pt.
1	--	4	,8	29	5	19	,2
2	--	9	,6	30	6	0	,—
3	--	14	,4	31	6	4	,8
4	--	19	,2	32	6	9	,6
5	1	0	,—	33	6	14	,4
6	1	4	,8	34	6	19	,2
7	1	9	,6	35	7	0	,—
8	1	14	,4	36	7	4	,8
9	1	19	,2	37	7	9	,6
10	2	0	,—	38	7	14	,4
11	2	4	,8	39	7	19	,2
12	2	9	,6	40	8	0	,—
13	2	14	,4	41	8	4	,8
14	2	19	,2	42	8	9	,6
15	3	0	,—	43	8	14	,4
16	3	4	,8	44	8	19	,2
17	3	9	,6	45	9	0	,—
18	3	14	,4	46	9	4	,8
19	3	19	,2	47	9	9	,6
20	4	0	,	48	9	14	,4
21	4	4	,8	49	9	19	,2
22	4	9	,6	50	10	0	,—
23	4	14	,4	51	10	4	,8
24	4	19	,2	52	10	9	,6
25	5	0	,—	53	10	14	,4
26	5	4	,8	54	10	19	,2
27	5	9	,6	55	11	0	,—
28	5	14	,4	56	11	4	,8

Nº.	Pwt.	gr.	Pt.	Nº.	Pwt.	gr.	Pt.	Nº.	Pwt.	gr.	Pt.
57	11	9	,6	72	14	9	,6	87	17	9	,6
58	11	14	,4	73	14	14	,4	88	17	14	,4
59	11	19	,2	74	14	19	,2	89	17	19	,2
60	12	0	,—	75	15	0	,—	90	18	0	,—
61	12	4	,8	76	15	4	,8	91	18	4	,8
62	12	9	,6	77	15	9	,6	92	18	9	,6
63	12	14	,4	78	15	14	,4	93	18	14	,4
64	12	19	,2	79	15	19	,2	94	18	19	,2
65	13	0	,—	80	16	0	,—	95	19	0	,—
66	13	4	,8	81	16	4	,8	96	19	4	,8
67	13	9	,6	82	16	9	,6	97	19	9	,6
68	13	14	,4	83	16	14	,4	98	19	14	,4
69	13	19	,2	84	16	19	,2	99	19	19	,2
70	14	0	,—	85	17	0	,—				
71	14	4	,8	86	17	4	,8				

Table I. | Table II. { Averdupois Weight, *one Pound the Integer.*

Nº.	dr.	Pts.	Nº.	oz.	dr.	Pt.	Nº.	oz.	dr.	Pt.
1	--	,025	1	--	2	,56	19	3	0	,64
2	--	,051	2	--	5	,12	20	3	3	,2
3	--	,076	3	--	7	,68	21	3	5	,76
4	--	,102	4	--	10	,24	22	3	8	,32
5	--	,128	5	--	12	,8	23	3	10	,88
6	--	,153	6	--	15	,36	24	3	13	,44
7	--	,179	7	1	1	,92	25	4	0	,—
8	--	,204	8	1	4	,48	26	4	2	,56
9	--	,23	9	1	7	,04	27	4	5	,12
10	--	,256	10	1	9	,6	28	4	7	,68
20	--	,512	11	1	12	,16	29	4	10	,24
30	--	,768	12	1	14	,72	30	4	12	,8
40	1	,024	13	2	1	,28	31	4	15	,36
50	1	,28	14	2	3	,84	32	5	1	,92
60	1	,536	15	2	6	,4	33	5	4	,48
70	1	,792	16	2	8	,96	34	5	7	,04
80	2	,048	17	2	11	,52	35	5	9	.6
90	2	,304	18	2	14	,08	36	5	12	,16

Nº.

N°.	*oz.*	*dr.*	*Pt.*	N°.	*oz.*	*dr.*	*Pt.*	N°.	*oz.*	*dr.*	*Pt.*
37	5	14	,72	58	9	4	,48	79	12	10	,24
38	6	1	,28	59	9	7	,04	80	12	12	,8
39	6	3	,84	60	9	9	,6	81	12	15	,36
40	6	6	,4	61	9	12	,16	82	13	1	,92
41	6	8	,96	62	9	14	,72	83	13	4	,48
42	6	11	,52	63	10	1	,28	84	13	7	,04
43	6	14	,08	64	10	3	,84	85	13	9	,6
44	7	0	,64	65	10	6	,4	86	13	12	,16
45	7	3	,2	66	10	8	,96	87	13	14	,72
46	7	5	,76	67	10	11	,52	88	14	1	,28
47	7	8	,32	68	10	14	,08	89	14	3	,84
48	7	10	,88	69	11	0	,64	90	14	6	,4
49	7	13	,44	70	11	3	,2	91	14	8	,96
50	8	0	,—	71	11	5	,76	92	14	11	,52
51	8	2	,56	72	11	8	,32	93	14	14	,08
52	8	5	,12	73	11	10	,88	94	15	0	,64
53	8	7	,68	74	11	13	,44	95	15	3	;2
54	8	10	,24	75	12	0	,—	96	15	5	,76
55	8	12	,8	76	12	2	,56	97	15	8	,32
56	8	15	,36	77	12	5	,12	98	15	10	,88
57	9	1	,92	78	12	7	,68	99	15	13	,44

Table I. | Table II. { Averdupois Weight, *one* C. *or* 112 lb. *the Int.*

N°.	*dr.*	*Pt.*	N°.	*oz.*	*dr.*	*Pt.*	N°.	*oz.*	*dr.*	*Pt.*
1	--	,02	1	--	2	,86	12	2	2	,4
2	--	,05	2	--	5	,73	13	2	5	,27
3	--	,08	3	--	8	,6	14	2	8	,13
4	--	,11	4	--	11	,46	15	2	11	,0
5	--	,14	5	--	14	,33	16	2	13	,87
6	--	,17	6	1	1	,2	17	3	0	,73
7	--	,2	7	1	4	,06	18	3	3	,6
8	--	,22	8	1	6	,93	19	3	6	,47
9	--	,25	9	1	9	,80	20	3	9	,34
10	--	,28	10	1	12	,67	21	3	12	,2
20	--	,57	11	1	15	,53	22	3	15	,7

N°.

Table I. | Table II. { Averdupois Weight, *one* C. *or* 112 lb. *the Int.*

Nº.	*dr.*	*Pt.*
30	--	,86
40	1	,14
50	1	,43
60	1	,72
70	2	,0
80	2	,29
90	2	,58

Nº.	*oz.*	*dr.*	*Pt.*	Nº.	*oz.*	*dr.*	*Pt.*
23	4	1	,94	50	8	15	,35
24	4	4	,8	51	9	2	,21
25	4	7	,67	52	9	5	,08
26	4	10	,54	53	9	7	,95
27	4	13	,4	54	9	10	,81
28	5	0	,27	55	9	13	,68
29	5	3	,14	56	10	0	,55
30	5	6	,01	57	10	3	,41
31	5	8	,87	58	10	6	,28
32	5	11	,74	59	10	9	,15
33	5	14	,61	60	10	12	,02
34	6	1	,47	61	10	14	,88
35	6	4	,34	62	11	1	,75
36	6	7	,21	63	11	4	,62
37	6	10	,07	64	11	7	,48
38	6	12	,94	65	11	10	,35
39	6	15	,81	66	11	13	,22
40	7	2	,68	67	12	0	,08
41	7	5	,54	68	12	2	,95
42	7	8	,41	69	12	5	,82
43	7	11	,28	70	12	8	,69
44	7	14	,14	71	12	11	,55
45	8	1	,01	72	12	14	,42
46	8	3	,88	73	13	1	,29
47	8	6	,74	74	13	4	,15
48	8	9	,61	75	13	7	,02
49	8	12	,48				

Table I. | Table II. { Averdupois Weight. *one* C. Wt. *the Integer.*

Nº.	*lb.*	*oz.*	*dr.*	*Pt.*	*Q.*	0	1	2	3	*lb.*	*oz.*	*dr.*	*Pt.*
76	—	13	9	,89		Nº	25	50	75	0	0	0	,—
77	—	13	12	,75		1	26	51	76	1	1	14	,7[illegible]
78	—	13	15	,62		2	27	52	77	2	3	13	,4[illegible]
79	—	14	2	,49		3	28	53	78	3	5	12	,16
80	—	14	5	,36		4	29	54	79	4	7	10	,88
81	—	14	8	,22		5	30	55	80	5	9	9	,6
82	—	14	11	,09		6	31	56	81	6	11	8	,32
83	—	14	13	,96		7	32	57	82	7	13	7	,04
84	—	15	0	,82		8	33	58	83	8	15	5	,76
85	—	15	3	,69		9	34	59	84	10	1	4	,48
86	—	15	6	,56		10	35	60	85	11	3	3	,2
87	—	15	9	,41		11	36	61	86	12	15	1	,92
88	—	15	12	,28		12	37	62	87	13	7	0	,64
89	—	15	15	,15		13	38	63	88	14	8	5	,36
90	1	0	2	,03		14	39	64	89	15	10	14	,08
91	1	0	4	,89		15	40	65	90	16	12	12	,8
92	1	0	7	,76		16	41	66	91	17	14	11	,52
93	1	0	10	,63		17	42	67	92	19	0	10	,24
94	1	0	13	,49		18	43	68	93	20	2	8	,96
95	1	1	0	,36		19	44	69	94	21	4	7	,68
96	1	1	3	,23		20	45	70	95	22	6	6	,4
97	1	1	6	,09		21	46	71	96	23	8	5	,12
98	1	1	8	,96		22	47	72	97	24	10	3	,84
99	1	1	11	,83		23	48	73	98	25	12	2	,56
						24	49	74	99	26	14	1	,28

Table I. | Table II. { Averdupois Weight, *one* Tun *or* Load *the Int.*

Nº.	*lb.*	*Pt.*	Nº.	*lb.*	*oz.*	*Pt.*	Nº.	*lb.*	*oz.*	*Pt.*
1	--	,03	1	0	3	,58	5	1	1	,9
2	--	,07	2	0	7	,16	6	1	5	,48
3	--	,1	3	0	10	,74	7	1	9	,06
4	--	,14	4	0	14	,32	8	1	12	,64

N Nº.

N°	oz.	Pt.
5	--	,18
6	--	,21
7	--	,25
8	--	,28
9	--	,32
10	--	,35
20	--	,41
30	1	,07
40	1	,43
50	1	,79
60	2	,14
70	2	,5
80	2	,86
90	3	,22

N°	lb.	oz.	Pt.
9	2	0	,22
10	2	3	,8
11	2	7	,38
12	2	10	,96
13	2	14	,54
14	3	2	,12
15	3	5	,7
16	3	9	,28
17	3	12	,86
18	4	0	,44
19	4	4	,02
20	4	7	,6
21	4	11	,18
22	4	14	,76
23	5	2	,34
24	5	5	,92
25	5	9	,5
26	5	13	,08
27	6	0	,66
28	6	4	,24
29	6	7	,82
30	6	11	,4
31	6	14	,98
32	7	2	,56
33	7	6	:14
34	7	9	,72
35	7	13	,3
36	8	0	,88
37	8	4	,46
38	8	8	,04
39	8	11	,62
40	8	15	,2
41	9	2	,78
42	9	7	,36
43	9	9	,94
44	9	13	,52
45	10	1	,1
46	10	4	,68
47	10	8	,26

N°.	lb.	oz.	Pt.
48	10	11	,84
49	10	15	,42
50	11	3	,—
51	11	6	,58
52	11	10	,16
53	11	13	,74
54	12	1	,32
55	12	4	,9
56	12	8	,48
57	12	12	,06
58	12	15	,64
59	13	3	,22
60	13	6	,8
61	13	10	,38
62	13	13	,96
63	14	1	,54
64	14	5	,12
65	14	8	,7
66	14	12	,28
67	14	15	,86
68	15	3	,44
69	15	7	,02
70	15	10	,6
71	15	14	,18
72	16	1	,76
73	16	5	,34
74	16	8	,92
75	16	12	,5
76	17	0	,08
77	17	3	,66
78	17	7	,34
79	17	10	,82
80	17	14	,4
81	18	1	,98
82	18	5	,56
83	18	9	,14
84	18	12	,72
85	19	0	,3
86	19	3	,88

N°.	*lb.*	*oz.*	*Pt.*	N°.	*lb.*	*oz.*	*Pt.*	N°.	*lb.*	*oz.*	*Pt.*
87	19	7	,46	92	20	9	,36	97	21	11	,26
88	19	11	,04	93	20	12	,94	98	21	14	,84
89	19	14	,62	94	21	0	,52	99	22	2	,42
90	20	2	,2	95	21	4	,1				
91	20	5	,78	96	21	7	,68				

Table III. Averdupois Weight, *one Tun the Integer.*

N°.	*C.*	*Q.*	*lb.*	*oz.*	*Pt.*	N°.	*C.*	*Q.*	*lb.*	*oz.*	*Pt.*
1	0	0	22	6	,4	29	5	3	5	9	,6
2	0	1	16	12	,8	30	6	0	0	0	,—
3	0	2	11	3	,2	31	6	0	22	6	,4
4	0	3	5	9	,6	32	6	1	16	12	,8
5	1	0	0	0	,—	33	6	2	11	3	,2
6	1	0	22	6	,4	34	6	3	5	9	,6
7	1	1	16	12	,8	35	7	0	0	0	,—
8	1	2	11	3	,2	36	7	0	22	6	,4
9	1	3	5	9	,6	37	7	1	16	12	,8
10	2	0	0	0	,—	38	7	2	11	3	,2
11	2	0	22	6	,4	39	7	3	5	9	,6
12	2	1	16	12	,8	40	8	0	0	0	,—
13	2	2	11	3	,2	41	8	0	22	6	,4
14	2	3	5	9	,6	42	8	1	16	12	,8
15	3	0	0	0	,—	43	8	2	11	3	,2
16	3	0	22	6	,4	44	8	3	5	9	,6
17	3	1	16	12	,8	45	9	0	0	0	,—
18	3	2	11	3	,2	46	9	0	22	6	,4
19	3	3	5	9	,6	47	9	1	16	12	,8
20	4	0	0	0	,—	48	9	2	11	3	,2
21	4	0	22	6	,4	49	9	3	5	9	,6
22	4	1	16	12	,8	50	10	0	0	0	,—
23	4	2	11	3	,2	51	10	0	22	6	,4
24	4	3	5	9	,6	52	10	1	16	12	,8
25	5	0	0	0	,—	53	10	2	11	3	,2
26	5	0	22	6	,4	54	10	3	5	9	,6
27	5	1	16	12	,8	55	11	0	0	0	,—
28	5	2	11	3	,2	56	11	0	22	6	4,

Nº.	C.	Q.	lb.	oz.	Pt.	Nº.	C.	Q.	lb.	oz.	Pt.
57	11	1	16	12	,8	79	15	3	5	9	,6
58	11	2	11	3	,2	80	16	0	0	0	,0
59	11	3	5	9	,6	81	16	0	22	6	,4
60	12	0	0	0	,—	82	16	1	16	12	,8
61	12	0	22	6	,4	83	16	2	11	3	,2
62	12	1	16	12	,8	84	16	3	5	9	,6
63	12	2	11	3	,2	85	17	0	0	0	,0
64	12	3	5	9	,6	86	17	0	22	6	,4
65	13	0	0	0	,—	87	17	1	16	12	,8
66	13	0	22	6	,4	88	17	2	11	3	,2
67	13	1	16	12	,8	89	17	3	5	9	,6
68	13	2	11	3	,2	90	18	0	0	0	,0
69	13	3	5	9	,6	91	18	0	22	6	,4
70	14	0	0	0	,—	92	18	1	16	12	,8
71	14	0	22	6	,4	93	18	2	11	3	,2
72	14	1	16	12	,8	94	18	3	5	9	,6
73	14	2	11	3	,2	95	19	0	0	0	,—
74	14	3	5	9	,6	96	19	0	22	6	,4
75	15	0	0	0	,—	97	19	1	16	12	,8
76	15	0	22	6	,4	98	19	2	11	3	,2
77	15	1	16	12	,8	99	19	3	5	9	,6
78	15	2	11	3	,2						

Table I. | Table II. { Apothecaries Weight, *one* ℔ *the Integer.*

Nº.	℈	gr.	Pt.	Nº.	℥	ʒ	℈	gr.	Pt.	Nº.	℥	ʒ	℈	gr.	Pt.
1	--	--	,56	1	--	--	2	17	,6	11	1	2	1	13	,6
2	--	1	,15	2	--	1	2	15	,2	12	1	3	1	11	,2
3	--	1	,72	3	--	2	2	12	,8	13	1	4	1	8	,8
4	--	2	,3	4	--	3	2	10	,4	14	1	5	1	6	,4
5	--	2	,88	5	--	4	2	8	,—	15	1	6	1	4	,—
6	--	3	,45	6	--	5	2	5	,6	16	1	7	1	1	,6
7	--	4	,03	7	--	6	2	3	,2	17	2	0	0	19	,2
8	--	4	,6	8	--	7	2	0	,8	18	2	1	0	16	,8
9	--	5	,19	9	1	0	1	18	,4	19	2	2	0	14	,4
10	--	5	,76	10	1	1	1	16	,—	20	2	3	0	12	,—

N°	℈	gr.	Pt.
20	—	11	,52
30	—	17	,28
40	1	3	,04
50	1	8	,8
60	1	14	,56
70	2	0	,28
80	2	6	,02
90	2	11	,75

N°	℥	ʒ	℈	gr.	Pt.
21	2	4	0	9	,6
22	2	5	0	7	,2
23	2	6	0	4	,8
24	2	7	0	2	,4
25	3	0	0	0	,—
26	3	0	2	17	,6
27	3	1	2	15	2,
28	3	2	2	12	,8
29	3	3	2	10	,4
30	3	4	2	8	,—
31	3	5	2	5	,6
32	3	6	2	3	,2
33	3	7	2	0	,8
34	4	0	1	18	,4
35	4	1	1	16	,—
36	4	2	1	13	,6
37	4	3	1	11	,2
38	4	4	1	8	,8
39	4	5	1	6	,4
40	4	6	1	4	,—
41	4	7	1	1	6,
42	5	0	0	19	,2
43	5	1	0	16	,8
44	5	2	0	14	,4
45	5	3	0	12	,—
46	5	4	0	9	,6
47	5	5	0	7	,2
48	5	6	0	4	,8
49	5	7	0	2	,4
50	6	0	0	0	,—
51	6	0	2	17	,6
52	6	1	2	15	,2
53	6	2	2	12	,8
54	6	3	2	10	,4
55	6	4	2	8	,—
56	6	5	2	5	,6
57	6	6	2	3	,2
58	6	7	2	0	,8
59	7	0	1	18	,4
60	7	1	1	16	,—

N°	℥	ʒ	℈	gr.	Pt.
61	7	2	1	13	,6
62	7	3	1	11	,2
63	7	4	1	8	,8
64	7	5	1	6	,4
65	7	6	1	4	,—
66	7	7	1	1	,6
67	8	0	0	19	,2
68	8	1	0	16	8
69	8	2	0	14	,4
70	8	3	0	12	,—
71	8	4	0	9	,6
72	8	5	0	7	,2
73	8	6	0	4	,8
74	8	7	0	2	,4
75	9	0	0	0	,—
76	9	0	2	17	,6
77	9	1	2	15	,2
78	9	2	2	12	,8
79	9	3	2	10	,4
80	9	4	2	8	,—
81	9	5	2	5	,6
82	9	6	2	3	,2
83	9	7	2	0	,8
84	10	0	1	18	,4
85	10	1	1	16	,—
86	10	2	1	13	,6
87	10	3	1	11	,2
88	10	4	1	8	,8
89	10	5	1	6	,4
90	10	6	1	4	,—
91	10	7	1	1	,6
92	11	0	0	19	,2
93	11	1	0	16	,8
94	11	2	0	14	,4
95	11	3	0	12	,—
96	11	4	0	9	,6
97	11	[illegible]	0	7	,2
98	11	6	0	4	,8
99	11	7	0	2	,4

Table

Table I. | Table II. { Wine Measure, *one Hogshead the Integer.*

Nº.	P.	Pt.
1	- -	,05
2	- -	,1
3	- -	,15
4	- -	,2
5	- -	,25
6	- -	,3
7	- -	,35
8	- -	,4
9	- -	,45
10	- -	,5
20	1	,—
30	1	,51
40	2	,01
50	2	,52
60	3	,02
70	3	,52
80	4	,02
90	4	,53

Nº.	G.	P.	Pt.	Nº.	G.	P.	Pt.
1	—	5	,04	36	22	5	,44
2	1	2	,08	37	23	2	,48
3	1	7	,12	38	23	7	,52
4	2	4	,16	39	24	4	,56
5	3	1	,2	40	25	1	,6
6	3	6	,24	41	25	6	,64
7	4	3	,28	42	26	3	,68
8	5	0	,32	43	27	0	,72
9	5	5	,36	44	27	5	,76
10	6	2	,4	45	28	2	,8
11	6	7	,44	46	28	7	,84
12	7	4	,48	47	29	4	,88
13	8	1	,52	48	30	1	,92
14	8	6	,56	49	30	6	,96
15	9	3	,6	50	31	4	,—
16	10	0	,64	51	32	1	,04
17	10	5	,68	52	32	6	,08
18	11	2	,72	53	33	3	,12
19	11	7	,76	54	34	0	,16
20	12	4	,8	55	34	5	,2
21	13	1	,84	56	35	2	,24
22	13	6	,88	57	35	7	,28
23	14	3	,92	58	36	4	,32
24	15	0	,96	59	37	1	,36
25	15	6	,—	60	37	6	,4
26	16	3	,04	61	38	3	,44
27	17	0	,08	62	39	0	,48
28	17	5	,12	63	39	5	,52
29	18	2	,16	64	40	2	,56
30	18	7	,2	65	40	7	,6
31	19	4	,24	66	41	4	,64
32	20	1	,28	67	42	1	,68
33	20	6	,32	68	42	6	,72
34	21	3	,36	69	43	3	,76
35	22	0	,4	70	44	0	,8

Nº	*G.*	*P.*	*Pt.*	Nº	*G.*	*P.*	*Pt.*	Nº	*G.*	*P.*	*Pt.*
71	44	5	,84	81	51	0	,24	91	57	2	,64
72	45	2	,88	82	51	5	,28	92	57	7	,68
73	45	7	,92	83	52	2	,32	93	58	4	,72
74	46	4	,96	84	52	7	,36	94	59	1	,76
75	47	2	,—	85	53	4	,4	95	59	6	,8
76	47	7	,04	86	54	1	,44	96	60	3	,84
77	48	4	,08	87	54	6	,48	97	61	0	,88
78	49	1	,12	88	55	3	,52	98	61	5	,92
79	49	6	,16	89	56	0	,56	99	62	2	,96
80	50	3	,2	90	56	5	,6				,—

Table I. | Table II. { Beer Measure, *one Hogshead the Integer.*

Table I.

Nº	*P.*	*Pt.*
1	—	,04
2	—	,08
3	—	,12
4	—	,17
5	—	,21
6	—	,25
7	—	,3
8	—	,34
9	—	,38
10	—	,43
20	—	,86
30	1	,29
40	1	,72
50	2	,16
60	2	,59
70	3	,02
80	3	,45
90	3	,88

Table II.

Nº	*G.*	*P.*	*Pt.*	Nº	*G.*	*P.*	*Pt.*
1	0	4	,32	23	12	3	,36
2	1	0	,64	24	12	7	,68
3	1	4	,96	25	13	4	,—
4	2	1	,28	26	14	0	,32
5	2	5	,6	27	14	4	,64
6	3	1	,92	28	15	0	,96
7	3	6	,24	29	15	5	,28
8	4	2	,56	30	16	1	,6
9	4	6	,88	31	16	5	,92
10	5	3	,2	32	17	2	,24
11	5	7	,52	33	17	6	,56
12	6	3	,84	34	18	2	,88
13	7	0	,16	35	18	7	,2
14	7	4	,48	36	19	3	,52
15	8	0	,8	37	19	7	,84
16	8	5	,12	38	20	4	,16
17	9	1	,44	39	21	0	,48
18	9	5	,76	40	21	4	,8
19	10	2	,08	41	22	0	,12
20	10	6	,4	42	22	5	,44
21	11	2	,72	43	23	1	,76
22	11	7	,04	44	23	6	,08

Nº.	G.	P.	Pt.	Nº.	G.	P.	Pt.	Nº.	G.	P.	Pt.
45	24	2	,4	64	34	4	,48	83	44	6	,56
46	24	6	,72	65	35	0	,8	84	45	2	,88
47	25	3	,04	66	35	5	,12	85	45	7	,2
48	25	7	,36	67	36	1	,44	86	46	3	,52
49	26	3	,68	68	36	5	,76	87	46	7	,84
50	27	0	,—	69	37	2	,08	88	47	4	,16
51	27	4	,32	70	37	6	,4	89	48	0	,48
52	28	0	,64	71	38	2	,72	90	48	4	,8
53	28	4	,96	72	38	7	,04	91	49	0	,12
54	29	1	,28	73	39	3	,36	92	49	5	,44
55	29	5	,6	74	39	7	,68	93	50	1	,76
56	30	1	,92	75	40	4	,—	94	50	6	,08
57	30	6	,24	76	41	0	,32	95	51	2	,4
58	31	2	,56	77	41	4	,64	96	51	6	,72
59	31	6	,88	78	42	0	,96	97	52	3	,04
60	32	3	,2	79	42	5	,28	98	52	7	,36
61	32	7	,52	80	43	1	,6	99	53	3	,68
62	33	3	,64	81	43	5	,92				
63	34	0	,16	82	44	2	,24				

Table I. Corn Measure, *one Bushel the Integer.*

Nº.	G.	P.	Pt.	Nº.	G.	P.	Pt.	Nº.	G	P.	Pt.
1	—	—	,64	16	1	2	,24	31	2	3	,84
2	—	1	,28	17	1	2	,88	32	2	4	,48
3	—	1	,92	18	1	3	,52	33	2	5	,12
4	—	2	,56	19	1	4	,16	34	2	5	,76
5	—	3	,2	20	1	4	,8	35	2	6	,4
6	—	3	,84	21	1	5	,44	36	2	7	,04
7	—	4	,48	22	1	6	,08	37	2	7	,68
8	—	5	,12	23	1	6	,72	38	3	0	,32
9	—	5	,76	24	1	7	,36	39	3	0	,96
10	—	6	,4	25	2	0	,—	40	3	1	,6
11	—	7	,04	26	2	0	,64	41	3	2	,24
12	—	7	,68	27	2	1	,28	42	3	2	,88
13	1	0	,32	28	2	1	,92	43	3	3	,52
14	1	0	,96	29	2	2	,56	44	3	4	,16
15	1	1	,6	30	2	3	,2	45	3	4	,8

Nº.

N°	G.	P.	Pt.	N°	G.	P.	Pt.	N°	G.	P.	Pt.
46	3	5	,44	64	5	0	,96	82	6	4	,48
47	3	6	,08	65	5	1	,6	83	6	5	,12
48	3	6	,72	66	5	2	,24	84	6	5	,76
49	3	7	,36	67	5	2	,88	85	6	6	,4
50	4	0	,—	68	5	3	,52	86	6	7	,04
51	4	0	,64	69	5	4	,16	87	6	7	,68
52	4	1	,28	70	5	4	,8	88	7	0	,32
53	4	1	,92	71	5	5	,44	89	7	0	,96
54	4	2	,56	72	5	6	,08	90	7	1	,6
55	4	3	,2	73	5	6	,72	91	7	2	,24
56	4	3	,84	74	5	7	,36	92	7	2	,88
57	4	4	,48	75	6	0	,—	93	7	3	,52
58	4	5	,12	76	6	0	,64	94	7	4	,16
59	4	5	,76	77	6	1	,28	95	7	4	,8
60	4	6	,4	78	6	1	,92	96	7	5	,44
61	4	7	,04	79	6	2	,56	97	7	6	,08
62	4	7	,68	80	6	3	,2	98	7	6	,72
63	5	0	,32	81	6	3	,84	99	7	7	,36

Table I.

N°.	P.	Pt.
1	—	,05
2	—	,1
3	—	,15
4	—	,2
5	—	,25
6	—	,3
7	—	,35
8	—	,4
9	—	,45
10	—	,51
20	1	,02
30	1	,53
40	2	,04

Table II. Corn Measure, *one Quarter the Integer.*

N°.	B.	G.	P.	Pt.	N°.	B.	G.	P.	Pt.
1	0	0	5	,12	14	1	0	7	,68
2	0	1	2	,24	15	1	1	4	,8
3	0	1	7	,36	16	1	2	1	,92
4	0	2	4	,48	17	1	2	7	,04
5	0	3	1	,6	18	1	3	4	,16
6	0	3	6	,72	19	1	4	1	,28
7	0	4	3	,84	20	1	4	6	,4
8	0	5	0	,96	21	1	5	3	,52
9	0	5	5	,08	22	1	6	0	,64
10	0	6	3	,2	23	1	6	5	,76
11	0	7	0	,32	24	1	7	2	,88
12	0	7	5	,44	25	2	0	0	,—
13	1	0	2	,56	26	2	0	5	,12

N°.	P.	Pt.
50	2	,55
60	3	,07
70	3	,58
80	4	,09
90	4	,6

N°.	B.	G.	P.	Pt.
27	2	1	2	,24
28	2	1	7	,36
29	2	2	4	,48
30	2	3	1	,6
31	2	3	6	,72
32	2	4	3	,84
33	2	5	0	,96
34	2	5	5	,08
35	2	6	3	,2
36	2	7	0	,32
37	2	7	5	,44
38	3	0	2	,56
39	3	0	7	,68
40	3	1	4	,8
41	3	2	1	,92
42	3	2	7	,04
43	3	3	4	,16
44	3	4	1	,28
45	3	4	6	,4
46	3	5	3	,52
47	3	6	0	,64
48	3	6	5	,76
49	3	7	2	,88
50	4	0	0	,—
51	4	0	5	,12
52	4	1	2	,24
53	4	1	7	,36
54	4	2	4	,48
55	4	3	1	,6
56	4	3	6	,72
57	4	4	3	,84
58	4	5	0	,96
59	4	5	5	,08
60	4	6	3	,2
61	4	7	0	,32
62	4	7	5	,44
63	5	0	2	,36

N°.	B.	G.	P.	Pt.
64	5	0	7	,68
65	5	1	4	,8
66	5	2	1	,92
67	5	2	7	,04
68	5	3	4	,16
69	5	4	1	,28
70	5	4	6	,4
71	5	5	3	,52
72	5	6	0	,64
73	5	6	5	,76
74	5	7	2	,88
75	6	0	0	,—
76	6	0	5	,12
77	6	1	2	,24
78	6	1	7	,36
79	6	2	4	,48
80	6	3	1	,6
81	6	3	6	,72
82	6	4	3	,84
83	6	5	0	,96
84	6	5	5	,08
85	6	6	3	,2
86	6	7	0	,32
87	6	7	5	,44
88	7	0	2	,56
89	7	0	7	,68
90	7	1	4	,8
91	7	2	1	,92
92	7	2	7	,04
93	7	3	4	,16
94	7	4	1	,28
95	7	4	6	,4
96	7	5	3	,52
97	7	6	0	,64
98	7	6	5	,76
99	7	7	2	,88

N°.

Table I. | Table II. { Wine Measure, *one Tun the Integer.*

Nº.	G.	Pts.
1	--	,025
2	--	,05
3	--	,075
4	--	,1
5	--	,126
6	--	,151
7	--	,176
8	--	,201
9	--	,226
10	--	,252
20	--	,504
30	--	,756
40	1	,008
50	1	,26
60	1	,512
70	1	,764
80	2	,016
90	2	,268

Nº.	P.	H.	T.	G.	Pt.
1	—	—	—	2	,52
2	—	—	—	5	,04
3	—	—	—	7	,56
4	—	—	—	10	,08
5	—	—	—	12	,6
6	—	—	—	15	,12
7	—	—	—	17	,64
8	—	—	—	20	,16
9	—	—	—	22	,68
10	—	—	—	25	,2
11	—	—	—	27	,72
12	—	—	—	30	,24
13	—	—	—	32	,76
14	—	—	—	35	,28
15	—	—	—	37	,8
16	—	—	—	40	,32
17	—	—	1	0	,84
18	—	—	1	3	,36
19	—	—	1	5	,88
20	—	—	1	8	,4
21	—	—	1	10	,92
22	—	—	1	13	,44
23	—	—	1	15	,96
24	—	—	1	18	,48
25	—	1	0	0	,
26	—	1	0	2	,52
27	—	1	0	5	,04
28	—	1	0	7	,56
29	—	1	0	10	,08
30	—	1	0	12	,6
31	—	1	0	15	,12
32	—	1	0	17	,64
33	—	1	0	20	,16
34	—	1	0	22	,68
35	—	1	0	25	,2

Nº.	P.	H.	T.	G.	Pt.
36	--	1	0	27	,72
37	--	1	0	30	,24
38	--	1	0	32	,76
39	--	1	0	35	,28
40	--	1	0	37	,8
41	--	1	0	40	,32
42	--	1	1	0	,84
43	--	1	1	3	,36
44	--	1	1	5	,88
45	--	1	1	8	,4
46	--	1	1	10	,92
47	--	1	1	13	,44
48	--	1	1	15	,96
49	--	1	1	18	,48
50	1	0	0	0	,—
51	1	0	0	2	,52
52	1	0	0	5	,04
53	1	0	0	7	,56
54	1	0	0	10	,08
55	1	0	0	12	,6
56	1	0	0	15	,12
57	1	0	0	17	,64
58	1	0	0	20	,16
59	1	0	0	22	,68
60	1	0	0	25	,2
61	1	0	0	27	,72
62	1	0	0	30	,24
63	1	0	0	32	,76
64	1	0	0	35	,28
65	1	0	0	37	,8
66	1	0	0	40	,32
67	1	0	1	0	,84
68	1	0	1	3	,36
69	1	0	1	5	,88
70	1	0	1	8	,4

N°.	P.	H.	T.	G.	Pt.	N°.	P.	H.	T.	G.	Pt.	N°.	P.	H.	T.	G.	Pt.
71	1	0	1	10	92	81	1	1	0	15	,12	91	1	1	0	40	,32
72	1	0	1	13	,44	82	1	1	0	17	,64	92	1	1	1	0	,84
73	1	0	1	15	,96	83	1	1	0	20	,16	93	1	1	1	3	,36
74	1	0	1	18	,48	84	1	1	0	22	,68	94	1	1	1	5	,88
75	1	1	0	0	,—	85	1	1	0	25	,2	95	1	1	1	8	,4
76	1	1	0	2	,52	86	1	1	0	27	,72	96	1	1	1	10	,92
77	1	1	0	5	,04	87	1	1	0	30	,24	97	1	1	1	13	,44
78	1	1	0	7	,56	88	1	1	0	32	,76	98	1	1	1	15	,96
79	1	1	0	10	,08	89	1	1	0	35	,28	99	1	1	1	18	,48
80	1	1	0	12	,6	90	1	1	0	37	,8						

Table I.

N°.	Q.	Pts.
1	- -	,004
2	- -	,009
3	- -	,014
4	- -	,019
5	- -	,024
6	- -	,028
7	- -	,033
8	- -	,038
9	- -	,043
10	- -	,048
20	- -	,096
30	- -	,144
40	- -	,192
50	- -	,24
60	- -	,288
70	- -	,336
80	- -	,384
90	- -	,432

Table II. { *One Foot, or one Shilling the Integer.*

N°.	In. P.	Q.	Pt.	N°.	In. P.	Q.	Pt.
1	—	—	,48	22	2	2	,56
2	—	—	,96	23	2	3	,04
3	—	1	,44	24	2	3	,52
4	—	1	,92	25	3	0	,—
5	—	2	,4	26	3	0	,48
6	—	2	,88	27	3	0	,96
7	—	3	,36	28	3	1	,44
8	—	3	,84	29	3	1	,92
9	1	0	,32	30	3	2	,4
10	1	0	,8	31	3	2	,88
11	1	1	,28	32	3	3	,36
12	1	1	,76	33	3	3	,84
13	1	2	,24	34	4	0	,32
14	1	2	,72	35	4	0	,8
15	1	3	,2	36	4	1	,28
16	1	3	,68	37	4	1	,76
17	2	0	,16	38	4	2	,24
18	2	0	,64	39	4	2	,72
19	2	1	,12	40	4	3	,2
20	2	1	,6	41	4	3	,68
21	2	2	,08	42	5	0	,16

Nº.	*In P.*	*Q.*	*Pt.*	Nº.	*In. P.*	*Q.*	*Pt.*	Nº.	*In. P.*	*Q.*	*Pt.*
43	5	0	,64	62	7	1	,76	81	9	2	,88
44	5	1	,12	63	7	2	,24	82	9	3	,36
45	5	1	,6	64	7	2	,72	83	9	3	,84
46	5	2	,08	65	7	3	,2	84	10	0	,32
47	5	2	,56	66	7	3	,68	85	10	0	,8
48	5	3	,04	67	8	0	,16	86	10	1	,28
49	5	3	,52	68	8	0	,64	87	10	1	,76
50	6	0	,—	69	8	1	,12	88	10	2	,24
51	6	0	,48	70	8	1	,6	89	10	2	,72
52	6	0	,96	71	8	2	,08	90	10	3	,2
53	6	1	,44	72	8	2	,56	91	10	3	,68
54	6	1	,92	73	8	3	,04	92	11	0	,16
55	6	2	,4	74	8	3	,52	93	11	0	,64
56	6	2	,88	75	9	0	,—	94	11	1	,12
57	6	3	,36	76	9	0	,48	95	11	1	,6
58	6	3	,84	77	9	0	,96	96	11	2	,08
59	7	0	,32	78	9	1	,44	97	11	2	,56
60	7	0	,8	79	9	1	,92	98	11	3	,04
61	7	1	,28	80	9	2	,4	99	11	3	,52

Table I.

Nº.	*P.*	*Pt.*
1	—	,03
2	—	,07
3	—	,11
4	—	,15
5	—	,19
6	—	,22
7	—	,26
8	—	,3
9	—	,34
10	—	,38
20	—	,76
30	1	,15
40	1	,53

Table II. { Liquid common Meaſ. *a Hogſh. of* 51 *Gall. Int.*

Nº.	*G.*	*P.*	*Pt.*	Nº.	*G.*	*P.*	*Pt.*
1	—	3	,84	14	6	5	,76
2	—	7	,68	15	7	1	,6
3	1	3	,52	16	7	5	,44
4	1	7	,36	17	8	1	,28
5	2	3	,2	18	8	5	,12
6	2	7	,04	19	9	0	,96
7	3	2	,88	20	9	4	,8
8	3	6	,72	21	10	0	,64
9	4	2	,56	22	10	4	,48
10	4	6	,4	23	11	0	,32
11	5	2	,24	24	11	4	,16
12	5	6	,08	25	12	0	,—
13	6	1	,92	26	12	3	,84

Nº.

N°	P.	Pt.
50	1	,92
60	2	,3
70	2	,68
80	3	,07
90	3	,45

N°	G.	P.	Pt.
27	12	7	,68
28	13	3	,52
29	13	7	,36
30	14	3	,2
31	14	7	,04
32	15	2	,88
33	15	6	,72
34	16	2	,56
35	16	6	,4
36	17	2	,24
37	17	6	,08
38	18	1	,92
39	18	5	,76
40	19	1	,6
41	19	5	,44
42	20	1	,28
43	20	5	,12
44	21	0	,96
45	21	4	,8
46	22	0	,64
47	22	4	,48
48	23	0	,32
49	23	4	,16
50	24	0	,—
51	24	3	,84
52	24	7	,68
53	25	3	,52
54	25	7	,36
55	26	3	,2
56	26	7	,04
57	27	2	,88
58	27	6	,72
59	28	2	,56
60	28	6	,4
61	29	2	,24
62	29	6	,08
63	30	1	,92

N°	G.	P.	Pt.
64	30	5	,76
65	31	1	,6
66	31	5	,44
67	32	1	,28
68	32	5	,12
69	33	0	,96
70	33	4	,8
71	34	0	,64
72	34	4	,48
73	35	0	,32
74	35	4	,16
75	36	0	,—
76	36	3	,84
77	36	7	,68
78	37	3	,52
79	37	7	,36
80	38	3	,2
81	38	7	,04
82	39	2	,88
83	39	6	,72
84	40	2	,56
85	40	6	,4
86	41	2	,24
87	41	6	,08
88	42	1	,92
89	42	5	,76
90	43	1	,6
91	43	5	,44
92	44	1	,28
93	44	5	,12
94	45	0	,96
95	45	4	,8
96	46	0	,64
97	46	4	,48
98	47	0	,32
99	47	4	,16

Table I. | Table II. { Corn Measure, *a* Tun *or* Load *the Integer*.

N°.	G.	Pt.
1	—	,03
2	—	,06
3	—	,09
4	—	,12
5	—	,16
6	—	,19
7	—	,22
8	—	,25
9	—	,28
10	—	,32
20	—	,64
30	—	,96
40	1	,28
50	1	,6
60	1	,92
70	2	,24
80	2	,56
90	2	,88

N°.	Q.	B.	G.	Pt.
1	--	--	3	,2
2	--	--	6	,4
3	-	1	1	,6
4	--	1	4	,8
5	--	2	0	,—
6	--	2	3	,2
7	--	2	6	,4
8	--	3	1	,6
9	--	3	4	,8
10	--	4	0	,—
11	--	4	3	,2
12	--	4	6	,4
13	--	5	1	,6
14	--	5	4	,8
15	--	6	0	,—
16	--	6	3	,2
17	--	6	6	,4
18	--	7	1	,6
19	--	7	4	,8
20	1	0	0	,—
21	1	0	3	,2
22	1	0	6	,4
23	1	1	1	,6
24	1	1	4	,8
25	1	2	0	,—
26	1	2	3	,2
27	1	2	6	,4
28	1	3	1	,6
29	1	3	4	,8
30	1	4	0	,—
31	1	4	3	,2
32	1	4	6	,4
33	1	5	1	,6
34	1	5	4	,8
35	1	6	0	,—
36	1	6	3	,2
37	1	6	6	,4
38	1	7	1	,6
39	1	7	4	,8
40	2	0	0	,—
41	2	0	3	,2
42	2	0	6	,4
43	2	1	1	,6
44	2	1	4	,8
45	2	2	0	,—
46	2	2	3	,2
47	2	2	6	,4
48	2	3	1	,6
49	2	3	4	,8
50	2	4	0	,—
51	2	4	3	,2
52	2	4	6	,4
53	2	5	1	,6
54	2	5	4	,8
55	2	6	0	,—
56	2	6	3	,2
57	2	6	6	,4
58	2	7	1	,6
59	2	7	4	,8
60	3	0	0	,—
61	3	0	3	,2
62	3	0	6	,4
63	3	1	1	,6
64	3	1	4	,8
65	3	2	0	,—
66	3	2	3	,2
67	3	2	6	,4
68	3	3	1	,6

N°.

N°.	Q.	B.	G.	Pt.	N°.	Q.	B.	G.	Pt.	N°.	Q.	B.	G.	Pt.
69	3	3	4	,8	80	4	0	0	,—	91	4	4	3	,2
70	3	4	0	,—	81	4	0	3	,2	92	4	4	6	,4
71	3	4	3	,2	82	4	0	6	,4	93	4	5	1	,6
72	3	4	6	,4	83	4	1	1	,6	94	4	5	4	,8
73	3	5	1	,6	84	4	1	4	,8	95	4	6	0	,—
74	3	5	4	,8	85	4	2	0	,—	96	4	6	3	,2
75	3	6	0	,—	86	4	2	3	,2	97	4	6	6	,4
76	3	6	3	,2	87	4	2	6	,4	98	4	7	1	,6
77	3	6	6	,4	88	4	3	1	,6	99	4	7	4	,8
78	3	7	1	,6	89	4	3	4	,8					
79	3	7	4	,8	90	4	4	0	,—					

Table I. | Table II. { Long Measure, *one Pole or Rod the Integer.*

N°.	In.	Pts.	N°.	Yd.	Ft.	In.	Pt.	N°.	Yd	Ft	In	Pt.
1	—	,019	1	—	—	1	,98	23	1	0	9	,54
2	—	,039	2	—	—	3	,96	24	1	0	11	,52
3	—	,059	3	—	—	5	,94	25	1	1	1	,5
4	—	,079	4	—	—	7	,92	26	1	1	3	,48
5	—	,099	5	—	—	9	,9	27	1	1	5	,46
6	—	,118	6	—	—	11	,88	28	1	1	7	,44
7	—	,138	7	—	1	1	,86	29	1	1	9	,42
8	—	,158	8	—	1	3	,84	30	1	1	11	,4
9	—	,178	9	—	1	5	,82	31	1	2	1	,38
10	—	,198	10	—	1	7	,8	32	1	2	3	,36
20	—	,396	11	—	1	9	,78	33	1	2	5	,34
30	—	,594	12	—	1	11	,76	34	1	2	7	,32
40	—	,792	13	—	2	1	,74	35	1	2	9	,3
50	—	,99	14	—	2	3	,72	36	1	2	11	,28
60	1	,188	15	—	2	5	,7	37	2	0	1	,26
70	1	,386	16	—	2	7	,68	38	2	0	3	,24
80	1	,584	17	—	2	9	,66	39	2	0	5	,22
90	1	,782	18	—	2	11	,64	40	2	0	7	,2
			19	1	0	1	,62	41	2	0	9	,18
			20	1	0	3	,6	42	2	0	11	,16
			21	1	0	5	,58	43	2	1	1	,14
			22	1	0	7	,56	44	2	1	3	,12

N°.

N°	*Yd*	*Ft*	*In*	*Pt.*	N°	*Yd*	*Ft*	*In*	*Pt.*	N°	*Yd*	*Ft*	*In*	*Pt.*
45	2	1	5	,1	64	3	1	6	,72	83	4	1	8	,34
46	2	1	7	,08	65	3	1	8	,7	84	4	1	10	,32
47	2	1	9	,06	66	3	1	10	,68	85	4	2	0	,3
48	2	1	11	,04	67	3	2	0	,66	86	4	2	2	,26
49	2	2	1	,02	68	3	2	2	,64	87	4	2	4	,26
50	2	2	3	,—	69	3	2	4	,62	88	4	2	6	,24
51	2	2	4	,98	70	3	2	6	,6	89	4	2	8	,22
52	2	2	6	,96	71	3	2	8	,58	90	4	2	10	,2
53	2	2	8	,94	72	3	2	10	,56	91	5	0	0	,18
54	2	2	10	,92	73	4	0	0	,54	92	5	0	2	,16
55	3	0	0	,9	74	4	0	2	,52	93	5	0	4	,14
56	3	0	2	,88	75	4	0	4	,5	94	5	0	6	,12
57	3	0	4	,86	76	4	0	6	,48	95	5	0	8	,1
58	3	0	6	,84	77	4	0	8	,46	96	5	0	10	,08
59	3	0	8	,82	78	4	0	10	,44	97	5	1	0	,06
60	3	0	10	,8	79	4	1	0	,42	98	5	1	2	,04
61	3	1	0	,78	80	4	1	2	,4	99	5	1	4	,02
62	3	1	2	,76	81	4	1	4	,38					
63	3	1	4	,74	82	4	1	6	,36					

Table I. | Table II. { Long Measure, *one Yard the Integer.*

N°.	*Q.*	*Pt.*
1	—	,014
2	—	,028
3	—	,042
4	—	,057
5	—	,072
6	—	,086
7	—	,1
8	—	,115
9	—	,129
10	—	,144
20	—	,288
30	—	,432
40	—	,576

N°.	*Ft*	*In*	*Q.*	*Pt.*	N°.	*Ft*	*In*	*Q.*	*Pt.*
1	—	—	1	,44	14	—	5	0	,16
2	—	—	2	,88	15	—	5	1	,6
3	—	1	0	,32	16	—	5	3	,04
4	—	1	1	,76	17	—	6	0	,48
5	—	1	3	,2	18	—	6	1	,92
6	—	2	0	,64	19	—	6	3	,36
7	—	2	2	,08	20	—	7	0	,8
8	—	2	3	,52	21	—	7	2	,24
9	—	3	0	,96	22	—	7	3	,68
10	—	3	2	,4	23	—	8	1	,12
11	—	3	3	,84	24	—	8	2	,56
12	—	4	1	,28	25	—	9	0	,—
13	—	4	2	,72	26	—	9	1	,44

N°	Q.	Pt.
50	—	,72
60	—	,864
70	—	,908
80	1	,152
90	1	,296

N°.	Ft	In.	Q.	Pt.
27	—	9	2	,88
28	—	10	0	,32
29	—	10	1	,76
30	—	10	3	,2
31	—	11	0	,64
32	—	11	2	,08
33	—	11	3	,52
34	1	0	0	,96
35	1	0	2	,4
36	1	0	3	,84
37	1	1	1	,28
38	1	1	2	,72
39	1	2	0	,16
40	1	2	1	,6
41	1	2	3	,04
42	1	3	0	,48
43	1	3	1	,92
44	1	3	3	,36
45	1	4	0	,8
46	1	4	2	,24
47	1	4	3	,68
48	1	5	1	,12
49	1	5	2	,56
50	1	6	0	,—
51	1	6	1	,44
52	1	6	2	,88
53	1	7	0	,32
54	1	7	1	,76
55	1	7	3	,2
56	1	8	0	,64
57	1	8	2	,08
58	1	8	3	,52
59	1	9	0	,96
60	1	9	2	,4
61	1	9	3	,84
62	1	10	1	,28
63	1	10	2	,72

N°.	Ft	In	Q.	Pt.
64	1	11	0	,16
65	1	11	1	,6
66	1	11	3	,04
67	2	0	0	,48
68	2	0	1	,92
69	2	0	3	,36
70	2	1	0	,8
71	2	1	2	,24
72	2	1	3	,68
73	2	2	1	,12
74	2	2	2	,56
75	2	3	0	,—
76	2	3	1	,44
77	2	3	2	,88
78	2	4	0	,32
79	2	4	1	,76
80	2	4	3	,2
81	2	5	0	,64
82	2	5	2	,08
83	2	5	3	,52
84	2	6	0	,96
85	2	6	2	,4
86	2	6	3	,84
87	2	7	1	,28
88	2	7	2	,72
89	2	8	0	,16
90	2	8	1	,6
91	2	8	3	,04
92	2	9	0	,48
93	2	9	1	,92
94	2	9	3	,36
95	2	10	0	,8
96	2	10	2	,24
97	2	10	3	,68
98	2	11	1	,12
99	2	11	2	,56

Table I.

Nº.	In.	Pts.
1	- -	,063
2	- -	,126
3	- -	,189
4	- -	,252
5	- -	,315
6	- -	,378
7	- -	,441
8	- -	,504
9	- -	,567
10	- -	,633
20	1	,266
30	1	,899
40	2	,532
50	3	,165
60	3	,789
70	4	,431
80	5	,064
90	5	,697

Table II. { Long Meaſure, *one Mile the Integer.*

Nº.	R.	Yd	Ft	In	Pt.	Nº.	R.	Yd	Ft	In	Pt.
1	—	—	—	6	,33	36	1	0	2	5	,88
2	—	—	1	0	,66	37	1	1	0	0	,21
3	—	—	1	6	,99	38	1	1	0	6	,54
4	—	—	2	1	,32	39	1	1	1	0	,87
5	—	—	2	7	,65	40	1	1	1	7	,2
6	—	1	0	1	,98	41	1	1	2	1	,53
7	—	1	0	8	,31	42	1	1	2	7	,86
8	—	1	1	2	,64	43	1	2	0	2	,19
9	—	1	1	8	,97	44	1	2	0	8	,52
10	—	1	2	3	,3	45	1	2	1	2	,85
11	—	1	2	9	,63	46	1	2	1	9	,18
12	—	2	0	3	,96	47	1	2	2	3	,51
13	—	2	0	10	,29	48	1	2	2	9	,84
14	—	2	1	4	,62	49	1	3	0	4	,17
15	—	2	1	10	,95	50	1	3	0	10	,5
16	—	2	2	5	,28	51	1	3	1	4	,83
17	—	2	2	11	,61	52	1	3	1	11	,16
18	—	3	0	5	,94	53	1	3	2	5	,49
19	—	3	1	0	,27	54	1	3	2	11	,82
20	—	3	1	6	,6	55	1	4	0	6	,15
21	—	3	2	0	,93	56	1	4	1	0	,48
22	—	3	2	7	,26	57	1	4	1	6	,81
23	—	4	0	1	,59	58	1	4	2	1	,14
24	—	4	0	7	,92	59	1	4	2	7	,47
25	—	4	1	2	,25	60	1	5	0	1	,8
26	—	4	1	8	,58	61	1	5	0	8	,13
27	—	4	2	2	,91	62	1	5	1	2	,46
28	—	4	2	9	,24	63	2	0	0	2	,79
29	—	5	0	3	,57	64	2	0	0	9	,12
30	—	5	0	9	,9	65	2	0	1	3	,45
31	0	5	1	4	,23	66	2	0	1	9	,78
32	1	0	0	4	,56	67	2	0	2	4	,11
33	1	0	0	10	,89	68	2	0	2	10	,44
34	1	0	1	5	,22	69	2	1	0	4	,77
35	1	0	1	11	,55	70	2	1	0	11	,1

N°.	R.	Yd	Ft	In	Pt.	N°.	R.	Yd	Ft	In	Pt.	N°.	R.	Yd	Ft	I	Pt.
71	2	1	1	5	,43	81	2	3	0	8	,73	91	2	5	0	0	,03
72	2	1	1	11	,76	82	2	3	1	3	,06	92	2	5	0	6	,36
73	2	1	2	6	,09	83	2	3	1	9	,39	93	2	5	1	0	,69
74	2	2	0	0	,42	84	2	3	2	3	,72	94	3	0	0	1	,02
75	2	2	0	6	,75	85	2	3	2	10	,05	95	3	0	0	7	,35
76	2	2	1	1	,08	86	2	4	0	4	,38	96	3	0	1	1	,68
77	2	2	1	7	,41	87	2	4	0	10	,71	97	3	0	1	8	,01
78	2	2	2	1	74	88	2	4	1	5	,04	98	3	0	2	2	,34
79	2	2	2	8	,07	89	2	4	1	11	,37	99	3	0	2	8	,67
80	2	3	0	2	,4	90	2	4	2	5	,7						

Table III. Long Meaſure, *one Mile the Integer*.

N°.	Fg	R.	Yd	Ft	In.	Pt.	N°.	Fg	R.	Yd	Ft	In.	Pt.
1	0	3	1	0	3	,6	23	1	33	3	0	10	,8
2	- -	6	2	0	7	,2	24	1	36	4	1	2	,4
3	- -	9	3	0	10	,8	25	2	0	0	0	0	,—
4	- -	12	4	1	2	,4	26	2	3	1	0	3	,6
5	- -	16	0	0	0	,—	27	2	6	2	0	7	,2
6	- -	19	1	0	3	,6	28	2	9	3	0	10	,8
7	- -	22	2	0	7	,2	29	2	12	4	1	2	,4
8	- -	25	3	0	10	,8	30	2	16	0	0	0	,—
9	- -	28	4	1	2	,4	31	2	19	1	0	3	,6
10	- -	32	0	0	0	,—	32	2	22	2	0	7	,2
11	- -	35	1	0	3	,6	33	2	25	3	0	10	,8
12	- -	38	2	0	7	,2	34	2	28	4	1	2	,4
13	1	1	3	0	10	,8	35	2	32	0	0	0	,—
14	1	4	4	1	2	,4	36	2	35	1	0	3	,6
15	1	8	0	0	0	,—	37	2	38	2	0	7	,2
16	1	11	1	0	3	,6	38	3	1	3	0	10	,8
17	1	14	2	0	7	,2	39	3	4	4	1	2	,4
18	1	17	3	0	10	,8	40	3	8	0	0	0	,—
19	1	20	4	1	2	,4	41	3	11	1	0	3	,6
20	1	24	0	0	0	,—	42	3	14	2	0	7	,2
21	1	27	1	0	3	,6	43	3	17	3	0	10	,8
22	1	30	2	0	7	,2	44	3	20	4	1	2	,4

Nº.	*Fg*	*R.*	*Yd*	*Ft*	*In.*	*Pt.*	Nº.	*Fg*	*R.*	*Yd*	*Ft*	*In.*	*Pt.*
45	3	24	0	0	0	,—	73	5	33	5	0	10	,8
46	3	27	1	0	3	,6	74	5	36	4	1	2	,4
47	3	30	2	0	7	,2	75	6	0	0	0	0	,—
48	3	33	3	0	10	,8	76	6	3	1	0	3	,6
49	3	36	4	1	2	,4	77	6	6	2	0	7	,2
50	4	0	0	0	0	,—	78	6	9	3	0	10	,8
51	4	3	1	0	3	,6	79	6	12	4	1	2	,4
52	4	6	2	0	7	,2	80	6	16	0	0	0	,—
53	4	9	3	0	10	,8	81	6	19	1	0	3	,6
54	4	12	4	1	2	,4	82	6	22	2	0	7	,2
55	4	16	0	0	0	,—	83	6	25	3	0	10	,8
56	4	19	1	0	3	,6	84	6	28	4	1	2	,4
57	4	22	2	0	7	,2	85	6	32	0	0	0	,—
58	4	25	3	0	10	,8	86	6	35	1	0	3	,6
59	4	28	4	1	2	,4	87	6	38	2	0	7	,2
60	4	32	0	0	0	,—	88	7	1	3	0	10	,8
61	4	35	1	0	3	,6	89	7	4	4	1	2	,4
62	4	38	2	0	7	,2	90	7	8	0	0	0	,—
63	5	1	3	0	10	,8	91	7	11	1	0	3	,6
64	5	4	4	1	2	,4	92	7	14	2	0	7	,2
65	5	8	0	0	0	,—	93	7	17	3	0	10	,8
66	5	11	1	0	3	,6	94	7	20	4	1	2	,4
67	5	14	2	0	7	,2	95	7	24	0	0	0	,—
68	5	17	3	0	10	,8	96	7	27	1	0	3	,6
69	5	20	4	1	2	,4	97	7	30	2	0	7	,2
70	5	24	0	0	0	,—	98	7	33	3	0	10	,8
71	5	27	1	0	3	,6	99	7	36	4	1	2	,4
72	5	30	2	0	7	,2							

Table I. | Table II. { Square Measure, *one Mile Square the Integer.*

Nº.	*Pq*	*Pts.*
1	—	,102
2	—	,204
3	—	,307
4	—	,409
5	—	,512

Nº.	*Aq*	*Rq*	*Pq*	*Pt.*	Nº.	*Aq*	*Rq*	*Pq*	*Pt.*
1	—	—	10	,24	6	—	1	21	·44
2	—	—	20	,48	7	—	1	31	,68
3	—	—	30	,72	8	—	2	1	,92
4	—	1	0	,96	9	—	2	12	,16
5	—	1	11	,2	10	—	2	22	,4

Nº.

N°.	Pq	Pts.
6	- -	,614
7	- -	,716
8	- -	,819
9	- -	,921
10	1	,02
20	2	,04
30	3	,07
40	4	,09
50	5	,12
60	6	,14
70	7	,16
80	8	,19
90	9	,21

N°.	Aq	Rq	Pq	Pt.
11	- -	2	32	,64
12	- -	3	2	,88
13	- -	3	13	,12
14	- -	3	23	,36
15	- -	3	33	,6
16	1	0	3	,84
17	1	0	14	,08
18	1	0	24	,32
19	1	0	34	,56
20	1	1	4	,8
21	1	1	15	,04
22	1	1	25	,28
23	1	1	35	,52
24	1	2	5	,76
25	1	2	16	,—
26	1	2	26	,24
27	1	2	36	,48
28	1	3	6	,72
29	1	3	16	,96
30	1	3	27	,2
31	1	3	37	,44
32	2	0	7	,68
33	2	0	17	,92
34	2	0	28	,16
35	2	0	38	,4
36	2	1	8	,64
37	2	1	19	,88
38	2	1	29	,12
39	2	1	39	,36
40	2	2	9	,6
41	2	2	19	,84
42	2	2	29	,08
43	2	2	39	,32
44	2	3	10	,56
45	2	3	20	,8
46	2	3	31	,04
47	3	0	1	,28
48	3	0	11	,52
49	3	0	21	,76
50	3	0	32	,—

N°.	Aq	Rq	Pq	Pt.
51	3	1	2	,24
52	3	1	12	,48
53	3	1	22	,72
54	3	1	32	,96
55	3	2	3	,2
56	3	2	13	,44
57	3	2	23	,68
58	3	2	33	,92
59	3	3	4	,16
60	3	3	14	,4
61	3	3	24	,64
62	3	3	34	,88
63	4	0	5	,12
64	4	0	15	,36
65	4	0	25	,6
66	4	0	35	,84
67	4	1	6	,08
68	4	1	16	,32
69	4	1	26	,56
70	4	1	36	,8
71	4	2	7	,04
72	4	2	17	,28
73	4	2	27	,52
74	4	2	37	,76
75	4	3	8	,—
76	4	3	18	,24
77	4	3	28	,48
78	4	3	38	,72
79	5	0	8	,96
80	5	0	19	,2
81	5	0	29	,44
82	5	0	39	,68
83	5	1	9	,92
84	5	1	20	,16
85	5	1	30	,4
86	5	2	0	,64
87	5	2	10	,88
88	5	2	21	,12
89	5	2	31	,36
90	5	3	1	,6

N°.	Aq	Rq	Pq	Pt.	N°.	Aq	Rq	Pq	Pt.	N°.	Aq	Rq	Pq	Pt.
91	5	3	11	,84	94	6	0	2	,56	97	6	0	33	,28
92	5	3	22	,08	95	6	0	12	,8	98	6	1	3	,52
93	5	3	32	,32	96	6	0	23	,04	99	6	1	13	,76

Table III. Square Meafure, *one Mile Square the Integ.*

N°.	Aq.	Rq	Pq	N°.	Aq.	Rq	Pq	N°.	Aq.	Rq	Pq
1	6	1	24	34	217	2	16	67	429	3	8
2	12	3	8	35	224	0	--,	68	435	0	32
3	19	0	32	36	230	1	24	69	441	2	16
4	25	2	16	37	236	3	8	70	448	0	--,
5	32	0	--,	38	243	0	32	71	454	1	24
6	38	1	24	39	249	2	16	72	460	3	8
7	44	3	8	40	256	0	--,	73	467	0	32
8	51	0	32	41	262	1	24	74	473	2	16
9	57	2	16	42	268	3	8	75	480	0	--,
10	64	0	--,	43	275	0	32	76	486	1	24
11	70	1	24	44	281	2	16	77	492	3	8
12	76	3	8	45	288	0	--,	78	499	0	32
13	83	0	32	46	294	1	24	79	505	2	16
14	89	2	16	47	300	3	8	80	512	0	--,
15	96	0	--,	48	307	0	32	81	518	1	24
16	102	1	24	49	313	2	16	82	524	3	8
17	108	3	8	50	320	0	--,	83	531	0	32
18	115	0	32	51	326	1	24	84	537	2	16
19	121	2	16	52	332	3	8	85	544	0	--,
20	128	0	--,	53	339	0	32	86	550	1	24
21	134	1	24	54	345	2	16	87	556	3	8
22	140	3	8	55	352	0	--,	88	563	0	32
23	147	0	32	56	358	1	24	89	569	2	16
24	153	2	16	57	364	3	8	90	576	0	--,
25	160	0	--,	58	371	0	32	91	582	1	24
26	166	1	24	59	377	2	16	92	588	3	8
27	172	3	8	60	384	0	--,	93	595	0	32
28	179	0	32	61	390	1	24	94	601	2	16
29	185	2	16	62	396	3	8	95	60	0	--,
30	192	0	--,	63	403	0	32	96	614	1	24
31	298	1	24	64	409	2	16	97	620	3	8
32	204	3	8	65	416	0	--,	98	627	0	32
33	211	0	32	66	422	1	24	99	633	2	16

Table I.

Nº.	Fq	Pt.
1	—	,02
2	—	,05
3	—	,08
4	—	,1
5	—	,13
6	—	,16
7	—	,18
8	—	,21
9	—	,24
10	—	,27
20	—	,54
30	—	,81
40	1	,08
50	1	,35
60	1	,62
70	1	,89
80	2	,16
90	2	,43

Table II. { Square Measure, one *Rod square the Integer.*

Nº.	Yq.	Fq	Pt	Nº.	Yq	Fq	Pt.
1	—	2	,72	36	10	7	,92
2	—	5	,44	37	11	1	,64
3	—	8	,16	38	11	4	,36
4	1	1	,88	39	11	7	,08
5	1	4	,6	40	12	0	,8
6	1	7	,32	41	12	3	,52
7	2	1	,04	42	12	6	,24
8	2	3	,76	43	12	8	,96
9	2	6	,48	44	13	2	,68
10	3	0	,2	45	13	5	,4
11	3	2	,92	46	13	8	,12
12	3	5	,64	47	14	1	,84
13	3	8	,36	48	14	4	,56
14	4	2	,08	49	14	7	,28
15	4	4	,8	50	15	1	,—
16	4	7	,52	51	15	3	,72
17	5	1	,24	52	15	6	,44
18	5	3	,96	53	16	0	,16
19	5	6	,68	54	16	2	,88
20	6	0	,4	55	16	5	,6
21	6	3	,12	56	16	8	,32
22	6	5	,84	57	17	2	,04
23	6	8	,56	58	17	4	,76
24	7	2	,28	59	17	7	,48
25	7	5	,—	60	18	1	,2
26	7	7	,72	61	18	3	,92
27	8	1	,44	62	18	6	,64
28	8	4	,16	63	19	0	,36
29	8	6	,88	64	19	3	,08
30	9	0	,6	65	19	5	,8
31	9	3	,32	66	19	8	,52
32	9	6	,04	67	20	2	,24
33	9	8	,76	68	20	4	,96
34	10	2	,48	69	20	7	,68
35	10	5	,2	70	21	1	,4

N°.	Yq	Fq	Pt.	N°.	Yq	Fq	Pt.	N°.	Yq	Fq	Pt.
71	21	4	,12	81	24	4	,32	91	27	4	,52
72	21	6	,84	82	24	7	,04	92	27	7	,24
73	22	0	,56	83	25	0	,76	93	28	0	,96
74	22	3	,28	84	25	3	,48	94	28	3	,68
75	22	6	,—	85	25	6	,2	95	28	6	,4
76	22	8	,72	86	25	8	,92	96	29	0	,12
77	23	2	,44	87	26	2	,64	97	29	2	,84
78	23	5	,16	88	26	5	,36	98	29	5	,56
79	23	7	,88	89	26	8	,08	99	29	8	,28
80	23	1	,6	90	27	1	,8				

Table I. | Table II. { Square Measure, *one Yard Square the Integer.*

N°.	Iq.	Pt.
1	--	,12
2	--	,25
3	--	,38
4	--	,51
5	--	,64
6	--	,77
7	--	,9
8	1	,03
9	1	,16
10	1	,29
20	2	,59
30	3	,88
40	5	,17
50	6	,46
60	7	,75
70	9	,04
80	10	,33
90	11	,62

N°.	Fq	Iq.	Pt.	N°.	Fq	Iq.	Pt.
1	--	12	,96	23	2	10	,08
2	--	25	,92	24	2	23	,04
3	--	38	,88	25	2	36	,—
4	--	51	,84	26	2	48	,96
5	--	64	,8	27	2	61	,92
6	--	77	,76	28	2	74	,88
7	--	90	,72	29	2	87	,84
8	--	103	,68	30	2	100	,8
9	--	116	,64	31	2	113	,76
10	--	129	,6	32	2	126	,72
11	--	142	,56	33	2	139	,68
12	1	11	,52	34	3	8	,64
13	1	24	,48	35	3	21	,6
14	1	37	,44	36	3	34	,56
15	1	50	,4	37	3	47	,52
16	1	63	,36	38	3	60	,48
17	1	76	,32	39	3	73	,44
18	1	89	,28	40	3	86	,4
19	1	102	,24	41	3	99	,36
20	1	115	,2	42	3	112	,32
21	1	128	,16	43	3	125	,28
22	1	141	,12	44	3	138	,24

Nº	Fq	Iq.	Pt.	Nº	Fq	Iq.	Pt.	Nº	Fq	Iq.	Pt.
45	4	7	,2	64	5	109	,44	83	7	67	,68
46	4	20	,16	65	5	122	,4	84	7	80	,64
47	4	33	,12	66	5	135	,36	85	7	93	,6
48	4	46	,08	67	6	4	,32	86	7	106	,56
49	4	59	,04	68	6	17	,28	87	7	119	,52
50	4	72	,—	69	6	30	,24	88	7	132	,48
51	4	84	,96	70	6	43	,2	89	8	1	,44
52	4	97	,92	71	6	56	,16	90	8	14	,4
53	4	110	,88	72	6	69	,12	91	8	27	,36
54	4	123	,84	73	6	82	,08	92	8	40	,32
55	4	136	,8	74	6	95	,04	93	8	53	,28
56	5	5	,76	75	6	108	,—	94	8	66	,24
57	5	18	,72	76	6	120	,96	95	8	79	,2
58	5	31	,68	77	6	133	,92	96	8	92	,16
59	5	44	,64	78	7	2	,88	97	8	105	,12
60	5	57	,6	79	7	15	,84	98	8	118	,08
61	5	70	,56	80	7	28	,8	99	8	131	,04
62	5	83	,52	81	7	41	,76				
63	5	96	,48	82	7	54	,72				

Table I. | Table II. { Square Measure, *one Foot Square the Integer.*

Nº	Iq.	q.q	Pt.	Nº	Iq.	q.q	Pt.	Nº	Iq.	q.q	Pt.
1	—	—	,23	1	1	7	,04	14	20	2	,56
2	—	—	,46	2	2	14	,08	15	21	9	,6
3	—	—	,69	3	4	5	,12	16	23	0	,64
4	—	—	,92	4	5	12	,16	17	24	7	,68
5	—	1	,15	5	7	3	,2	18	25	14	,72
6	—	1	,38	6	8	10	,24	19	27	5	,76
7	—	1	,61	7	10	1	,28	20	28	12	,8
8	—	1	,84	8	11	8	,32	21	30	3	,84
9	—	2	,07	9	12	15	,36	22	31	10	,88
10	—	2	,3	10	14	6	,4	23	33	1	,92
20	—	4	,6	11	15	13	,44	24	34	8	,96
30	—	6	,9	12	17	4	,48	25	36	0	,—
40	—	9	2,	13	18	11	,52	26	37	7	,84

N°.	lq.	q.q	Pt.
50	—	11	,5
60	—	13	,8
70	1	0	,1
80	1	2	,4
90	1	4	,7

N°.	lq.	q.q	Pt.
27	38	14	,08
28	40	5	,12
29	41	12	,16
30	43	3	,2
31	44	10	,24
32	46	1	,28
33	47	8	,32
34	48	15	,36
35	50	6	,4
36	51	13	,44
37	53	4	,48
38	54	11	,52
39	56	2	,56
40	57	9	,6
41	59	0	,64
42	60	7	,68
43	61	14	,72
44	63	5	,76
45	64	12	,8
46	66	3	,84
47	67	10	,88
48	69	1	,92
49	70	8	,96
50	72	0	,—
51	73	7	,04
52	74	14	,08
53	76	5	,12
54	77	12	,16
55	79	3	,2
56	80	10	,24
57	82	1	,28
58	83	8	,32
59	84	15	,36
60	86	6	,4
61	87	13	,44
62	89	4	,48
63	90	11	,52

N°.	lq.	q.q	Pt.
64	92	2	,56
65	93	9	,6
66	95	0	,64
67	96	7	,68
68	97	14	,72
69	99	5	,76
70	100	12	,8
71	102	3	,84
72	103	10	,88
73	105	1	,92
74	106	8	,96
75	108	0	,—
76	109	7	,04
77	110	14	,08
78	112	5	,12
79	113	12	,16
80	115	3	,2
81	116	10	,24
82	118	1	,28
83	119	8	,32
84	120	15	,36
85	122	6	,4
86	123	13	,44
87	125	4	,48
88	126	11	,52
89	128	2	,56
90	129	9	,6
91	131	0	,64
92	132	7	,68
93	133	14	,72
94	135	5	,76
95	136	12	,8
96	138	3	,84
97	139	10	,88
98	141	1	,92
99	142	8	,96

Table I. | Table II. { Cubic *or* Solid Measure, *one Solid Yard the Integer.*

Nº.	F.c.	P.c.
1	4	,66
2	9	,32
3	13	,98
4	18	,65
5	23	,31
6	27	,98
7	32	,64
8	37	,31
9	41	,97
10	46	,65
20	93	,3
30	139	,98
40	186	,6
50	233	,25
60	279	,9
70	326	,62
80	373	,2
90	419	,94

Nº.	Fc	I. c.	Pt.	Nº.	Fc	I. c.	Pt.
1	—	466	,56	36	9	1244	,16
2		933	,12	37	9	1710	,72
3	—	1399	,68	38	10	449	,28
4	1	138	,24	39	10	915	,84
5	1	604	,8	40	10	1382	,4
6	1	1071	,36	41	11	120	,96
7	1	1537	,92	42	11	587	,52
8	2	276	,48	43	11	1054	,08
9	2	743	,04	44	11	1520	,64
10	2	1209	,6	45	12	259	,2
11	2	1676	,16	46	12	725	,76
12	3	414	,72	47	12	1192	,32
13	3	881	,28	48	12	1658	,88
14	3	1347	,84	49	13	397	,44
15	4	86	,4	50	13	864	,—
16	4	552	,96	51	13	1330	,56
17	4	1019	,52	52	14	69	,12
18	4	1486	,08	53	14	535	,68
19	5	224	,64	54	14	1002	,24
20	5	691	,2	55	14	1468	,8
21	5	1157	,76	56	15	207	,36
22	5	1624	,32	57	15	673	,92
23	6	362	,88	58	15	1140	,48
24	6	829	,44	59	15	1607	,04
25	6	1296	,—	60	16	345	,6
26	7	34	,56	61	16	812	,16
27	7	501	,12	62	16	1278	,72
28	7	967	,68	63	17	17	,28
29	7	1434	,24	64	17	483	,84
30	8	172	,8	65	17	950	,4
31	8	639	,36	66	17	1416	,96
32	8	1105	,92	67	18	155	,52
33	8	1572	,48	68	18	622	,08
34	9	311	,04	69	18	1088	,64
35	9	777	,6	70	18	1555	,2

Nº.	*Fc*	*I. c.*	*Pt.*	Nº.	*Fc.*	*I. c.*	*Pt.*	Nº.	*Fc*	*I. c.*	*Pt.*
71	19	293	,76	81	21	1503	,36	91	24	984	,96
72	19	760	,32	82	22	241	,92	92	24	1451	,52
73	19	1226	,88	83	22	708	,48	93	25	190	,08
74	19	1693	,44	84	22	1175	,04	94	25	656	,64
75	20	432	,—	85	22	1641	,6	95	25	1123	,2
76	20	898	,56	86	23	380	,16	96	25	1589	,76
77	20	1365	,12	87	23	846	,72	97	26	328	,32
78	21	103	,68	88	23	1313	,28	98	26	794	,88
79	21	570	,24	89	24	51	,84	99	26	1261	,44
80	21	1036	,8	90	24	518	,4				

Table I.

Nº.	*I. c*	*Qc*	*Pt.*
1	—	11	,05
2	—	22	,11
3	—	33	,17
4	—	44	,23
5	—	55	,29
6	1	2	,35
7	1	13	,41
8	1	24	,47
9	1	35	,53
10	1	46	,59
20	3	29	,18
30	5	11	,77
40	6	58	,36
50	8	40	,95
60	10	23	,54
70	12	6	,13
80	13	52	,72
90	15	35	,31

Table II. { Cubic *or* Solid Measure, *one Solid Foot the Integer.*

Nº.	*I. c.*	*Qc*	*Pt.*	Nº.	*I. c.*	*Qc*	*Pt.*
1	17	17	,92	23	397	28	,16
2	34	35	,84	24	414	46	,08
3	51	53	,76	25	432	00	,—
4	69	7	,68	26	449	17	,92
5	86	25	,6	27	466	35	,84
6	103	43	,52	28	483	53	,76
7	120	61	,44	29	501	7	,68
8	138	15	,36	30	518	25	,6
9	155	33	,28	31	535	43	,52
10	172	51	,2	32	552	61	,44
11	190	5	,12	33	570	15	,36
12	207	23	,04	34	587	33	,28
13	224	40	,96	35	604	51	,2
14	241	58	,88	36	622	5	,12
15	259	12	,8	37	639	23	,04
16	276	30	,72	38	656	40	,96
17	293	48	,64	39	673	58	,88
18	311	2	,56	40	691	12	,8
19	328	20	,48	41	708	30	,72
20	345	38	,4	42	725	48	,64
21	362	56	,32	43	743	2	,56
22	380	10	,24	44	760	20	,48

N°.	I. c.	Qc	Pt.	N°.	I. c.	Qc	Pt.	N°.	I. c.	Qc	Pt.
45	777	38	,4	64	1105	58	,88	83	1434	15	,36
46	794	56	,32	65	1123	12	,8	84	1451	33	,28
47	812	10	,24	66	1140	30	,72	85	1468	51	,2
48	829	28	,16	67	1157	48	,64	86	1486	5	,12
49	846	46	,08	68	1175	2	,56	87	1503	23	,04
50	864	0	,—	69	1192	20	,48	88	1520	40	,96
51	881	17	,92	70	1209	38	,4	89	1537	58	,88
52	898	35	,84	71	1226	56	,32	90	1555	12	,8
53	915	53	,76	72	1244	10	,24	91	1572	30	,72
54	933	7	,68	73	1261	28	,16	92	1589	48	,64
55	950	25	,6	74	1278	46	,08	93	1607	2	,56
56	967	43	,52	75	1296	0	,—	94	1624	20	,48
57	984	61	,44	76	1313	17	,92	95	1641	38	,4
58	1002	15	,36	77	1330	35	,84	96	1658	56	,32
59	1019	33	,28	78	1347	53	,76	97	1676	10	,24
60	1036	51	,2	79	1365	7	,68	98	1693	28	,16
61	1054	5	,12	80	1382	25	,6	99	1710	46	,08
62	1071	23	,04	81	1399	43	,52				
63	1088	40	,96	82	1416	61	,44				

Table I.

N°.	Pq	Yq	Pt.
1	—	—	,48
2	—	—	,96
3	—	1	,44
4	—	1	,92
5	—	2	,4
6	—	2	,88
7	—	3	,36
8	—	3	,84
9	—	4	,32
10	—	4	,84
20	—	9	,68
30	—	14	,52
40	—	19	,36

Table II. { Land Measure, *one Acre the Integer.*

Rods	0	1	2	3	Pq	Yq	Pt.
N°.	0	25	50	75	0	0	,0
	1	26	51	76	1	18	,15
	2	27	52	77	3	6	,05
	3	28	53	78	4	24	,2
	4	29	54	79	6	12	,1
	5	30	55	80	8	0	,—
	6	31	56	81	9	18	,15
	7	32	57	82	11	6	,05
	8	33	58	83	12	24	,2
	9	34	59	84	14	12	,1
	10	35	60	85	16	0	,—
	11	36	61	86	17	18	,15
	12	37	62	87	19	6	,05

N°.

N°.	*Pq*	*Yq.*	*Pt.*
50	—	24	,2
60	—	29	,04
70	1	3	,88
80	1	8	,72
90	1	13	,56

Rods	0	1	2	3	*Pq*	*Yq*	*Pt.*
N°.	13	38	63	88	20	24	,2
	14	39	64	89	22	12	,1
	15	40	65	90	24	0	,—
	16	41	66	91	25	18	,15
	17	42	67	92	27	6	,05
	18	43	68	93	28	24	,2
	19	44	69	94	30	12	,1
	20	45	70	95	32	0	,—
	21	46	71	96	33	18	,15
	22	47	72	97	35	6	,05
	23	48	73	98	36	24	,2
	24	49	74	99	38	12	,1

Table I. | Table II. { Time. *One Year the Integer.*

N°.	*D.*	*H.*	*Pt.*
1	—	—	,87
2	—	1	,74
3	—	2	,62
4	—	3	,5
5	—	4	,38
6	—	5	,25
7	—	6	,12
8	—	7	,—
9	—	7	,88
10	—	8	,76
20	—	17	,52
30	1	2	,28
40	1	11	,04
50	1	19	,8
60	2	4	,56
70	2	13	,32
80	2	22	,08
90	3	6	,84

N°.	*M*	*W*	*D.*	*H.*	*Pt.*
1	—	—	3	15	,6
2	—	1	0	7	,2
3	—	1	3	22	,8
4	—	2	0	14	,4
5	—	2	4	6	,—
6	—	3	0	21	,6
7	—	3	4	13	,2
8	1	0	1	4	,8
9	1	0	4	20	,4
10	1	1	1	12	,—
11	1	1	5	3	,6
12	1	2	1	19	,2
13	1	2	5	10	,8
14	1	3	2	2	,4
15	1	3	6	18	,—
16	2	0	2	9	,6
17	2	0	6	1	,2
18	2	1	3	16	,8

N°.	*M*	*W.*	*D.*	*H.*	*Pt.*
19	2	1	6	8	,4
20	2	2	3	0	,—
21	2	2	6	15	,6
22	2	3	3	7	,2
23	2	3	6	22	,8
24	3	0	3	14	,4
25	3	1	0	6	,—
26	3	1	3	21	,6
27	3	2	0	13	,2
28	3	2	4	4	,8
29	3	3	0	20	,4
30	3	3	4	12	,—
31	4	0	1	3	,6
32	4	0	4	19	,2
33	4	1	1	10	,8
34	4	1	5	2	,4
35	4	2	1	18	,6
36	4	2	5	9	,6

N°.	M.	W.	D.	H.	Pt.	N°.	M.	W.	D.	H.	Pt.
37	4	3	2	1	,2	69	8	3	6	20	,4
38	4	3	5	16	,8	70	9	0	3	12	,—
39	5	0	2	8	,4	71	9	1	0	3	,6
40	5	0	6	0	,—	72	9	1	3	19	,2
41	5	1	2	15	,6	73	9	2	0	10	,8
42	5	1	6	7	,2	74	9	2	4	2	,4
43	5	2	2	22	,8	75	9	3	0	18	,—
44	5	2	6	14	,4	76	9	3	4	9	,6
45	5	3	3	6	,—	77	10	0	1	1	,2
46	5	3	6	21	,6	78	10	0	4	16	,8
47	6	0	3	13	,2	79	10	1	1	8	,4
48	6	1	0	4	,8	80	10	1	5	0	,—
49	6	1	3	20	,4	81	10	2	1	15	,6
50	6	2	0	12	,—	82	10	2	5	7	,2
51	6	2	4	3	,6	83	10	3	2	22	,8
52	6	3	0	19	,2	84	10	3	5	14	,4
53	6	3	4	10	,8	85	11	0	2	6	,—
54	7	0	1	2	,4	86	11	0	6	21	,6
55	7	0	4	18	,—	87	11	1	2	13	,2
56	7	1	1	9	,6	88	11	1	6	4	,8
57	7	1	5	1	,2	89	11	2	2	20	,4
58	7	2	1	16	,8	90	11	2	6	12	,—
59	7	2	5	8	,4	91	11	3	3	3	,6
60	7	3	2	0	,—	92	11	3	6	19	,2
61	7	3	5	15	,6	93	12	0	3	10	,8
62	8	0	2	7	,2	94	12	1	0	2	,4
63	8	0	6	22	,8	95	12	1	3	18	,—
64	8	1	2	14	,4	96	12	2	0	9	,6
65	8	1	6	6	,—	97	12	2	4	1	,2
66	8	2	2	21	,6	98	12	3	0	16	,8
67	8	2	6	13	,2	99	12	3	4	8	,4
68	8	3	3	4	,8						

N°.

Table I. | Table II. { Time. *One Month the Integer.*

Nº.	*H.*	*Pt.*
1	—	,06
2	—	,13
3	—	,2
4	—	,26
5	—	,33
6	—	,4
7	—	,46
8	—	,53
9	—	,6
10	—	,67
20	1	,34
30	2	,01
40	2	,68
50	3	,35
60	4	,02
70	4	,69
80	5	,36
90	6	,03

W.	0	1	2	3	*D.*	*H.*	*Pt.*
Nº.	0	25	50	75	0	0	,—
	1	26	51	76	0	6	,72
	2	27	52	77	0	13	,44
	3	28	53	78	0	20	,16
	4	29	54	79	1	2	,88
	5	30	55	80	1	9	,6
	6	31	56	81	1	16	,32
	7	32	57	82	1	23	,04
	8	33	58	83	2	5	,76
	9	34	59	84	2	12	,48
	10	35	60	85	2	19	,2
	11	36	61	86	3	1	,92
	12	37	62	87	3	8	,64
	13	38	63	88	3	15	,36
	14	39	64	89	3	22	,08
	15	40	65	90	4	4	,8
	16	41	66	91	4	11	,52
	17	42	67	92	4	18	,24
	18	43	68	93	5	0	,96
	19	44	69	94	5	7	,68
	20	45	70	95	5	14	,4
	21	46	71	96	5	21	,12
	22	47	72	97	6	3	,84
	23	48	73	98	6	10	,56
	24	49	74	99	6	17	,28

Table I. | Table II. { Time. *One Day the Integer.*

Nº.	*M'*	*Pt.*
1	—	,14
2	—	,28
3	—	,43
4	—	,57
5	—	,72

Nº.	*H.*	*M'*	*Pt.*	Nº.	*H.*	*M'*	*Pt.*
1	0	14	,4	6	1	26	4,
2	0	28	,8	7	1	40	,8
3	0	43	,2	8	1	55	,2
4	0	57	,6	9	2	9	,6
5	1	12	,—	10	2	24	,—

Nº	M	Pt.
6	—	,86
7	1	,—
8	1	,15
9	1	,29
10	1	,44
20	2	,88
30	4	,32
40	5	,76
50	7	,2
60	8	,64
70	10	,08
80	11	,52
90	12	,96

Nº.	H.	M	Pt.
11	2	38	,4
12	2	52	,8
13	3	7	,2
14	3	21	,6
15	3	36	,—
16	3	50	,4
17	4	4	,8
18	4	19	,2
19	4	33	,6
20	4	48	,—
21	5	2	,4
22	5	16	,8
23	5	31	,2
24	5	45	,6
25	6	0	,—
26	6	14	,4
27	6	28	,8
28	6	43	,2
29	6	57	,6
30	7	12	,—
31	7	26	,4
32	7	40	,8
33	7	55	,2
34	8	9	,6
35	8	24	,—
36	8	38	,4
37	8	52	,8
38	9	7	,2
39	9	21	,6
40	9	36	,—
41	9	50	,4
42	10	4	,8
43	10	19	,2
44	10	33	,6
45	10	48	,—
46	11	2	,4
47	11	16	,8
48	11	31	,2
49	11	45	,6
50	12	0	,—

Nº	H.	M	Pt.
51	12	14	,4
52	12	28	,8
53	12	43	,2
54	12	57	,6
55	13	12	,—
56	13	26	,4
57	13	40	,8
58	13	55	,2
59	14	9	,6
60	14	24	,—
61	14	38	,4
62	14	52	,8
63	15	7	,2
64	15	21	,6
65	15	36	,—
66	15	50	,4
67	16	4	,8
68	16	19	,2
69	16	33	,6
70	16	48	,—
71	17	2	,4
72	17	16	,8
73	17	31	,2
74	17	45	,6
75	18	0	,—
76	18	14	,4
77	18	28	,8
78	18	43	,2
79	18	57	,6
80	19	12	,—
81	19	26	,4
82	19	40	,8
83	19	55	,2
84	20	9	,6
85	20	24	,—
86	20	38	,4
87	20	52	,8
88	21	7	,2
89	21	21	,6
90	21	36	,—

N°.	*H.*	*M.*	*Pt.*	N°.	*H.*	*M.*	*Pt.*	N°.	*H.*	*M.*	*Pt.*
91	21	50	,4	94	22	33	,6	97	23	16	,8
92	22	4	,8	95	22	48	,—	98	23	31	,2
93	22	19	,2	96	23	2	,4	99	23	45	,6

Table I. | Table II. { Time *or* Motion. *One Hour, or Degree the Integer.*

Table I.

N°.	''	*Pt.*
1	—	,36
2	—	,72
3	1	,08
4	1	,44
5	1	,8
6	2	,16
7	2	,52
8	2	,88
9	3	,24
10	3	,6
20	7	,2
30	10	,8
40	14	,4
50	18	,—
60	21	,6
70	25	,2
80	28	,8
90	32	,4

Table II.

N°.	'	''	N°.	'	''
1	0	36	29	17	24
2	1	12	30	18	,—
3	1	48	31	18	36
4	2	24	32	19	12
5	3	,—	33	19	48
6	3	36	34	20	24
7	4	12	35	21	,—
8	4	48	36	21	36
9	5	24	37	22	12
10	6	,—	38	22	48
11	6	36	39	23	24
12	7	12	40	24	,—
13	7	48	41	24	36
14	8	24	42	25	12
15	9	,—	43	25	48
16	9	36	44	26	24
17	10	12	45	27	,—
18	10	48	46	27	36
19	11	24	47	28	12
20	12	,—	48	28	48
21	12	36	49	29	24
22	13	12	50	30	,—
23	13	48	51	30	36
24	14	24	52	31	12
25	15	,—	53	31	48
26	15	36	54	32	24
27	16	12	55	33	,—
28	16	48	56	33	36

N°.	′	″	N°.	′	″	N°.	′	″
57	34	12	72	43	12	87	52	12
58	34	48	73	43	48	88	52	48
59	35	24	74	44	24	89	53	24
60	36	,—	75	45	,—	90	54	,—
61	36	36	76	45	36	91	54	36
62	37	12	77	46	12	92	55	12
63	37	48	78	46	48	93	55	48
64	38	24	79	47	24	94	56	24
65	39	,—	80	48	,—	95	57	,—
66	39	36	81	48	36	96	57	36
67	40	12	82	49	12	97	58	12
68	40	48	83	49	48	98	58	48
69	41	24	84	50	24	99	59	24
70	42	,—	85	51	,—			
71	42	36	86	51	36			

Table I. | Table II. { Motion. / *the Integer.* *One Sign*

N°.	′	″	N°.	gr°.	′	N°.	gr°.	′
1	—	10	1	0	18	19	5	42
2	—	21	2	0	36	20	6	,--
3	—	32	3	0	54	21	6	18
4	—	43	4	1	12	22	6	36
5	—	54	5	1	30	23	6	54
6	1	4	6	1	48	24	7	12
7	1	15	7	2	6	25	7	30
8	1	26	8	2	24	26	7	48
9	1	37	9	2	42	27	8	6
10	1	48	10	3	,--	28	8	24
20	3	36	11	3	18	29	8	42
30	5	24	12	3	36	30	9	,--
40	7	12	13	3	54	31	9	18
50	9	,--	14	4	12	32	9	36
60	10	48	15	4	30	33	9	54
70	12	36	16	4	48	34	10	12
80	14	24	17	5	6	35	10	30
90	16	12	18	5	24	36	10	48

N°.

N°.	gr°.	'	N°.	gr°.	'	N°.	gr°.	'
37	11	6	58	17	24	79	23	42
38	11	24	59	17	42	80	24	,--
39	11	42	60	18	,--	81	24	18
40	12	,--	61	18	18	82	24	36
41	12	18	62	18	36	83	24	54
42	12	36	63	18	54	84	25	12
43	12	54	64	19	12	85	25	30
44	13	12	65	19	30	86	25	48
45	13	30	66	19	48	87	26	6
46	13	48	67	20	6	88	26	24
47	14	6	68	20	24	89	26	42
48	14	24	69	20	42	90	27	,--
49	14	42	70	21	,--	91	27	18
50	15	,--	71	21	18	92	27	36
51	15	18	72	21	36	93	27	54
52	15	36	73	21	54	94	28	12
53	15	54	74	22	12	95	28	30
54	16	12	75	22	30	96	28	48
55	16	30	76	22	48	97	29	6
56	16	48	77	23	6	98	29	24
57	17	6	78	23	24	99	29	42

CHAP.

CHAP. VI.

The Use of DECIMALS *in all the Rules of Proportion*, viz. *The Rule of Three direct and Inverse*; *and the double Rule of Five Numbers.*

DIRECT *Proportion* is, when of *four* Numbers, the *first* is in the ſame *Proportion bigger* or *leſs* than the *ſecond*, as the *third* is *bigger* or *leſs* than the *fourth* Number. Thus as 4 is to 12 ſo is 6 to 18; conſequently the *two middle* Numbers multiplied together, are *equal* to the Product of the *two extreme* ones; as $12 \times 6 = 18 \times 4 = 72$. Hence is deduced the *general Rule* for working all Queſtions in the *Rule* of *Three direct*, viz.

Multiply the ſecond Number by the third, and divide by the firſt; and the Quotient will be the fourth Number ſought, or Anſwer.

Now as theſe Rules of Proportion have ſome certain Numbers given to find others in the ſame Proportion, and their Subject being generally *Trade* and *Merchandiſe*, thoſe given Numbers often conſiſt of *diverſe Parts* and *Denominations*, which therefore are to be reduced to *Decimals*, in order for the Queſtion to be wrought in the ſimpleſt Manner, and with the greateſt eaſe and expedition; which ought to be the *Aim* of every *Artiſt*.

But as the Manner of Reducing the Parts of *Coins*, *Meaſures*, *Weights*, &c. hath been fully taught already; I ſhall only here expreſs the Queſtion *vulgarly*, but ſtate and work it decimally.

Queſtion 1. If $7\frac{3}{4}$ *Yards* of Cloth coſt 2*l.* 12*s.* 9*d.* What will $140\frac{1}{2}$ *Yards* coſt at that Rate?

Thus

		L.	*s.*	*d.*	*Yds*
Thus stated	Vulgarly	$7\frac{3}{4}$:	2—12—9	: :	140 $\frac{1}{2}$.
	Decimally	7,75 :	2,6375	: :	140,5

```
          2,6375
           140,5
          ------
          131875
         105500
         26375                    L.  s.  d.
        ---------
  7,75) 370,56875 (47,8153=47 : 16 : 3¾
        3100
        ----
        ·6056
         5425
         ----
         ·6318
          6200
          ----
          ·1187
            775
           ----
           4125
           3875
           ----
           2500
           2325
           ----
            175
```

Question 2. At the Rate of 5 *l.* 8 *s.* 4 *d.* per *C. Weight*, What will 19 *C.* 2 *qr.* 14 *lb.* cost?

		C.	*l.*	*s.*	*d*	*C.*	*Q.*	*lb.*
Stated thus	Vulgarly	1 :	5 —	8 —	4 : :	19 :	2 :	14
	Decimally	1 :	5,416		: :	19,625		

```
                     19,625
                      5,416
                    -------
                 9) 117750
                    -------
                     130833
                     19625
                    78500
                   98125
                  ----------
The Answer, 106 l. 6 s. 0½ = 106,302083
                  ----------
```

Question 3. Suppose four *Hogsheads*, three *Firkins*, and five *Gallons* of *Beer* cost 6 *l.* 14 *s.* 8 *d.* How much is that *per Hogshead*, and *per Gallon?*

		Hds.	*Fr.*	*G.*		*l.*	*s.*	*d.*		
Stated thus	Vulgarly	4 —	3 —	5	:	6 —	14 —	8	::	1
	Decimally	4,5925			:	6,73			::	1

OPERATION.

```
4,5925) 6,73333 (
  45       673
-------  -------   54
4,588 ) 6,7266 ( 1,4661 ( ,0271
        4588   ( 108
        -----    ---
        21386   ·386
        18352    378
        -----    ---
        ·30340   ··81
         27528     54
         -----     --
         28120     27
         27528
         -----
         ··5920
           4588
           ----
           1332
```

	l.		*l.*		*s.*		*d.*	
The Answer	1,4661	=	1	:	9	:	$3\frac{3}{4}$	*per Hogshead.*
	0,0271	=	0	:	0	:	$6\frac{1}{2}$	*per Gallon.*

Question 4. The *mean Motion* of the *Sun* being known to be 59′ 8″ each Day, 'tis required to know in what time He performs one intire Revolution through the whole Circle of the Ecliptic, or 360 Degrees.

		°	′	″		*D.*		°	′	″
This Question is thus stated.	Vulgarly	0—	59—	8	:	1	::	360—	00—	00
	Decimally	,985			:	1	::	360		

OPE-

OPERATION.

```
,98̇ ) 360 (
 98    360           Days.      D.  H.  '  "
,887 ) 324,000 (365,2762=365 : 6 : 37 : 43
       2661
       ·5790
        5322
        ·4680
         4435
         ·2450
          1774
           6760
           6269
            5510
            5322
            ·1880
             1774
              106
```

These are the various Cases which may happen in the *Rule* of *Three Direct*; by which any one may observe the *Advantage* of *Decimals*, and the absolute Necessity of understanding the Management of *circulating* or *repeating Decimals*.

The Rule *of* Three Inverse.

Inverse Proportion is, when of *four* Numbers, the *third* bears the *same Ratio* or Analogy to the *first*, as the *second* does to the *fourth*.

Whence the *Rule* is; to multiply the first and second of the given Numbers, and divide that Product by the third; the Quotient will be the fourth Number, or Answer.

To know when the Terms of a proposed Question are in this *reciprocal* or *inverse Proportion*, observe this Rule; *viz.* When the third Number is $\left\{\begin{array}{l}\textit{Bigger}\\ \textit{Lesser}\end{array}\right\}$ than the *first*, and requires $\left\{\begin{array}{l}\textit{Less,}\\ \textit{More,}\end{array}\right\}$ The

Terms are in the *inverſe Ratio*; and are to be worked by the *Rule* above; as in the following Inſtances.

Queſtion 1. If when Wheat is ſold for 6 *s.* 6 *d. per* Buſhel, the Penny White Loaf ought to weigh eight Ounces *Troy*; What muſt it weigh when it is at 4 *s. per* Buſhel?

Thus ſtated { Vulgarly 6 *s.* 6 *d.* : 8 *oz.* : : 4 *s.*
{ Decimally 6,5 : ,$\dot{6}$ *lb.* : : 4 *s.*

,$\dot{6}$

9) 3,90 *lb.*   *lb.* *oz.*
4) 4,$\dot{3}$ (1,08$\dot{3}$ = 1 : 1. or 13 *Oun.*
4
33
32
13
12 } *ad infinitum.*
1

Queſtion 2. Two Equal *Parallelograms A, B* given, the Length of *A* is 8 Feet 8 Inches, and its Breadth 4 Feet and 3 Inches; the Breadth of *B* is 2 Feet 10 Inches, Quere its Length?

F. In. F. In. F. In.
Thus ſtated { Vulgarly 4 — 3 : 8—8 : : 2—10
{ Decimally 4,25 : 8,$\dot{6}$: : 2,8$\dot{3}$

8,$\dot{6}$

4,25 [8,$\dot{6}$ A]   9) 2550   [B] 2,8$\dot{3}$
28$\dot{3}$3   13
3400
2,8$\dot{3}$) 36,8$\dot{3}$ (
28) 368 (13 Inches the Anſwer.
2,55) 3315
255
765
765
...

Queſtion

Question 3. A Piece of Land 4 Rod broad and 40 long, being a Statute-Acre; tis required to know what Length, with 10 Rod and 2 Yards Breadth, will make an Acre?

		R.		R. Y.
Stated thus	Vulgarly	4 : 40	::	10 — 2
	Decimally	4 : 40	::	10,36

```
                 4
10,36) 160,0
       10    16          Rod.        R. Y. F. In.
10,26) 158,4 (15,438 = 15 : 2 : 1 : 2
       1026
       5580
       5130
        4500
        4104
         396
```

The Double Rule of Three; or Rule of Five Numbers.

In this Rule of Proportion there are *Five* Numbers given to find a *Sixth* in Proportion; which is either *Direct* or *Inverse*, according to the Nature of the Question.

Questions in this Rule are performed at two Operations, that is, by a *double stating* the Question, most generally.

Question 1. What is the *Interest* of 364 *l.* 5 *s.* for seven Months, three Weeks, at the Rate of 4 *l.* 10 *s.* *per Cent. per Annum?*

		l.	*M.*	*l.* *s.*		*l.* *s.*	*M.* *W.*
Thus stated	Vulgarly	100 :	12 :	4 — 10	::	364 — 5 :	7 — 3
	Decimally	100 :	12 :	4,5	::	364,25	: 7,75

```
                       l.      l.        l.
First Operation,     100  :  4,5  ::  364,25
                                         4,5
                                      182125
                                      145700
                                                   l.
                               100) 1639,125 (16,39125
```

Hence the Interest of 364,25 *l.* for one Year, is 16,39125 *l.*

Then

Then, Second Operation

	Mo.		l.		Mo.
Second Operation	12	:	16,39125	:	7,75

```
              16,39125
                  57,7
              ________
              114739
               11474
                 819     l.      l.   s.  d.
              ________
    12 ) 127,032 (10,586 = 10 : 11 : 8½
                   The Answer.
```

Question 2. Suppose it were required to know what *Principal* would gain 15 *l.* 14 *s.* 8 *d.* in nine Months and three Days, at the Rate of 4 *l.* 10*s.* *per Cent, per Annum, Quere.* the Answer ?

		l. s.	Mo.	l.	l. s. d.	Mo D.
Thus stated	Vulgarly	4 : 10	—12—	100—	15 : 14 : 8	—9 : 3
	Decimally	4,5	—12—	100—	15,73	—9,107

The First Operation *Direct.*

```
  l.
  4,5 : 100 : : 15,73
                  100
                 ______
          4,5 ) 1573,3 (349,62
                 135
                 ___
                 223
                 180
                 ___
                 433
                 405
                 ___
                  283
                  270      Ad infinitum.
                  ___
                  133
                   90
                  ___
                   43
```

The Principal from whence 15,73 *l.* comes in 12 Months is 349,62 *l.*

The Second Operation therefore must be wrought *inversely.*

Thus,

```
          Mº.            Mº.
Thus,     12 : 349,62 : : 9,107
                  12
               ------
               69924
              34962                l.          l.     s.    d.
               ------
    9,107)  4195,5  (460,6956 = 460  :  13  :  11
            36428
            ------
             55275
             54642
            ------
             ··63355
               54642
              ------
              ·87135
               81963
               ------
                5172
```

But any Question in this Rule may be answered at *one Operation*, by the following *Rule*:

Make the	1st 2d 3d	Number of the Five, the same kind with the	4th 5th 6th	Number given. the Num. sought.

Then, Multiply the *Three* Numbers to the Right-hand together, and the *two first* to the Left-hand; and divide the *first Product* by this *last*, and the *Quotient* will be the sixth Number, or *Answer*, if the Proportion be *Direct*.

Question 3. If a 1000 Men can dig a Trench 500 Feet Long in 24 Hours, what Length of such a Trench can 9800 Men dig in 10 Hours?

Thus

	Men	*Hours*	*Feet*		*Men*	*Hours*
Thus stated,	1000.	24 :	500	::	9800.	10

```
1000.                          9800.
  24                              10
------                         ------
24000                          98000
                                 500
            24|000)49000|000(2041,6
                   48...
                    100
                     96
                    ---
                    .40
                     24
                    ---
                    160 }
                    144 } In Infinitum.
                    --- }
                     16 }
```

The Answer is
Feet *Feet* *In.*
2041,6 = 2041 : 8
The Length required.

If any Part of the Question be in *reciprocal or inverse* Proportion; place the Three first Numbers as in the last Question; and of the other two, place *That* the fourth, which is of the same kind as the second; and consequently the other must be the fifth Number.

Question 4. If 1000 Men can dig a Trench in 24 Hours 500 Feet Long, How many Men will dig 2041,6 Feet in 10 Hours?

	Men	*H.*	*Feet.*		*H.*	*Feet.*
Stated thus,	1000 :	24 :	500	::	10 :	2041,6

```
 500                 2041,6
  10                   1000
-----              ----------
5000               2041666,6
                          24
                   ----------
                   81666666,6
                   40833333,3
        5|000)49000|000(9800
              45
              --
               40
               40
               --
              ..00
```

Rule. Multiply the 1st, 2d, and 5th Numbers, and the 3d, and 4th; then divide the first Product by the last, the Quotient is the Answer; *viz.* 9800 Men.

But for the greater readiness and ease of the *ingenious Arithmetician*, I shall transcribe that famous *general Theorem* in

in Mr. *Ward*'s *Young Mathematician*'s *Guide*, which ſhews *at once* how to anſwer any Queſtion of *Five Numbers* at one Operation, without regard to the Proportion of the Terms; be that *Direct* or *Indirect* as it will.

The *Theorem* is this, $TgP = Gpt$. In this *Theorem* you obſerve three *Capital Letters*, viz. *T*, *P*, *G*, and the ſame three Letters in *ſmall* Characters, *g*, *p*, *t*. The three *Capitals* ſignify the *Three firſt conditional Terms* of the Queſtion,

Thus, $\begin{cases} P\text{, Is the } \textit{Principal} \text{ Cauſe of } \textit{Gain, Loſs, Action}\text{, \&c.} \\ T\text{, Is the } \textit{Time, Space, Diſtance}\text{, \&c.} \\ G\text{, Is the } \textit{Gain, Loſs, Action}\text{, \&c.} \end{cases}$

Of the *Three ſmall Letters*, (which correſpond to, and ſignify the ſame with the *Capitals*) *two* always move the Queſtion, the other ſhews the *Anſwer*; which, as the Letters are three, is *threefold*; and anſwered by the ſame *Theorem* diſpoſed in theſe three proper *Terms*.

Viz. If $\left\{\begin{matrix} p \\ t \\ g \end{matrix}\right\}$ be ſought, the *Theorem* is $\begin{cases} \dfrac{TgP}{tG} = P. \\ \dfrac{TgP}{Gp} = t. \\ \dfrac{GtP}{TP} = g. \end{cases}$

Or thus, $TgP \div Gt = p$. $TgP \div Gp = t$. $Gpt \div TP = g$.

If any *Arithmetician* ſhould complain he does not underſtand ſuch *Algebraic Forms and Characters*, all that I have to anſwer is, That 'tis a very *neceſſary Part* of his Buſineſs and Profeſſion, and *highly concerns him to learn it*.

CHAP.

CHAP. VII.

A New Decimal Practice; or a short Way of computing all kind of Merchandise by DECIMALS.

THO' there is scarce *any Part of Arithmetick* in which *Decimals* are of greater (or indeed so great) Service, as *Practice*; yet this of all others has been the *least improv'd* by it; hardly an Author can be met with on this Head; and those who have undertaken it, have presented us with but *imperfect Sketches*, and left the Matter unfinished. I hope what follows in this Chapter will give Satisfaction in this Point.

The *Tables of aliquot Parts* commonly used, are these

s.	*d.*	*Parts.*	
10	0	2	The *even* or *aliquot Parts* of a Pound *Sterling*. By which *dividing*, gives an Answer in *Pounds*.
6	8	3	
5	0	4	
4	0	5	
3	4	6	
2	6	8	
2	0	10	
1	8	12	

d.	*Parts.*	
6	2	The *even* or *aliquot Parts* of a *Shilling Sterling*. By which *dividing*, gives an Answer in *Shillings*.
4	3	
3	4	
2	6	
$1\frac{1}{2}$	8	
1	12	

But the *Table* following is far more *general*, *expeditious* and *useful*; and has not yet been applied to *Decimal Practice*.

A General Table *for* Decimal Practice.

Price. *d. q.*	*Divisors.*	Price. *d. q.*	*Divisors.*	Price *s.*	*Divisors.*
0 : 1	3,4,80	6 : 1	80,×2,+12	1	20
0 : 2	6,80	6 : 2	40,+12	2	10
0 : 3	4,80	6 : 3	40,+8	3	10,+2
1 : 0	4,60	7 : 0	40,+6	4	10,×2
1 : 1	4,6,8	7 : 1	40,+6,4	5	4
1 : 2	2,80	7 : 2	40,+4	6	10,×3
1 : 3	3,40,—8	7 : 3	40,+4,6	7	10,×3,+2
2 : 0	3,40	8 : 0	30.	8	10,×4
2 : 1	3,40,+8	8 : 1	80,×3,—4	9	10,×4,+2
2 : 2	3,40,+4	8 : 2	60,×2,+8	10	2
2 : 3	80,—12	8 : 3	80,×3,-12	11	2,+10
3 : 0	80.	9 : 0	40,+2	12	10,×6
3 : 1	80,+12	9 : 1	80,×3,+12	13	10,×6,+2
3 : 2	80,+6	9 : 2	80,×3,+6	14	10,×7
3 : 3	80,+4	9 : 3	80,×3,+4	15	2,+2
4 : 0	60.	10 : 0	3,8	16	10,×8
4 : 1	3,40,×2,+8	10 : 1	30,+4,8	17	10,×8,+2
4 : 2	60,+8	10 : 2	20,—8	18	10,×9
4 : 3	60,+8,2	10 : 3	40,+2,2,6	19	1,—20
5 : 0	6,8	11 : 0	20,—12		
5 : 1	40,-8	11 : 1	40,×2,—8		
5 : 2	40,—12	11 : 2	40,×2,—12		
5 : 3	80,×2,—12	11 : 3	30,+2,—16		
6 : 0	40.				

An Explanation *of the preceeding* Table.

The *first Column* shews the *Price* of the *Commodity*, either in *Pence* and *Farthings*, or in *Shillings*, for one of a sort; as one *Pound*, *Yard*, *Piece*, &c.

Against the *Piece*, you observe in the second *Column*, several Numbers, of which those which *stand first*, and have nothing prefixt to them, are *Divisors*; by which any given *Quantity* or Number of *Yards*, *Ells*, *Pounds*, are to be *divided*.

If any Number follow these with any *Character prefixt* to them as 3,40,×2,+8. 80,×3,—12, *&c.* They are to be *understood*, and *read*, as in the following Examples.

3,4,—8	From a 4th of a 3d, take one 8th of that 4th
3,4,+8	To a 4th of a 3d add an 8th of that 4th.
3,40,×2,+8	To a 40th of a 3d, multiplied by 2, add an 8th of that 40th.
60,+8,2	To a 60th add an 8th of that 60th, and half of that 8th.
3,+2,—16	To a 3d add an half of that 3d, then substract a 16th of that 2d.
1,—20	From the given Number take one 20th Part.

These being well understood, 'twill not be difficult to use the Table on all occasions with ease; especially after perusing the Examples ensuing, which are chosen for the more difficult Parts thereof.

Note, When the *Price* consists of *Shillings* only, the Number may be multiplied by the *Decimal*, that is, *half the Number of Shillings*, and the Answer will be the same.

Ex. 1. At 1 *q. per* Yard, What cost 144 Yards?
One 3d = 48
One 4th of that = 12
One 80th of that = 0,15=3 *s.* Answer.

Ex. 2. At 3 *q. per* Yard, What cost 172,5 Yards?
One 4th = 43,125
One 80th of that = 0,53907=10 *s.* 9 *d* ¼.

Ex. 3. At 1 *d.* 1 *q.* What cost 1792,25
One 4th = 448,0625
A 6th of that = 74,67708
An 8th of that = 9,33463=9 *l.* 6 *s.* 8½ *d.*

Ex. 4. At 1 *d.* 3 *q.* What cost 9742,6
One 3d = 3247,8

A 40th of that = 81,18888
From which take an 8th = 10,14361

		l.	*s.*	*d.*
Remains the Answer =	71,04527=	71	00	10

Ex.

Ex. 5. At 2 *d.* 1 *q.* What coſt 3696,24
One 3d = 1232,08

A 40th of that = 30,80205
To which add one 8th = 3,85025

l. *s.* *d.*
The Sum the Anſwer = 34,6523 = 34 : 13 : 0½

Ex. 6. At 2 *d.* 3 *q.* What coſt 4130,21 ?

One 80th = 51,627
From which take a 12th = 4,302

l. *s.* *d.*
Remains the Anſwer = 47,325 = 47 : 6 : 6

Ex. 7. At 4 *d.* 1 *q.* What coſt 1932,49 ?
One 3d = 644,16

A 40th of that = 16,104
Ditto = 16,104
Of which add an 8th = 2,013

l. *s.* *d.*
The Sum is the Anſwer = 34,221 = 34 : 4 : 5

Ex. 8. At 4 *d.* 3 *q.* What coſt 948,46 ?

One 60th = 15,807
One 8th of that = 1,975
One half of that = 0,987

l. *s.* *d.*
The Sum is the Anſwer = 18,771 = 18 : 15 : 5

Ex. 9. At 5 *d.* 3 *q.* What coſt 1012,2 ?
One 80th = 12,6
2

One 80th, × 2 = 25,33
Subſtract a 12th of an 80th = 1,05

l. *s.* *d.*
Remains the Anſwer = 24,27 = 24 : 5 : 6½

Ex. 10. At 8*d.* 1*q.* What coſt 2640?

One 80th = 33
3

That multiplied by 3 = 99
Subſtract a 4th of an 80th = 8,25

	l.	*s.*	*d.*
Remains the Anſwer = 90,75 =	90	15	0

Ex. 11. At 9*d.* 1*q.* What coſt 961,92?

One 80th = 12,024
3

An 80th × 3 = 36,072
To which add 12th of an 80th = 1,002

	l.	*s.*	*d.*
The Sum is the Anſwer = 37,074 =	37	1	$5\frac{1}{4}$

Ex. 12. At 10*d.* 3*q.* What coſt 1600

One 40th = 40
Add an half = 20
Add half that = 10
Add a ſixth of that = 1,6

	l.	*s.*	*d.*
The Sum is the Anſwer = 71,6 =	71	13	4

Ex. 13. At 11*d.* 3*q.* What coſt 906,18?

One 30th = 30,206
To which add an half = 15,103

From the Sum = 45,309
Subſtract a 16th = 0,943

	l.	*s.*	*d.*
Remains the Anſwer = 44,368 =	44	7	$3\frac{1}{3}$

Ex. 14. At 7*s.* What coſt 365,25?

One 10th = 36,525
3

One 10th × 3 = 109,575
To which add half that 10th = 18,2625

	l.	*s.*	*d.*
The Sum is the Anſwer = 127,8375 =	127	16	9

Ex.

Ex. 15. At 19 *s.* What cost 257,6?
Substract a 20th = 12,88ȝ

		l.	*s.*	*d.*
Remains the Answer =	244,78ȝ =	244 :	15 :	8

Ex. 16. At 13 *s.* 9¼ *d.* What cost 96x,9z?

	96x,9z	For 13 *Shil.*
10th =	96,x9z	
	6	
10th × 6 =	577,x53	
½ of 10th =	48,096	
The Sum =	625,z49	

80th =	12,024	For the 9¼ *d.*
	3	
80th × 3 =	36,072	
12 of 80 =	1,002	

		s. *d.*
Add	37,074	for 0:9¼
	625,z49	for 13:00
The Answer =	662,ȝ2ȝ	for 13:9¼

In this Example (and any other) the Answer for the *Shillings* is found with least trouble, and Figures, by *Multiplying the given Number* by the *Decimal* of the *Shillings*,

Thus,

96x,9z	The given Number.
,65	The Decimal of a Pound for 13 *s.*
480960	
577x531	
625,z49	The same as before.

If the *Price* or *Value* consist of *Pounds*, *Shillings*, *Pence*, &c. the most *ready* and *practical* way, is to turn the *whole* into *Decimals*, then multiply the given Number (turn'd into *Decimals* also, if express'd in *diverse Parts*;) and the *Product* will be the *Answer*.

Ex. 17. At 5 *l.* 16 *s.* 8 *d.* What cost 270? *l.* *s.* *d.*

	5,8ȝ =	5 : 16 : 8
	9) 810	
	900	
	2160	
	1350	
The Answer =	1575,00	

Ex.

		l.	s.	d.		C.	Q.	lb.	oz.
Ex. 18.	At	1 :	17 :	$3\frac{3}{4}$	What cost	14 :	1 :	14 :	10?

$$
\begin{array}{rl}
 & \text{C.} \\
\text{Then,} & \left\{\begin{array}{r} 14,38058 \\ \underline{526568,1} \end{array}\right. \text{ The Multiplier } \textit{inverted.} \\
 & 143805 \\
 & 115044 \\
 & 8628 \\
 & 719 \\
 & 86 \\
 & \underline{3} \\
\text{The Answer } l. & \underline{26,8285} = 26\,l.\ 16\,s.\ 6\tfrac{3}{4}\,d.
\end{array}
$$

These Examples are sufficient to the ingenious Practical Student of *Decimal Arithmetick*; who with those Instructions will easily (*proprio Marte*) apply this noble Art to *all Cases* of *Common Trade* and *Merchandise*.

CHAP. VIII.

The Use of DECIMALS *in the Rules of* Fellowship, Tare *and* Trett, Barter, Gain *and* Loss, Exchange, Alligation, Rule of False Position, Extraction of Roots.

Single Fellowship; *or That without* Time.

THE *Rules* of *Fellowship* are proper to *Merchants* and those who *Trade in Company*, or *Partnership*; where they have a *common Joint-Stock* to traffick withal; for to every one of the Company is distributed his *due share* of *Gain* or *Loss* acquired by *Trading*, in proportion to his *Stock laid out*, by this following

Rule

Rule.

As the total Sum of the Stocks, is to the total Gain or Loss; so is each Man's particular Stock, to his particular Gain or Loss.

Example. Suppose *Four* Merchants, *A, B, C, D,* make a *Joint Stock* of 421 *l.* 8 *s.* 6 *d.*

$$\text{Thus,}\left\{\begin{array}{llcl} A \text{ puts in} & 154 : 13 : 4 & = & 154,6666 \\ B \quad — & 110 : 18 : 6 & = & 110,9250 \\ C \quad — & 95 : 00 : 8 & = & 95,0833 \\ D \quad — & 60 : 16 : 0 & = & 60,8 \end{array}\right.$$

$$\text{The whole Stock} = 421 : 08 : 6 = 421,425$$

The Trade and Gain 88 *l.* 17 *s.* 10 *d.* = 88,8916 *l.* Tis required to find each Man's Part or Share of that Gain.

$$\text{Then as } \overset{l.}{421,425} : \overset{l.}{88,8916} \left\{\begin{array}{lll} :: & \overset{l.}{154,6} & : \overset{l.}{32,6192} \\ :: & 110,925 & : 23,394 \\ :: & 95,08 & : 20,0425 \\ :: & 60,8 & : 12,8227 \end{array}\right.$$

The Sum of the several Shares — — 88,8784

Which being the same with, or equal to, the *total Gains* always proves the *Truth* of the Work.

But all Cases in the Rules of Fellowship are soonest and easiest answer'd by finding the *Proportional Part* of the *Gain* or *Loss* due to *one Pound*; and then *by that* to multiply *each Man's particular Part* of the *Stock*; for the *several Products* in such Cases, are the *several Answers*; *viz.* each Man's *Part* of the *Gain* or *Loss*.

$$\text{Thus, As } \overset{l.}{421,425} : \overset{l.}{88,8916} :: \overset{l.}{1} : \overset{l.}{,2109} \text{ the common}$$
Multiplier.

$$\begin{array}{lr} A\text{'s Part of Stock} = & \overset{l.}{154,6} \\ \text{Multiply by} & ,2109 \\ \hline & 13919 \\ & 154666 \\ & 3093333 \\ \hline A\text{'s Part of Gain} = & 32,6192 \end{array} \qquad \begin{array}{lr} B\text{'s Part of Stock} = & \overset{l.}{110,925} \\ \text{Multiply by} & ,2109 \\ \hline & 998325 \\ & 1109250 \\ & 221850 \\ \hline B\text{'s P. of Gain} = & 23,394, \text{ \&c.} \end{array}$$

C's

	l.		*l.*
C's Part of Stock =	95,03	*D*'s Part of Stock =	60,8
Multiply by	,2109	Multiply by	,2109
	8552$\cancel{9}$		5472
	950333		6080
	1900$\cancel{6}$666		1216
C's Part of Gain =	20,04253	*D*'s Part of Gain =	12,82272

Here every Man's Share is the ſame as before.

Double Fellowſhip, *or That with* Time.

Fellowſhip with Time conſiders the *Share* of the *Gain* or *Loſs* with regard to the *Money*, and the *Time* it was imployed, and *proportionates it to both* by the following

Rule.

Multiply each Man's *Stock by the Time it was employed; then ſay, As the ſum of thoſe Products, is to the whole Gain or Loſs; ſo is every one of the Products, to its proportional Part of the Gain or Loſs.*

Example. Three Merchants *A*, *B*, and *C*, enter into *Partnerſhip*, thus;

	l.		
A puts in	65,5	for	8 Months, 2 Weeks, and 3 Days.
B —	78,6	—	12 Months, 3 Weeks, and 1 Day.
C —	84	—	6 Months, and 6 Days.

They *traffick* and *gain* 140,016 *l.* 'Tis required to find each Man's Share proportional to his Stock, and Time 'twas in.

		l.		*Months.*		*Products.*
First	*A*'s Stock	65,5	×	8,607	=	563,7585
	B's Stock	78,6	×	12,3357	=	970,4084
	C's Stock	84,	×	6,214	=	521,976
	The Sum of the Products				=	2056,1429

Then

$$\text{Then, As } 2056{,}1429 : 140{,}018 \left\{ \begin{array}{l} :: 563{,}7585 : \overset{l.}{38{,}3918} = A \\ :: 970{,}4084 : 66{,}0846 = B \\ :: 521{,}976 \ \ : \underline{35{,}5465} = C \end{array} \right.$$

The whole Gain very near = $\underline{140{,}0229}$ *l.*

Questions in this Rule also are much better answer'd *by finding the Proportional Part* to *one Pound*, for a *common Multiplier*, as before.

Thus, as 2056,1429 : $\overset{l.}{140{,}018}$ :: 1 : ,0681 Common Multiplier.

	For *A*	For *B*
The Operation for *A*, =	563,7585.	= 970,4084
The Multiplier inverted	1860,0	1860,0
	338255	582244
	45100	77632
	563	970
	38,3918	66,0806

	For *C*
For *C* =	521,9760
	1860,0
	313185
	41758
	522
	35,5465

35,5465 { Their several Parts of the Gain, as before.

Thus appears the *excellent Use* of *Decimals* in the *Rules* of *Fellowship*.

Tare *and* Trett.

Tare is the *Weight* of the *Hogshead*, *Chest*, *Bag*, *Cask*, &c. which contain the *Goods bought* or *sold*.

Trett is an *Allowance* of 4 *lb.* in 100, or 104 *lb.* for Goods wherein is *Loss* by *refuse*, &c.

Cloff is an *Allowance* of 2 Pound upon every *Draught* which exceedeth 300 *Gross Weight*.

 Sub-

Subtile is the *Weight* when the *Tare* is *deducted*, but not the *Trett*.

Neat Weight is the *Remainder* when *Tare*, *Trett*, and *Cloff*, if all are allowed, are *taken away*.

For resolving Questions in this Rule there are *several Methods*; but *those by Decimals* are much the shortest and best, and are as follow.

Question 1. What is the *Neat* Weight of 9 *C.* 2 *qr.* 7 *lb.* *Tare* at 14 *lb. per Cent.* to be deducted.

First, This may be answered by the *Golden Rule* in *Decimals*.

$$\begin{array}{l} \quad\quad\quad\quad lb. \quad\quad\quad\quad\quad C. \\ \text{If 1 } C. \text{ allow } 14, \text{ What will } 9{,}5625 \text{ allow for Tare?} \end{array}$$

$$\begin{array}{r} 14 \\ \hline 382500 \\ 95625 \\ \hline \end{array}$$

Total Tare $lb. = 133{,}875 = 1{,}1953$ *C.*

$$\text{Then, }\left\{\begin{array}{l} \text{The } \textit{Gross Weight} = 9{,}5625 \; C. \\ \text{The } \textit{Whole Tare} = \underline{1{,}1953} \text{ to be substracted.} \\ \text{The } \textit{Difference} \text{ is } = 8{,}3672 = 8\,C.\ 1\,qr.\ 13\,lb. \\ \quad \text{the Neat Weight.} \end{array}\right.$$

A *Second* Way, is to Multiply the *Gross Weight* by the *Decimal* of *C.* Weight, equal to the *Tare* allow'd.

$$\begin{array}{lr} & C. \\ \text{The } \textit{Gross Weight} = & 9{,}5625 \\ \text{The } \textit{Decimal} \text{ of } 14\,lb. = & {,}125 \\ \hline & 478125 \\ & 191250 \\ & 95625 \\ \hline \text{The } \textit{Tare} \text{ (as before)} = & 1{,}1953125 \text{ to be substracted.} \end{array}$$

A *Third* Way, is to multiply the *Gross Weight* by the *Decimal* of the *Neat Part* of a *Hundred Weight*.

Thus,

	C.		C.	
Thus, from	1,000	Then	9,5625	
Subſtract	,125		578,0	Multiplier inverted.
The *Neat* of *C.*	,875		76500	
			6693	
			478	

The *Neat Weight* total = 8,3671 as above.

A *Fourth Way*, is to work by *Aliquot Parts* as in *Practice*. Thus 14 being the 8th Part of 112; if you take an 8th of 9,5625, that will be the *Tare* of the Whole.

	C.	
Thus,	9,5625	the *Groſs Weight.*
	$\frac{1}{8}$=1,1953	the *Tare*, as before.

For the more *expeditious* finding the *Tare* by *Aliquot Parts*, I have inſerted the following *Table* of *Tare* and *proper Diviſors*.

Tare.	*Diviſors.*	*Tare.*	*Diviſors.*	*Tare.*	*Diviſors.*
lb.					
1	2,7,8	8	2,7	15	2,8,×2,+7
2	7,8	9	2,7,+8	16	7
3	7,8,+2	10	2,7,+4	17	2,7,×2,+8
4	4,7	11	2,7,+4,2	18	7,+8
5	4,7,+4	12	2,7,+2	19	7,+4,−20
6	4,7.+2	13	2,7,+2,4	20	7,+4
7	2,8	14	8	21	8,+2

The Conſtruction and Uſe of this Table of Tare is the ſame with the Table of Prices or Values in Practice, which ſee there taught.

Having ſhewn how to find the *Tare*, the next Buſineſs is to find the *Trett*, or the *Neat Weight* when the *Trett* is deducted from the *Subtile*;

Thus Multiply	,0384	the Product is the *Trett.*
the *Subtile* by	,9616	the Product is the *Neat Weight.*

Question 2. In 72 *C.* 3 *qr.* 12 *lb.* Groſs, *Tare* at 12 *lb.* *per C. Trett* 4 *lb. per* 104. How many C. *neat?*

	C.
The Groſs =	72,8571
Multiply by the Decimal of 12 *lb.* inverted =	1701,0
	72857
	5100
	72
The Tare (ſubtile) =	7,8029
The Subtile =	65,0542
Multiplier inverted =	4830,0
	19516
	5204
	260
The Trett to be ſubtile =	2,4980
The Neat Wt. =	62,5562

A *Shorter* Way, thus;

	C.
The Groſs =	72,8571
Mult. by the Neat Dec. of 112 *lb.* inverted =	8298,0
	582856
	65571
	1457
	582
The Subtile =	65,0466
The Multiplier inverted =	6169,0
	585419
	39027
	650
	390
The Neat Weight =	62,5486

Theſe are the *beſt Methods* for finding *Tare* and *Trett*; and that I have here given for finding the *Trett* is *new* to me, not having ſeen it in any Author I have met with.

Barter.

Barter.

By the *Rule* of *Barter*, *Merchants* and *Traders* exchange Goods of *different Values*, *Kinds*, and *Quantities*, ſo as to ſuſtain no *Loſs* or *Diſadvantage* by ſuch a *Barter* or *Change*.

Queſtion 1. Two *Merchants*, *A* and *B* barter; *A* would exchange 5 *C.* 3 *qr.* 14 *lb.* of *Pepper*, worth 3 *l.* 10 *s. per C.* with *B* for *Cotton* worth 10 *d. per lb.* How much *Cotton* muſt B give *A* for his *Pepper*?

Proceed thus by *Decimals* to find the Value of the *Pepper.*

```
              C.   l.      C.
Say, As  1 : 3,5 : : 5,875
                       3,5
                     -----
                     29375
                    17625
                    -----                 l.—s.—d.
The Value of the Pepper = 20,5625 = 20 : 11 : 3
```

Then to find the Quantity of *Cotton* equal to the Value of the *Pepper*;

```
              l.        C.        l.
Say,   As ,0416 : ,00892 : : 20,5625
                             29800,0
                             -------
                             1645
                              185
                                4
                            -------
                ,0416)      ,1834
                 041)        1834
               ------      ------       l.
                ,0375)      16506     (4,4016
                            1500
                            -----
                             1506
                             1500
                             -----
                             ···600
                                375
                             ------
                               2250
                               2250
                               ----
                               ····
```

Thus *B* muſt give *A*
C.   *C.*   *qr.*   *lb.*
4016 = 4 : 1 : 17½
f Cotton for his Pep-
r.

Queſtion

Question 2. *A* has 52 *Dozen of Hats*, worth in ready Money 2 *s.* 6 *d.* but *barters* at 2 *s.* 9 *d.* *per Hat*. B has *Cotton* at 10 *d.* *per Pound*, ready Money. *Quere* at *what Rate per Pound* B must *barter* his *Cotton*, and how much he must give for the *Hats*?

First; say, As 2,5 s. : ,25 s. :: ,83 s. : 083 s. = One Penny.

So that B's Cotton is to be advanced a Penny a Pound in Barter.

Secondly, to find the Value of the (624) *Hats* in *barter*;

Say, As 1 *Hat* : ,1375 *l.* :: 624 *Hats* : 85,8 *l.*

```
,1375
  624
-----
 5500
2750
8250
---------
85,8000 = 85 l. : 16 s. the Price of all the Hats.
```

Thirdly to know what *Cotton* at 11 *d.* *per Pound* can be had for that Money;

Say, As ,04583 *l.* : ,00892 *C.* :: 85,8 *l.* : 16,6982 *C.*

```
            ,00892
              85,8
           -------
              7136
             4460
            7136
           -------
,04583)    ,765336
  0458      765336
-------    -------
,04125)    ,6888024 (16,6982 C.
            4125
            -----
            27630
            24750
            -----
            ·28802
             24750
            ------
            ·40524
             37125
            ------
             33990
             33000
            ------
              9900
              8250
            ------
              1650
```

Hence it appears that *B* must give in Exchange 16,6982 *C.* = 16 *C.* : 2 *Q.* : 24 *lb.* of *Cotton* at 11 *d.* *per lb.* for 52 Dozen of *Hats* at 2 *s.* 9 *d.* *per Hat*.

Theſe two Queſtions well underſtood, are ſufficienr for all other Caſes in *this Rule.*

Gain *and* Loſs.

By *this Rule* Men of *Trade* and *Buſineſs* know what they get by *Retailing Goods*; and in *caſe of Damage*, what they *loſe*, by ſelling it at any given Rate; and whether they *gain* or *loſe*, to know at *what Rate per Cent.*

Queſtion 1. If I buy *Cambricks* at 5 *s.* 6 *d. per* Yard, and ſell them at 8 *s.* 9 *d.* What is the Gain *per Cent?*

		s. d.	*s. d.*	*l.*	*l. s. d.*
Say	Vulg.	5—6 :	8—9 : :	100 :	159—1—$9\frac{1}{4}$
	Dec.	,275 :	4375 : :	100 :	159,09

		s. d.	*s. d.*	*l.*	*l. s. d.*	
Or, thus,	Vulg.	5—6 :	3—3 : :	100 :	59—1—$9\frac{1}{4}$	The Gain
	Dec.	275 :	1625 : :	100 :	59,09	*per Cent.*

By the very ſame Manner of Working you find the *Loſs.*

Queſtion 2. If I buy *Cambrick* at 5 *s.* 6 *d. per* Yard, How muſt I ſell it to Gain 59 *l.* 1 *s.* $9\frac{1}{2}$ *d. per Cent?*

The *Converſe* of the *laſt Method* ſolves this Queſtion.

		l.	*l. s. d.*	*s. d.*	*s. d.*	
Thus ſay,	Vulg.	100 :	159—1—$9\frac{3}{4}$: :	5—6 :	8—9	The Price
	Dec.	100 :	159,09 : :	,275 :	,4375	*per Yard.*

Queſtion 3. If I buy a C. Weight of *Tobacco* for 4 *l.* 13 *s.* 4 *d.* and ſell it at 11 *d. per Pound*; What do I *Gain* or *Loſe*, and at *what per Cent?*

Firſt find what a *C. Weight* will amount to at 11 *d. per Pound.*

		lb.	*d.*	*lb.*	*l. s. d.*	
Thus,	Vulg.	1 :	11 : :	112 :	5—2—8	Anſw.
		C.	*l.*	C.	*lb.*	
	Dec.	,00892 :	,04583 : :	1 :	5,13	

		l. s. d.	*l. s. d.*	*l.*	*l.*	
Then ſay,	Vulg.	4—13—4 :	5—2—8 : :	100 :	110	*viz.* 10 *l.*
	Dec.	4,6 :	5,13 : :	100 :	110	*per Cent.*

Then

	l.	*s.*	*d.*		*l.*
Then from	5	2	8	And from	110
Take	4	13	4	take	100
Remains	0	9	4	the *Gain*, at the Rate of	10 *l. per C.*

The *Converse* of this needs no Example.

Question 4. If I buy $5\frac{1}{2}$ *Loads of Wheat* for 45 *l.* 16 *s.* 8 *d.* For how much must I sell it *per* Quarter, to gain 6 *l.* 10 *s.* by the Bargain?

		l.	*s.*	*d.*
First	To the given Price =	45	16	8
	Add the designed Gains =	6	10	0
	The Sum is =	52	6	8

for which the said Wheat must be sold. $5\frac{1}{2}$ Load = 27,5 Quarters.

Therefore say 27,5 *Qrs.* : 52,3 *l.* :: 1 *Q.* : 1,903 *l.* = 1 *l.* — 18 *s.* — $0\frac{1}{4}$ *d.* the Answer.

These being the *principal Cases* of this Rule, are sufficient if well understood; and the Operations at large are omitted for the Exercise of the Ingenious.

Exchange.

Both the *Name* and *Business* of *Exchange* is *analogous* to that of *Barter*; only that relates to *Goods* and *Commodities*; whereas this is concern'd in *Foreign Coins*, *Weights*, and *Measures*.

Exchange then consists in finding the *true Sum or Value* of one *Country Coin*, &c. *equivalent* to any given *Sum* or *Value* of that of any *other Country*.

The *Par of Exchange* is the *fixt* and *Standard Value* of *Foreign Coins*, &c. express'd in *Sterling Money* of our own; and is that in the Tables. 'Tis so called because in *Exchange*, *Par pro Pari*, i. e. One equal Value for another, is given.

The *Course of Exchange* is the *current Price* of *Exchange*, always unsettled, being sometimes *above* and sometimes *below* the *Par*; according to the various Circumstances and Accidents of *Trade* and *Nations*.

The

The *Course of Exchange* is published in the Weekly Papers and Pamphlets, which compar'd with the *Par* in the *Tables*, it appears whether it be above or below it at any Time.

EXAMPLE.

		Course of Exchange. s.	d.		*The Par.* s.	d.	*Difference.* s.	d.	
June 1733. at	*Amsterdam*	35	0	*Shil. d.*	—33	4	—1	8	above.
	Rotterdam	35	1		—33	4	—1	7	
	Hamburgh	34	1		—33	4	—0	7	
	Antwerp	35	4		—33	4	—2	0	
	Madrid	41	$\frac{7}{8}$	*Pence.*	—54	$\frac{1}{4}$	—0	$12\frac{3}{8}$	below.
	Legborn	50	$\frac{3}{4}$		—54	$\frac{1}{4}$	—0	$3\frac{1}{2}$	
	Genoa	53	$\frac{5}{8}$		—54	$\frac{1}{4}$	—0	$0\frac{5}{8}$	
	Venice	48	$\frac{7}{8}$		—52	$\frac{1}{2}$	—0	$3\frac{5}{8}$	

It is to be observed, that when the *Course* of *Exchange* is *above the Par*, tis a general Indication that our Trade is *prosperous*, and the Nation on the *Gainful* Side; as on the contrary, if tis *below the Par*, the *Trade* is *bad*, and the Nation *looser*.

The Par of Exchange in *Coins*, *Measures*, *Weights*, &c. between Us and Foreigners, are express'd in the following Tables, and which I have reduced to *Decimals* for more convenient and ready resolving of Questions in this Rule.

Low Country *Coins*.

	l.	*s.*	*d.*	*l.*
A Stiver — —	0	0	$1\frac{1}{5}$	0,005
A Flemish Shilling (= 6 Stivers)	0	0	$7\frac{1}{5}$	0,03
A Gilder (= 20 Stivers) —	0	2	0	0,1
A Flem. Pound (=33 *s.* 4 *d.* Flemish)	1	0	0	1,0
An Emblem Doller —	0	2	$3\frac{3}{5}$	0,115
A Campen Doller —	0	7	$7\frac{1}{5}$	0,13
A Zeland Doller — —	0	3	0	0,15
A Lyons Doller — —	0	4	0	0,2
A Specie Doller — —	0	5	0	0,25
A Duccatoon — —	0	6	$3\frac{3}{5}$	0,315

German *Coins.*

	l.	*s.*	*d.*	*l.*
A Rix Doller *of the* Empire —	0	4	3 $\frac{1}{4}$	0,2156
A Gilder *of* Noremberg —	0	7	1	0,354

French *Coins.*

A Denier — —	0	0	0 4 $\frac{3}{0}$	0,00031
A Soulz (= 12 Deniers) —	0	0	0 1 $\frac{9}{0}$	0,00374
A Livre (= 20 Soulz) —	0	1	6	0,075
A Crown (= 3 Livres) —	0	4	6	0,225

Spanish *Coins.*

Malvadies 13,$\frac{7}{0}$ make —	0	0	0 $\frac{1}{4}$	0,00104
A Rial (=372 Malvadies) —	0	0	6 $\frac{3}{4}$	0,02812
A Piece *of* 8 (Rials) Pillar —	0	4	6 $\frac{3}{4}$	0,22812
A Piece *of* 8 Mexico —	0	4	6	0,225
A Piece *of* 8 Peru — —	0	4	5	0,22083
A Piece of 8 Seville — —	0	4	6	0,225

Portugal *Coins.*

Rees, 12,$\frac{4}{0}$ *of which make* —	0	0	1	0,00416
Mill Ree (= 1000 Rees) —	0	6	8 $\frac{1}{2}$	0,33541
A Testoon — —	0	1	3	0,0625

Italian *Coins.*

The Livre *at* Leghorn —	0	0	9	0,0375
Crown currant *at* Florence —	0	5	3	0,2625
Ducat de Banco, *at* Venice —	0	4	4	0,216
The Courrant Du at —	0	3	4	0,16
Ducat *at* Naples —	0	5	0	0,25
A St. Mark — —	0	2	10	0,1416
A Palermo Florin — —	0	2	6	0,125

Deci-

Decimal Tables of Foreign Long Meaſures.

London	Foot	1,000
Paris	—	1,068
Amſterdam	—	0,942
Brill	—	1,103
Antwerp	—	0,946
Dort	—	1,184
Leyden	—	1,033
Lorrain	—	0,958
Mechlin	—	0,919
Middleburgh	—	0,991
Strasburgh	—	0,920
Bremen	—	0,964
Cologn	—	0,954
Frankford ad Mœn		0,948
Spaniſh	—	1,001
Toledo	—	0,900
Roman	—	0,967
Bononia	—	1,204
Mantua	—	1,569
Venice	—	1,162
Dantzick	—	0,944
Copenhagen	—	0,965
Prague	—	1,026
Riga	—	1,831
Turin	—	1,062
Greek	—	1,007

Lyons	— Ell	3,976
Bologn	—	2,056
Amſterdam	—	2,269
Antwerp	—	2,273
Leyden	— —	2,260
Frankford	—	1,826
Hamburgh	—	1,905
Leipſick	—	2,260
Lubeck	— —	1,908
Noremburgh	—	2,227
Bavaria	—	0,954
Vienna	—	1,054
Bononia	—	2,147
Dantzick	—	1,903
Florence	— Brace	1,913
Spaniſh	— Palm	0,751
Spaniſh	— Vare	3,001
Lisbon	—	2,750
Gibraltar	—	2,760
Toledo	—	2,685
Naples	— Canna	6,880
Genoa	— Palm	0,830
Milan	— Calamus	6,544
Parma	— Cubit	1,866
China	—	1,016
Cairo	— --	1,824
Turkiſh	— Pike	2,200
Perſian	— Araſh	3,197

	lb.
London, The Pound Averdupois	1,00
Paris —	0,93
Lyons — —	1,09
Bologn —	0,89
Amſterdam —	0,93
Antwerp —	0,98
Leyden —	0,96
Lorain —	0,98
Mechlin —	0,98

	lb.
London, The Pound Averdupois	1,00
Middleburgh —	0,98
Strasburgh —	0,93
Bremen —	0,94
Cologn —	0,97
Frankford —	0,93
Hamborough —	0,95
Leipſick —	1,15
Noremburgh —	0,94

	l.		l.
London, the Pound Averdupois	1,00	London, the Pound Averdupois	1,00
Copenhagen —	0 94	Genoa —	1,42
Vienna —	0,83	Mantua —	1,43
Castile —	0,99	Milan —	1,40
Lisbon —	1,06	Parma —	1,43
Gibraltar —	1,03	Venice —	1,53
Toledo —	1,00	Dantzick —	1,19
Rome —	1,23	Prague —	1,06
Bononia —	1,27	Cairo —	1,61
Florence —	1,23	Constantinople —	0,86
Naples —	1,43		

Having presented the Reader with large Tables of the *Par of Exchange*; I shall next exhibit a *Table* of the *Course of Exchange*, in *Pence*, and *Shillings* and *Pence*, (into which *Foreign Coins* are reduced) in *Decimal Parts* of a *Pound Sterling*.

Decimal Tables of the Course of Exchange.

P.	l.	P.	l.	P.	l.	P.	l.
1/4	,001	16	,06	37	,15416	58	,2416
1/2	,002	17	,07083	38	,1583	59	,24583
3/4	,0031	18	,075	39	,1625	60	,25
3/8	,0016	19	,07916	40	,16	61	,25416
5/8	,0026	20	,083	41	,17083	62	,2583
7/8	,0036	21	,0875	42	,175	63	,2625
1	,00416	22	,0916	43	,17916	64	,26
2	,0083	23	,09583	44	,183	65	,27083
3	,0125	24	,1	45	,1875	66	,275
4	,016	25	,10416	46	,1916	67	,27916
5	,02083	26	,1083	47	,19583	68	,283
6	,025	27	,1125	48	,2	69	,2875
7	,02916	28	,116	49	,20416	70	,2916
8	,03	29	,12083	50	,2083	71	,29583
9	,0375	30	,125	51	,2125	72	,3
10	,0416	31	,12916	52	,216	73	,30416
11	,04583	32	,13	53	,22083	74	,3083
12	,05	33	,1375	54	,225	75	,3125
13	,05416	34	,1416	55	,22916	76	,316
14	,0583	35	,14583	56	,23	77	,32083
15	,0625	36	,15	57	,2375	78	,325

P.	l.	P.	l.	P.	l.	P.	l.
79	,32916	85	,35416	91	,37916	97	,40416
80	,3	86	,3583	92	,383	98	,4083
81	,3375	87	,3625	93	,3875	99	,4125
82	,3416	88	,36	94	,3916	100	,416
83	,34583	89	,37083	95	,39583		
84	,35	90	,375	96	,4		

S. P.	Flem. P.	Eng. P.	S. P.	Flem. P.	Eng. P.
32.0	1,6	,625	8	1,73	,576923
1	1,60416	,623376	9	1,7375	,575539
2	1,6083	,621761	10	1,7416	,574162
3	1,6125	,620155	11	1,74583	,572792
4	1,616	,618556	35.0	1,75	,571428
5	1,62083	,616966	1	1,75416	,570071
6	1,625	,615384	2	1,7583	,568720
7	1,62916	,613810	3	1,7625	,567375
8	1,63	,612244	4	1,76	,566037
9	1,6375	,610687	5	1,77083	,564705
10	1,6416	,609137	6	1,775	,563380
11	1,64583	,607594	7	1,77916	,562060
33.0	1,65	,606060	8	1,783	,560747
1	1,65416	,604534	9	1,7875	,559440
2	1,6583	,603015	10	1,7916	,558139
3	1,6625	,601503	11	1,79583	,556844
4	1,66	,6	36.0	1,8	,5
5	1,67083	,598503	1	1,80416	,554272
6	1,675	,597014	2	1,8083	,552995
7	1,67916	,595533	3	1,8125	,551724
8	1,683	,594059	4	1,816	,550458
9	1,6875	,592592	5	1,82083	,549199
10	1,6916	,591133	6	1,825	,547945
11	1,69583	,589680	7	1,82916	,546697
34.0	1,7	,588235	8	1,83	,54
1	1,70416	,586797	9	1,8375	,544217
2	1,7083	,585365	10	1,8416	,542986
3	1,7125	,583941	11	1,84583	,541760
4	1,716	,582524	37.0	1,85	,540
5	1,72083	,581113	1	1,85416	,539325
6	1,725	,579710	2	1,8583	,538116
7	1,72916	,578313	3	1,8625	,536912

S. P.	Flem. P.	Eng. P.	S. P.	Flem. P.	Eng. P.
4	1,86	,535714	9	1,8875	,529801
5	1,87083	,534521	10	1,8916	,528634
6	1,875	,53	11	1,89583	,527472
7	1,87916	,532150	38.0	1,9	,526315
8	1,883	,530973			

From these Sets of Tables of the *Par* and *Course* of *Exchange*, (which are more compleat than any I have yet seem in the *Common Books* of *Arithmetick*,) the ingenious Accomptant will readily cast up any *Bill of Exchange*; or convert the *Coins*, *Weights* and *Measures* of any other Country into *the same* of our *own*. And by comparing the *Course* with the *Par*, may see whether our *Nation Gains* or *Loses* by trading to any *Foreign Parts*, and in what Proportion.

Question 1. Suppose at *Venice* I would exchange 175 *l.* 12 *s.* 6 *d.* for their *Ducats de Banco* at 4 *s.* 4 *d per Piece*, How many must I have?

First, 175 *l.* — 12 *s.* — 6 *d.* = 175,625 *l.*; and 4 *s.* — 4 *d.* = ,216 *l.*

Then ,216) 175,625 (759,3 fere. Ducats } The Answer.

```
,216)  175,625
 21    175625
----  -----------
,195)  158,0625
       1365
       ----
        1156
         975
        ----
         1812
         1755
         ----
          575
```

Question 2. The *Course* of *Exchange* at *Madrid* being now 41 $\frac{7}{8}$ *d. per Piece* of 8 *Mexico*, what Number of those *Pieces* may I have in exchange for 533,766 *l.*?

Per Table the First, *of the Course* of *Exchange* 41 $\frac{7}{8}$ *d.* = ,17443 *l.*

Then ,17443) 533,766 (=3060 Pieces, the Answer.

Quest-

Question 3. A *Bill of Exchange* was accepted at *London* for the Payment of 933,93 *l.* for the same value delivered at *Lisbon* in *Milrees*; Exchange at 5 *s.* 4 *d.* *per Piece.* How many of those *Milrees* was paid at *Lisbon*?

First 64 *d.* (= 5 *s.* 4 *d.*) = ,26 *l.*

Then ,26) 933,93 (=3502,25 *Milrees*, the Answer.

Now had these 3502,25 *Milrees* been exchanged at *Par* (viz. 6 *s.* 8 ½ *d.*) they would have amounted to 1174,6895 *l.* which is above 240 *l.* more; and consequently there was so much *loss*.

Question 4. In 1421 *Pieces* of 8 *Peru*, How many *English* Pounds *Sterling*, *Exchange at Par*?

Multiply —	1421
By the Par =	,22083
9)	4263
	4736
	11368
	28420
	2842
Answer *l.*	313,80416

Another Way.

			l.
For	1	=	0,22083
	20	=	4,41666
	400	=	88,33333
	1000	=	220,83333
	1421	=	313,80416

Question 5. When the *Exchange* from *Antwerp* to *London* is at 1 *l.* 4 *s.* 7 *d.* (=34 *s.* 7 *d.*) *Flemish*; How many Pounds *English* at *London* will ballance 236 *l.* *Flemish* at *Antwerp*?

Proceed thus,	Multiply the Tabular Number for *English* Pounds	,57831, &c.
	By the given Number —	236
	The Product is the Answer	*l.* 136,4819, &c.

Question 6. What Number of *Flemish Pounds* will be equivalent to 400 *l.* *Sterling*, *Exchange* at 1 *l.* 13 *s.* 6 *d.* (= 33 *s.* 6 *d.*)?

Multiply the Pound *Flemish*	1,675
By the Number of Pounds *Sterling*	400
The Product is the Answer =	670,000 Pounds *Sterl.*

Question 7. A *Dutch* Man sells 2050 *Flemish Ells* of *Holland* to an *English* Man, a *Spaniard*, a *Venetian*, an *Italian*, and

and a *Portuguese*; who are to have *each a like Quantity*; *Quere* how much in their *own Country Measure*?

First 2050 *Flemish Ells* are equal to 1230 *Ells English*, equal to 4612,5 Feet, which divided by 5 quotes 922,5 Feet each.

Then by the Table of Measure 922,5 Feet are equal to —	307,5	Yards for the *Briton*.	Ans.
	793,89	Feet for the *Venetian*.	
	1228,36	Palms for the *Spaniard*.	
	482,2	*Braces* for the *Italian*.	
	335,45	*Vares* for the *Portuguese*.	

Question 8. What Number of *Pounds Averdupois* at *Vienna* will *Equiponderate* 270 Pounds *Averdupois Weight* at *London*?

Divide 270 by ,83. the Quotient 325,3 *l.* is the Answer.

But if 'twas required to know what Number of Pounds Averdupois Weight at *London* would equal any given Number at any other place, then you must multiply by the *Tabular Number*.

Alligation, *or* Rule *of* Composition.

Alligation (so called of the Latin Word *Alligo*, to *bind* or *tie together*; because the vulgar Way is to *tie* or *connect together* the Numbers concern'd in the Work,) is a *Rule for Compounding or Mixing several Ingredients of different Sorts together, in any Manner or Proportion.* And is divided into *Alligation*, *Medial* and *Alternate*.

Alligation Medial is that by which the *Mean Rate* or *Price* of any *Mixture* is found when the *particular Quantities*, and their *Prices*, are given; and it is perform'd by this

Rule,

Multiply each Quantity by its Price; then say, As the Sum of all the Quantities, is to the Sum of the said Products, so is any Part of the Mixture, to the Mean Price of of that Part.

Question 1. A *Tobacconist* would mix 20 *lb.* of *Tobacco* at 9 *d. the Pound* with 60 *lb.* at 14 *d. per lb.* with 40 *lb.* at 18 *d. per lb.* and with 12 ¾ *lb.* at 2 *s. per lb. Quere* what a *Pound* of such a *Mixture* is worth?

First

	lb.	*Rate.*			*Products.*
First,	20	× ,0375	produceth	—	0,75
	60	× ,0583	produceth	—	3,5
	40	× ,075	produceth	—	3,0
	12,75	× ,1	produceth	—	1,275

The Sum 132,75 of the Quan. The Sum 8,525 the Prod.

Then say, As 132,75 *lb.* : 8,525 *l.* : : 1 *lb.* : 0642 *l.*

```
132,75) 8,5250 (,0642 = 1 s. 3 1/4 d. per lb. Answer.
        79650
        -----
        56000
        53100
        -----
         29000
         26550
         -----
          2450
```

Question 2. A *Goldsmith* hath *Gold* 12 *oz.* worth 4 *l.* *per oz.* 8,3 *oz.* at 4 *l.* 5 *s.*; 3 *oz.* at 4 *l.* 6 *s.* 8 *d.*; and 9 *oz.* at 4 *l.* 13 *s.* 4 *d.* Suppose these all melted down together, *Quere* what an *Ounce* of that *Mixture* would be worth?

	oz.	*l.*		*l.*
First,	12	× 4	produceth	48
	83	× 4,25	produceth	35,275
	3	× 4,3	produceth	13
	9	× 4,6	produceth	42

The Sum 32,3 The Sum 138,275 of the Prod.

Then say, As 32,3 *oz.* : 138,275 *l.* : : 1 *oz.* : 4,2809 *l.*

```
32,3) 138,275 (4,2809 = 4 l. 5 s. 7 1/4 d.
      1292
      ----
       907
       646
       ----
       2615
       2584
       -----
        3100
        2907
        ----
         193
```

Answer 4 *l.* 5 *s.* 7 $\frac{1}{4}$ *d.*

Alli-

Alligation Alternate is that by which the *particular Quantities* of every Ingredient in any *Mixture* are found; when the *particular Rates* of every one of the Ingredients; and the *Mean Rates* are given.

This is (as it were) the *Converse* of the former, and admits of *three Cases*.

Case 1. The *particular Rates* and the *Mean Rate* being given, to find the *Quantity* of each Ingredient for the *Mixture* proposed.

Question 1. A *Vintner* would make a Mixture of *Malaga* at 7 *s.* 6 *d. per* Gallon; with *Canary* at 6 *s.* 9 *d. per* Gallon; *Sherry* at 5 *s. per* Gallon, and *White Wine* at 4 *s.* 3 *d. per* Gallon: What *Quantity* of each Sort must he take, that the *Whole Measure* may be sold for 5 *s.* 10 *d. per* Gallon?

Note, In all Questions of this Nature, where two or four Things are mixt together, when *one half* of the Prices are *Greater*, and the *other half lesser* than the *Mean Rate*, you must set a *greater* and *lesser* Price *above*, and the same *below* the *mean Price*; then take the *Difference* between the *mean Rate* and the *particular Rates*, and place them *alternately*, and they will be the Quantities requir'd.

	Rates.		*Differences.*	
	s. 7,5	*Malaga*	1,583	Gal. of Malaga.
	4,25	*White*	1,6	Gal. of White.
Mean Rate = 5,83	5	*Sherry*	0,916	Gal. of Sherry.
	6,75	*Canary*	0,83	Gal. of Canary.

The Sum of those Differences is = 5,0 Gallons the whole Mixture.

Note, The *Differences* are not only the *Quantities*, which answer the Question, but *any other Numbers*, in the *same Proportion* as they are, will answer the Question as well.

For — —	1,583	1,6	0,916	0,83
All multiplied by	—	—	—	3
Produce the Proportionals	4,75	5	2,75	2,5
These multiplied by	—	—	—	4
Produce these whole Numbers in rhe same *Ratio*, and so on *In infinitum*.	19.	20.	11.	10.

In *Case* one of the *Given Rates* (when more than two) be {Greater / Lesser} and all the rest {Lesser / Greater} than the *mean Rate*.

Then

Then the *mean Rate*, *particular Rates*, and Differences must stand as in the following Examples.

$$\textit{Mean Rate } 20\left\{\begin{array}{ll} \textit{P. R.} & \textit{Differences.} \\ 18 & 4+2 \\ 22 & 2 \\ 24 & 2 \end{array}\right\} \textit{Mean Rate } 20\left\{\begin{array}{ll} \textit{P. R.} & \textit{Differ.} \\ 14 & 4 \\ 18 & 4 \\ 24 & 6+2 \end{array}\right\}$$

$$\textit{M. Rate } 4\left\{\begin{array}{ll} \textit{R.} & \textit{Differences.} \\ 2 & 1+5+13 \\ 5 & 2 \\ 9 & 2 \\ 17 & 2 \end{array}\right\} \text{Or } 40\left\{\begin{array}{ll} \textit{R.} & \textit{Differences.} \\ 10 & 8 \\ 25 & 8 \\ 36 & 8 \\ 48 & 30+15+4 \end{array}\right\}$$

The Method is the same for any other given Rates, or Prices.

Case 2. When the *Particular Rates*, the *Mean Rate*, and the Quantity of one Ingtedient is given; to find the *Quantity of all the rest* of the Ingredients.

This is call'd *Alligation Partial*, because a *Part* of the Mix'd Ingredients only are known.

In this Case you must set down the *mean Rate*, the *particular Rates*, and their *Differences* just as before; then say,

Rule.

As the Difference opposite to the known Quantity, is to the known given Quantity; so is any other Difference, to the Quantity of its opposite Name.

Question 2. How much *Malaga* at 7 *s.* 6 *d.*; *Sherry* at 5 *s.*; *White Wine* at 4 *s.* 3 *d.* the Gallon, must be mixt with eighteen Gallons of *Canary* at 9 *s.* 9 *d. per* Gallon, that the Whole may be sold for 5 *s.* 10 *d. per* Gallon?

$$\textit{Mean Rate } 5{,}83\left\{\begin{array}{ll} \textit{Rates.} & \\ 7{,}5 & \textit{Malaga} \\ 5 & \textit{Sherry} \\ 6{,}75 & \textit{Canary} \\ 4{,}25 & \textit{White} \end{array}\right. \begin{array}{l} \\ 0{,}83 \\ 1{,}6 \\ 1{,}583 \\ 0{,}916 \end{array}\left\}\textit{Differences.}\right.$$

$$\text{Then, As } 1{,}583 : 18 :: \left\{\begin{array}{ll} 0{,}83 & \text{to the Gallons of } \textit{Malaga.} \\ 1{,}6 & \text{to the Gallons of } \textit{Sherry.} \\ 0{,}916 & \text{to the Gallons of } \textit{W. Wine.} \end{array}\right.$$

I leave the Work to exercise the Learner.

Case 3. The *particular Rates*, the *mean Rate*, and the *Sum* of all the Quantities of the Ingredients given; hence to find the particular Quantities of the Mixture.

This is call'd *Alligation Total*; becauſe the whole Quantity of the Mixture is given.

It is thus perform'd; ſet down the *mean Rate*, the *particular Rates*; and find their *Differences* as before. Then ſay,

Rule. *As the Sum of all the Differences, is to the Sum of all the Quantities; ſo is each particular Difference, to its particular Quantity.*

Queſtion 3. Suppoſe it required to mix *Malaga* at 7 *s.* 6 *d.* with *Canary* at 6 *s.* 9 *d.* *Sherry* at 5 *s.* and *White Wine* at 4 *s.* 3 *d.* *per* Gallon; and the *whole Quantity* to be $84\frac{5}{8}$ Gallons, and to be ſold at 5 *s.* 10 *d.* *per* Gallon; Quere the Quantity of each Sort for the Mixture?

$$\text{Mean Rate } 5{,}8\dot{3}\left\{\begin{array}{ll}7{,}5 & \textit{Malaga}\\ 5 & \textit{Sherry}\\ 6{,}75 & \textit{Canary}\\ 4{,}25 & \textit{White}\end{array}\right\}\begin{array}{l}0{,}8\dot{3}\\ 1{,}\dot{6}\\ 1{,}58\dot{3}\\ 0{,}91\dot{6}\end{array}\left.\vphantom{\begin{array}{l}0\\0\\0\\0\end{array}}\right\}\text{ Differences.}$$

$$5 = \text{The Sum.}$$

$$\begin{array}{c}\textit{Gallons.}\\ \text{Then, As } 5 : 84{,}625 ::\end{array}\left\{\begin{array}{l}0{,}8\dot{3}\\ 1{,}\dot{6}\\ 1{,}58\dot{3}\\ 0{,}91\dot{6}\end{array}\right\}\text{ to the Quantity in Gallons of }\left\{\begin{array}{l}\textit{Malaga.}\\ \textit{Sherry.}\\ \textit{Canary.}\\ \textit{White.}\end{array}\right.$$

Note, The Work of theſe, and ſuch like Proportions, may be very much ſhortened, and eaſierly perform'd by a *common Multiplicator* as in *Fellowſhip*.

Now becauſe *Alligation alternate* anſwers not Queſtions compleatly, that is, does not give all the Anſwers ſuch Queſtions are capable of; and ſo perhaps not always thoſe which beſt ſuit the occaſion; I ſhall ſhew (from Mr. *Ward*) how this Imperfection of *common Arithmetick* is ſupplied by *Algebra*, and all the *poſſible* Anſwers to any Queſtions may be clearly and eaſily diſcover'd.

Queſtion 4. A *Tobacconiſt* hath three Sorts of Tobacco, *viz.* one of 2 *s.* 8 *d. per* Pound; another of 20 *d. per* Pound; a third ſort of 16 *d. per* Pound; of theſe he would make a *Mixture* to contain 56 Pound that may be ſold for 22 *d. per* Pound; *How much* of each Sort may he take?

Let

Let $\begin{cases} a = \text{the Quantity of that worth } 2\,s.\ 8\,d. = 32\,d. \\ e = \text{that of } 20\,d. \text{ per Pound.} \\ y = \text{that of } 16\,d. \text{ per Pound.} \end{cases}$

Then	1.	$a+e+y=56$
And	2.	$32a+20e+16y=1232$
$1-a$	3.	$e+y=56-a$
$2-32a$	4.	$20e+16y=1232-32a$
3×16	5.	$16e+16y=896-16a$
$4-5$	6.	$4e=336-16a$
$6\div4$	7.	$e=84-4a$
$3-7$	8.	$y=3a-28$

Hence 'tis evident from the 7th Step that the Quantity signified by a must be less than 21, and (by the 8th) Step greater than $9\frac{1}{3}$. That is a may be any Number between 21 and $9\frac{1}{3}$.

If there be *more* than *three Quantities* concerned in the Question, the Work will be more large; because the *Limits* of all the Quantities above *two*, must be found.

Question 5. Suppose it were required to mix *four Sorts* of *Wine* together; *viz.* one worth 7 *s.* 4 *d. per* Gallon; a second worth 4 *s.* 7 *d.* a third worth 3 *s.* 8 *d.* and a fourth worth 2 *s.* 9 *d. per* Gallon. How much of each Sort must be taken to make a Mixture of 63 Gallons, to be sold for 5 *s.* 6 *d. per* Gallon, without Loss?

			s.	d.	d.
First let	a =	that Quantity worth	7	4	= 88
	e =	that worth —	4	7	= 55
	y =	that worth —	3	8	= 44
	u =	that worth —	2	9	= 33
		the mean Rate —	5	6	= 66

Then	1.	$a+e+y+u=63$
And	2.	$88a+55e+44y+33u=4158.$
$1-a$	3.	$e+y+u=63-a$
$2-88a$	4.	$55e+44y\ 33u=4158-88a$
3×33	5.	$33e+33y+33y=2079-33a$
$4-5$	6.	$22e+11y=2079-55a$
$6\div11$	7.	$2e+y=189-5a$
3×55	8.	$55e+55y+55u=3465-55a$
$8-4$	9.	$11y+22u=33a-693$
$9\div11$	10.	$y+2u=3a-63$
Suppose	11.	$a=22.$ Then $5a$ 110, and $3a=66$
Per 7th	12.	$2e+y=189-5a=79$

$12-2e$

$12-2e$	13	$y=79-2e$
Per 3d.	14	$e+y+u=63-a=41$
$14-e$	15	$y+u=41-e$
$15-13$	16	$u=e-38$

From the ſeventh and tenth Steps it appears, that the Quantity denoted by a, muſt be leſs than $37\frac{4}{5}$, and greater than 21 Gallons; whence 16 anſwer flow from the Limits of a only. Then if a be put $=22$, by the thirteenth and ſixteenth Steps it appears $e=39$. $y=1$, and $u=1$. And thus proceeding with each ſingle value of a, above 120 Anſwers may be found to this Queſtion in whole Numbers; in Fractions, infinite.

Poſition, *or* Rule *of* Falſe.

This Rule of *Poſition*, or rather *Suppoſition*, is ſo call'd, becauſe we *ſuppoſe* or make a *Poſition* of ſome *uncertain* Numbers, in order that by reaſoning from them we may gain the *true Number* ſought; and becauſe thoſe *Poſitions* are altogether at *random* or *adventure*, the *Rule* is alſo call'd *Falſe*.

The Uſe of this *Rule*, before the common Knowledge of *Algebra*, was much more conſiderable than ſince; becauſe that Art ſupplies *Theorems* for reſolving all kind of Queſtions in this Rule in a better and more curious a manner than here; Yea ſome of the *beſt Pieces* of *Arithmetick* have intirely diſcarded it, and others poſt-pone it, as *obſolete* and of little uſe, ſince *Algebra*.

Queſtions in this are moſtly perform'd by one or two *Suppoſitions*; if by one, the Rule is ſaid to be of *Single Poſition*; if two Suppoſitions are neceſſary, 'tis called *Double Poſition*.

Single Poſition.

Queſtion 1. *Three Merchants A*, *B*, *C* trade in Company, and gain 100 *l.* of which *A* had a certain Part, *B* had twice as much, and *C* had thrice as much as *B*; How much had each Man?

Suppoſe *A* had 4 *l.* then *B* muſt have 8 *l.* and *C* would have 24 *l.* which together make 36 *l.* but ſhou'd have been an 100 *l.*

There-

Therefore Reason by Proportion

$$\text{Thus, As } \overset{l.}{36} : \overset{l.}{100} :: \begin{cases} 4 : 11,1 = A \\ 8 : 22,2 = B \\ 24 : 66,6 = C \end{cases}$$

(the column $11,1$, $22,2$, $66,6$ headed *l.*)

Their several Parts added make — 100 for Proof.

Question 2, A *Schoolmaster* being asked how many *Scholars* he had; answer'd, if I had as many, and $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many, I should have *99*. How many had he?

Suppose he had 40; Then $40 + 40 + 20 + 10 = 110$. but it should have been but *99*. Therefore say

As 110 : 40 :: *99* : 36 Scholars, the Answer.

Question 3. *Three* Men *A*, *B*, *C* buy a Ship for 310 *l.* 15 *s.* of which *A* paid an unknown Sum; *B* paid $2\frac{1}{2}$ as much: and *C* $3\frac{1}{3}$ as much: How much did each Man pay?

Suppose *A* paid 48 *l.* then *B* paid $48 \times 2,5 = 120$ *l.* and *C* must pay $48 \times 3,3 = 160$ *l.* But $48 + 120 + 160 = 328$ instead of 310,75 *l.*

Say therefore, As 328 : 48 :: 310,75 : 45,4756, &c.

		l.
Then *A* paid	— —	45,4756
B paid	($45,4756 \times 2,5 =$)	113,689
C paid	($45,4756 \times 3,3 =$)	151,5853
Proof is the Sum	—	310,75

Double Position.

In the *Double Rule*, *two* Suppositions are used, because here the Numbers cannot be parted to find the Answer by Proportion as before.

Therefore when we make *two Suppositions*, and miss in both, observe the *Nature* of the *Errours*, whether they be *Greater* or *Lesser* than the *Number proposed*; and accordingly mark them with the *Signs More* or *Less*, viz. +, —; and place them precisely against their *proper Suppositions*; then observe the general

Rule,

Rule,

Viz. *As the Difference of the Errours if alike, (or their Sum if unlike) is to the Difference of the Suppositions; so is either of the Errours, to a fourth Number.*

The *fourth* Number *add* to, or *substract* from, the Supposition opposite to it; and you have the Number sought.

Question 1. Admit *three Merchants* build a Ship which cost 1360 Pounds. *A* pays a certain Part unknown; *B* paid $2\frac{1}{2}$ as much, wanting 15,5 *l.* and *C* paid as much as both *A* and *B*, and 75,25 *l.* over; How much did each Man pay?

First, Suppose *A* paid 200 *l.* then *B* must have paid 484,5 *l.* and *C* paid 759,75 *l.* But those three Sums, *viz.* 200 + 484,5 + 759,75 = 1444,25 *l.* which is more than 1360 by 84,25 *l.* Wherefore the first Errour is — + 84,25 *l.*

Secondly, Suppose *A* paid 180 *l.* then *B* paid 434,5 *l.* and *C* paid 689,75 *l.* But 180 + 434,5 + 689,75 = 1304,25 *l.* which is too little by 55,75 *l.* therefore the Suppositions and their Errours will stand thus,

The *First* Supposition	200,	+ 84,25	Errour.
The *Second* Supposition	180,	— 55,75	Errour.
The *Difference* of Supposi. =	20	140 =	Sum of Errours.

Then by the General Rule, say,

$$\begin{matrix} \text{As,} \\ \text{Or,} \end{matrix} \left\{ \begin{matrix} 140 : 20 :: 55,75 : 7,964 \ \&c. \\ 140 : 20 :: 84,25 : 12,035 \ \&c. \end{matrix} \right\}$$

$$\text{Then,} \left\{ \begin{matrix} 200 - 12,035 = \\ 180 + \ \ 7,964 = \end{matrix} \right\} \overset{l.}{187,964} = A\text{'s Part.}$$

Then *B* must have paid	—	454,410 =	*B*'s Part.
And *C* must have paid	—	717,624 =	*C*'s Part.
The Sum of which is	=	1360	for Proof.

Note, When the *Errours* are *equal* and have *unlike* Signs; *half the Sum* of the *Suppositions*, is the Number sought.

Ex-

Extraction *of* Roots.

The *extream Use of Decimals* in all kinds of *Extractions* is sufficiently known to all versed in *Arithmetical Knowledge*; and its *absolute Necessity* in *some Parts* of Arithmetick, and its *Excellency* beyond even *Logarithms* themselves in others, is also as well known.

I would know what is the *Square Root* of 161,29?

```
Thus       161,29 (12,7 = the Answer.
           1
        -----
     22) 61
         44
        -----
    247) 1729
         1729
        ------
         ....
```

What is the *Square Root* of 3477?

In such Cases as this, you must add *twice* as many *Cyphers* to the given Number, as you design to have *Decimal Places* in the *Root* of the above Number to three Places of *Decimals.*

```
Then        3477,000000 ( 58,881 the Root requir'd.
            25
         ------
    108)  977
          864
         ------
   1168)  10300
           9344
         -------
  11768)  95600
          94144
         --------
 117761)  145600
          117761
         --------
           27839
```

Required the *Square Root* of 2, to 6 Places of Decimals.

```
             . . . . . . .
             2,000000000000 ( 1,414213 the Root required.
             1
          24) 100
               96
        281) ··400
               281
       2824 ) 11900
              11296
       2828 ) ··604
                565
                ·38
                 28
                 10
                  8
                 (2)
```

Having here got 3 Places of the 6, I work by the *contracted way of Division*, and gain the other 3 as truly as if wrought at *large*.

What is the *Square Root* of ,4489 ?

```
               . .
              ,4489 (,67 the Root requir'd.
               36
        127) ·889
              889
              ...
```

What is the *Square Root* of ,00576 ?

```
              . . .
              00576 (,024 the Root sought.
              004
        ,044) ·· 176
                 176
                 ...
```

What is the *Square Root* of, 00005625 ?

```
  .  .  .  .
,00005625 (,0075 = the Root sought.
 ....49
 ---------
145) .725
      725
     -----
      ...
```

To Extract *the* Roots *of* Single Repetends.

What is the *Square Root* of ,1 *or Unity* infinitely repeated ?

```
      .  .  .  .
     ,11111111 &c.
      9          (,3333 &c. the Square Root.
     ---
 63)  211
      189
     -----
663) .2211
      1989
     ------
6663)  22211  { So downward
       19989  {
      -------
      222211  { Ad Infinitum.
```

What is the *Square Root* of ,44,4 &c. ?

```
     .  .  .
    44,4444 &c. (6,66 &c. the Root.
    36
   ----
126)  844
      756
1326) .8844  { So downward
       7956  {
      ------
       88844 { Ad Infinitum.
```

If the *Root* does not *repeat* in the *first* Figure, 'twill be very *uncertain* when it will *repeat*.

Note, Only these *two Digits* 1 and 4 (of all the *Nine*) when *infinitely repeated* ; have their *Roots*, *pure single Repetends*.

To Extract *the* Square Root *of* Compound Repetends.

What is the *Square Root* of 196,85?

```
Thus,        196,85 (14,030568658 the Root.
             1
           -----
      24) ·96
           96
          -------
    2803) ··8568
            8409
           ---------
 280605) ·1595685
          1403025
          --------
  280611) 192660
          168366
          -------
          ·24294
           22448
           -----
           ·1846
            1683
            ----
            ·163
             140
             ---
             ·23
              22
             ---
             (1)
```

In this Manner the *skilful Artist* may proceed and gain the *Root* of any *Repetend* to what Number of Places he pleaseth. I omit the *Extraction* of the *Cube Root* here; because I shall have occasion to shew the Method and *Rationale* of that, and of the *Square Root*. both, when I come to shew the Use of Decimals in Algebra.

CHAP.

CHAP. IX.

The Use of DECIMALS *in the Business of Interest both Single and Compound; Of* Annuities, Pensions, *&c. their Value in Present Worth, and in Arrears; Of* Rebate *or* Discount; *Of* Free-hold *or* Real Estates.

INTEREST is a *small Sum of Money* paid for the Use of any *greater Sum*, according to any *Rate* agreed on; as 5 *l. per* 100 *l.* &c. for a Year; and it is either *Simple* or *Compound*.

Simple Interest is that which arises only from the *Principal* or *Sum of Money* lent; and both *Interest* and *Principal* are always the same as at first.

Compound Interest is that which ariseth from the Principal and its *Simple Interest* (when *due* and *forborn*) reckoned together as a *New Sum*, so that both *Principal* and *Interest* here are always increasing.

Annuities, *Pensions*, *Salaries*, &c. are *Rents*, *Profits*, and *Payments* made *Yearly*, or *Half Yearly*, &c. and they are said to be in *Arrears*, when they are due and unpaid for any Number of Payments.

Rebate or *Discount* is an *Abatement* of *Part* of a *Sum of Money* due sometime hence, in Consideration of *prompt* or present Payment of the Remainder; and this is done at any Rate of Interest.

In exemplifying the *wonderful Use of Decimals* in the *Affair* of *Interest*, &c. I need only shew the Reader the Solutions of those *admirable Theorems* in Numbers at large, which Mr. *Ward* (in his *Mathematician's Guide*) has with *great Invention* contriv'd from the following *Data*, and Method of Reasoning from thence.

In

In *Simple Interest.*

He puts
- $P =$ Any *Principal* or *Sum* put to Interest.
- $R =$ The *Ratio* of the *Rate per Cent. per Annum.*
- $T =$ The *Time* of the *Principal* at Interest.
- $A =$ The *Amount* of the *Principal* and its Interest.

In *Annuities*, &c. at Simple Interest.

He puts
- $U =$ The *Annuity*, *Pension*, or *Yearly Rent*, &c.
- $T =$ Time of *Forbearance*, or being Unpaid.
- $R =$ The *Ratio of Interest*, as before.
- $A =$ The *Amount* of the *Annuity* and its Interest.
- $\frac{2}{R} - 1 = x$ By way of *Substitution*, in *Arrears.*
- $\frac{2}{R} - \frac{2P}{U} + 1 = x$ By *Substitution* also, in pres. worth.
- $P =$ The *Present Worth* of *Annuities*, &c.

In *Compound Interest* these *Characters* signify the same Things as here; only *T* here, is there wrote in a small Letter (*t*) and denotes the *Power* of *R*; or is the *Index* of the Power to which *R* is to be involved; and $R =$ Amount of 1 *l.* and its Interest one Year. From the above *Data*, he makes the following Proportions.

Simple Interest.

The *Ratio* of the *Rate* of *Interest* signified by *R* is thus found (for 'tis only the Interest of 1 *l.* for 1 Year.)

	l.		*In.*		*l.*		*Ratio.*		
As	100	:	4	::	1	:	,04	=	*R* at 4 *per Cent.*
	100	:	4,5	::	1	:	,045	=	*R* at 4 ½ *per Cent.*
	100	:	5	::	1	:	,05	=	*R* at 5 *per Cent.*
	100	:	5,5	::	1	:	,055	=	*R* at 5 ½ *per Cent.*
	100	:	6	::	1	:	,06	=	*R* at 6 *per Cent.*

And	1.	2.	3.	4.	5.	6.	&c.	= *Years.*
	R.	2*R.*	3*R.*	4*R.*	5*R*	6*R.*		= *Interest.*

Hence 'tis evident the *Simple Interest* of 1 *l.* is a *Series* of *Terms* in *Arithmetical Progression* increasing.

Wherein
- $R =$ The *first Term*, and also the *Common Differ.*
- $T =$ The *Number* of all the Terms.
- $TR =$ The *last Term* of the *Series.*

Then

I. Int.

Then, As $1 : TR :: P : TRP =$ Interest of P. But the *Principal* and its *Interest* added is equal to the *Amount*. Whence this *General Theorem*,

$$TRP = P = A.$$

Annuities, Simple Interest.

Here $U =$ the *Yearly* Rent, and $R =$ the *Ratio* of Interest. Then $2U =$ the *Rent*, and $RU =$ the *Interest* for the second Year; and thus the following *Progressions* for five Years.

$$\text{Thus} \begin{cases} 1. \quad 2. \quad 3. \quad 4. \quad 5. \quad \&c. \text{ The Years.} \\ U. \quad 2U. \quad 3U. \quad 4U. \quad 5U. \quad \&c. \text{ The Rent.} \\ \quad RU. \quad 2RU. \quad 3RU. \quad 4RU. \quad \&c. \text{ The Interest.} \end{cases}$$

Hence 'tis plain, that $RU + 2RU + 3RU + 4RU + 5U = A$. The *Sum* of all the *Rents* and their *Interests* being *forborn* five Years. From whence it follows

That $RU + 2RU + 3RU + 4RU = A - TU$. For here $T = 5$.

Divide all by U. Then $R + 2R + 3R + 4R = \frac{A - TU}{U}$

Then by *Substitution*, put $R + 2R + 3R + 4R = Z$.

Then $1 + 2 + 3 + 4 = \frac{Z}{R}$ Now the *first* and *last* Terms of the *Progression* are $1 + 4 = 5 = T$. Therefore $\frac{T - 1}{2} \times T$ = Sum of all the Terms.

Now $\frac{TT - T}{2} = \frac{Z}{R}$ Hence $\frac{TTR - TR}{2} = Z$. Consequently $\frac{TTR - TR}{2} = \frac{A - TU}{U}$ The *General Theorem* for *Annuities* in *Arrears*.

But because $P =$ the *Present Worth*, is not in the *last General Theorem*, That will answer no Questions relating thereto; Wherefore a *New one* must be contriv'd: Now because A denotes the same Thing, *viz.* the *Amount*, in *both* the *last General Theorems*; and because *any two Quantities, equal to one and the same thing, are equal to one another*. And $PTR + P = A$ in the *first General Theorem*; And $\frac{TTRU - TRU + 2TU}{2} = A$ in the *latter General Theor.*

There-

Therefore $PTR + P = \frac{TTRU - TRU + 2TU}{2}$ is the *General Theorem* for Questions about the *present Worth* or *Purchasing* of *Annuities.*

Compound Interest.

The *Proportion* for finding R the *Ratio* of the *Rate* of *Compound Interest*, (which is only the *Amount* of 1 *l.* and its *Interest* for one *Year*,) is This

$$\text{As} \begin{cases} \overset{l.}{100} : 105 :: 1 : 1{,}05 = R \text{ at } 5 \textit{ per Cent.} \\ 100 : 106 :: 1 : 1{,}06 = R \text{ at } 6 \textit{ per Cent}, \text{ \&c.} \end{cases}$$

But as *one Pound*, is to the *Amount* of *one Pound*, at *one Year's End*; so is that *Amount*, to the *Amount* of *one Pound* at two *Year's End*; and so on continually.

That is, $1 : R :: R : RR :: RR : RRR :: R^3 : R^4 :: R^4 :: R^5 ::$ &c.

$$\text{Then} \begin{cases} 1. \quad 2. \quad 3. \quad 4. \quad 5. = \text{Years.} \\ R.\ R^2.\ R^3.\ R^4.\ R^5. = \text{The } \textit{Amount} \text{ of } 1\ l. \text{ at} \end{cases}$$

any Rate.

Hence 'tis evident the *Amount* proceeds in a *Geometrical Proportion*, wherein the *Time* ($= t$), or *Number* of *Years*, is always equal to, or the same with, the *Index* of the Power of the *last* and *highest Term* of the *Series*; viz. R^5, or R^t.

But, as *one Pound* : is to the *Amount* of *one Pound* for *any given Time* :: so is *any proposed Principal* or *Sum* : to its *Amount* for the *same Time*.

That is, As $1 : R^t :: P : PR^t$.

But $PR^t = A$ The *General Theorem.*

Annuities. Compound Interest.

$$\text{Here} \begin{cases} R = \textit{One Pound} \text{ and its } \textit{Interest} \text{ for } \textit{one Year}, \text{ as before.} \\ U = \text{The } \textit{first Year's} \text{ Rent } \textit{without} \text{ Interest.} \end{cases}$$

Then $RU =$ The *Amount* of the *first Year*'s Rent, and its Interest.

And hence is form'd the following *Progression* of *Amounts* in continued *Geometrical Proportion.*

$$\text{Thus} \begin{cases} 1. \qquad 2. \qquad 3. \qquad 4. \qquad 5. \quad \text{\&c. The Years.} \\ U + UR + UR^2 + UR^3 + UR^4, \text{ \&c. The Amounts.} \end{cases}$$

Hence

Hence $U + UR + UR^2 + UR^3 + UR^4 = A$ The *Amount* of any *Yearly Rent* or *Annuity* forborn five Years.

Now the last Term in the above Series is $UR^4 = UR^{t-1}$.

Therefore $A - UR^{t-1} =$ The *Sum* of all the *Antecedents*.

And $A - U =$ The *Sum* of all the *Consequents* in the *Series*.

So that it will be, $U : RU :: A - UR^{t-1} : A - U$.

Therefore $AU - UU = RUA - UUR^t$. Divide all by U.

Then $A - U = RA - UR^t$. The *General Theorem*.

For the *present Worth*, we must proceed as in *Simple Interest* in this Case, to gain an *Equation* or *general Theorem*, wherein shall be P.

The *Theorem* for *Interest* is $PR^t = A$.

And in the *last Theorem* $\frac{UR^t - U}{R - 1} = A$.

Consequently, $PR^t = \frac{UR^t - U}{R - 1}$ The *General Theorem*.

Free-hold Estates, *Compound Interest*.

Free-hold or *Real Estates* are supposed to be purchased for ever. And the *Computation* of the *Value* of such *Estates* is grounded on a *Series* of *Geometrical Proportionals* decreasing *ad Infinitum*.

Let P, U, R, denote the same as before; then the Series will be $\frac{U}{R}.\ \frac{U}{R^2}.\ \frac{U}{R^3}.\ \frac{U}{R^4}.\ \frac{U}{R^5} (= P)$ &c. till the last Term be $= 0$. Then will $P - 0 =$ *Sum* of all the *Antecedents*; and $P - \frac{U}{R} =$ *Sum* of all the *Consequents*.

Therefore, as $\frac{U}{R} : \frac{U}{R^2} :: P : P - \frac{U}{R}$ which gives $PR - U = P$. The *general Theorem*.

Theorems *Resolving all Questions concerning* Simple Interest.

Given P, R, T; To find A?

Theorem 1. $TRP + P = A$.

Given T, R, A; To find P?

Theorem 2. $\frac{A}{TR + 1} = P$.

 Given

Given *A*, *P*, *T*; To find *R*?

Theorem 3. $\left\{\frac{A-P}{TP}=R.\right.$

Given *P*, *R*, *A*; To find *T*?

Theorem 4. $\left\{\frac{A-P}{RP}=T.\right.$

Question 1. What will 256*l.* 10*s.* *Amount* to in three *Years*, one *Quarter*, two *Months*, and eighteen *Days*, at 6*l.* *per Cent. per Ann.*

Here is given $\left\{\begin{array}{l}P=256{,}5\\R=0{,}06\\T=3{,}46598\end{array}\right\}$ To find *A*, *per The.* 1.

Multiply	—	3,46598	$= T$
By	—	0,06	$= R$
Product	—	0,2079588	$= TR$
Mult. by Inversion		5,652	$= P$.
		4159176	
		1039794	
		124775	
		10397	
The Product	—	53,34142	$= TRP$
Add	—	+256,5	$= P$
Answer	—	*l.* 309,84142	$= A =$ 309*l.* 16*s.* 10*d.*

Question 2. What *Principal* or *Sum of Money* put to Interest, will raise a *Stock* of (or be *worth*) 405 *l.* 6 *s.* in five *Years*, and eight *Months*, at the Rate of 5 *l.* *per Cent. per Ann.*?

Here is given $\left\{\begin{array}{l}A=405{,}30016\\R=0{,}05\\T=5{,}613698\end{array}\right\}$ To find *P*, *per The.* 2.

Multi-

Multiply	5,613698	$= T$
By —	0,05	$= R$
Product	0,2806849	$= TR$
Add Unity	1,	

$TR + 1 =$ 1,2806849) 405,30016 (316,47144 *l.* $= P$.

38420547
·2109469
1280684
·828785
768410
·60375
51227
·9148
8964
·184
128
·56
51
5
5
·

Hence the *Principal* is 316,47144 *l.* which, in common Coin, is 316 *l.* 9 *s.* 5 *d.* for the Answer.

Note, By this Theorem 'tis you find what *present Money*, or *prompt Payment*, will satisfy a *Debt* due any Time hereafter, *Abating* or *Discounting at any Rate per Cent.*

Question 3. At what *Rate of Interest per Cent.* will 36 *l.* *amount* to 36 *l.* 18 *s.* 11 *d* $\frac{1}{4}$ *d.* in six *Months*, three *Weeks*, and three *Days*?

Here is given $\left\{\begin{array}{l} P = 36 \\ T = ,526 \\ A = 36,9468 \end{array}\right\}$ To find *R*, *per The.* 3.

Multiply ,526 = T
By — 36 = P

3156
1578

Prod. 18,936 = TP

From 36,9468 = A
Subst. 36, = P
18,936),94680(,05 = R
94680

Then, As 1 : 0,05 : : 100 : 5 *l.* Answer 5 *per Cent.*

Question 4. In what *Time* will 200 *l.* 1 *s.* 8 *d.* *amount* to 250 *l.* at 4 *l.* 10 *s.* *per Cent. Interest?*

$$\text{Here is given} \left\{ \begin{array}{l} P = 200{,}083 \\ R = 0{,}045 \\ A = 250, \end{array} \right\} \text{To find } T, \textit{ per Theo. } 4.$$

Mult. 200,083 = P
By — 0,045 = R

1000416
800333

Prod. 9,00375 = PR

From 250,000 = A
Subst. 200,083 = P

9,00375) 49,91666(5,54399 = T
4501875

489791
450187

39604
36012

3592
2701

891
810

81
81

The *Time* then is
Years
5,54399 = 5 *Years,*
7 *Months,* and 3
Days.

Theorems *resolving all Questions concerning* Annuities, Pensions, &c. *in* Arrears, Simple Interest.

Given U, T, R; to find A?

Theorem 1. $\left\{ \dfrac{TTU - TU}{2} R + TU = A. \right.$

Gi

Given A, T, R; To find U?

Theorem 2. $\left\{\frac{2A}{TTR - TR + 2T} = U.\right.$

Given A, T, U; To find R?

Theorem 3. $\left\{\frac{2A - 2TU}{TTU - TU} = R.\right.$

Given U, R, A; To find T?

Theorem 4. $\left\{\sqrt{\frac{2A}{RU} + \frac{xx}{4}} - \frac{1}{2}x = T\right.$

Question 1. If 250*l.* *Yearly Rent* (or *Annuity*, &c.) be *forborn* or unpaid 7 *Years*; what will it *amount* to in that *Time*, at the *Rate* of 6 *l. per Cent. per Annum*?

Here is given $\left\{\begin{array}{l} U = 250 \\ T = 7 \\ R = 0{,}06 \end{array}\right\}$ To find *A*, *per Theorem* 1.

Multiply —	250	$= U$
By —	7	$= T$
Product —	1750	$= TU.$
By	7	$= T.$
	12250	$= TTU.$
Substract	1750	$= TU.$
Remains	10500	$= TTU - TU.$
Halve	5250	$= TTU - TU \div 2$
Multiply by	0,06	$= R$
	315,00	
Add	1750,	$= TU$
Sum	2065 *l.*	$= A$ The Answer.

If the *Payment* of the aforesaid *Annuity* had been made *half Yearly*, then would $U = 125 = \frac{250}{2}$, and $T = 14 =$ *Number* of Payments; and $R = 0{,}03 = \frac{0{,}06}{2}$, and working as *per Theorem*; *A* will be found $=$ 2091 *l.* 5 *s.* which

is

is more than the *Yearly* Payment by 26 *l.* 5 *s.* Hence the *oftner* the *Payment*, the *more advantagious.*

Question 2. What *Annuity*, or *Yearly* Payment, being *unpaid* 8 ½ *Years*, will raise a *Stock* of 572 *l.* 12 *s.* 8 *d.* at 5 *per Cent*, *per Annum.*

Here is given $\left\{ \begin{array}{l} A = 572,63 \\ T = 8,5 \\ R = 0,05 \end{array} \right\}$ To find *U*, *per Theor.* 2.

Multiply	—	$8,5 = T$
By	—	$,05 = R$
		$,425 = TR$
Again by	—	$8,5 = T$
		2125
		3400
Product	—	$3,6125 = TTR$
Substract	—	$0,4250 = TR$
Remains	—	$3,1875 = TTR - TR$
Add $8,5 \times 2$	=	$17,0000 = 2T$

$TTR - TR + 2T = 20,1875$) 1145,2666 &c. $= 2A$ (56,73148 *l.* $(= U$

1009375

·1358916
1211250

·147666
141312

··6354
6056

:298
201

·97
80

·17
16

··1

The Annuity therefore is 56,73148 *l.* = 56 *l.* 14 *s.* 7 ½ *d.* The Answer.

Quest.

Question 3. At what *Rate* of *Interest, per Cent. per Ann.* will 40 *l.* 13 *s.* 4 *d. Yearly Rent, amount* to 450 *l.* 13 *s.* 4 *d.* in 9 *Years*?

Here is given $\left\{\begin{array}{l} A = 450,\dot{6} \\ T = 9 \\ U = 40,\dot{6} \end{array}\right\}$ To find R, *per Theorem* 3.

Multiply —	$40,\dot{6}$	$= U$
By —	9	$= T$
Product —	366	$= TU$
Multiply again by	9	$= T$
From that Prod.	3294	$= TTU$
Substract —	366	$= TU$
Remains —	2928	$= TTU - TU$ The Divisor.
Then from	$901,\dot{3}$	$= 2A$
Substract —	732	$= 2TU$
Remains —	$169,\dot{3}$	$= 2A - 2TU$ The Dividend.

Then 2928) 169,33 (0,05784 = R

```
14640
-----
 2293
 2049
-----
  244
  233
-----
   11
   11
-----
   ..
```

Therefore as 1 *l.* : 0,05784 *l.* : : 100 *l.* : 5,784 *l.* = 5 *l.* : 15 *s.* : 8 $\frac{1}{4}$ *d.* the Rate *per Cent.* required.

Question 4. In what *Time* will 250 *l. Yearly Rent*, raise a Stock of 2065 *l.* allowing 6 *per Cent.* &c. for the *Forbearance* of the Payments as they become *due*?

Here is given $\left\{\begin{array}{l} U = 250 \\ A = 2065 \\ R = 0,06 \end{array}\right\}$ To find T, *per Theorem* 4.

First

First	—	4130	$= 2A$
Then multiply	—	250	$= U$
By	—	0,06	$= R$
Product is	—	15	$= UR$
Then	—	$33,\dot{3}$	$= \frac{2}{R}$
And $33,\dot{3} - 1$		$= 32,\dot{3}$	$= \frac{2}{R} - 1 = x.$
And $32,\dot{3} \div 2$		$= 16,1\dot{6}$	$= \frac{1}{2}x$
The Square of it is		$261,36x$	$= \frac{1}{4}xx = \frac{xx}{4}$
To which add		$275,\dot{3}$	$= \frac{2A}{RU}$
The Sum is		$536,694$	$= \frac{2A}{RU} + \frac{xx}{4}$
The Square Root of which is		$23,1\dot{6}$	$= \sqrt{\frac{2A}{RU} + \frac{xx}{4}}$
From which take		$16,1\dot{6}$	$= \frac{1}{2}x$
There remains		7	$= T$ The Time required, *viz.* 7 Years.

The *Divisions* and *Extractions* at large I have omitted for the *Learner*'s Exercise; but I have represented all the Numbers in One; which Mr. *Ward*'s Method could not do as being *deficient* in the *Doctrine of Calculating Numbers*, as may be observ'd in his Work of this and other Questions of Interest.

N. B. In all Questions about *Yearly*, or *stated Rents* and *Payments*, the *Interest* is reckoned for *every Payment* after it becomes *due*, thro' the whole Time of *Forbearance*.

Theorems *resolving all Questions concerning the* Present Worth *of* Annuities, Pensions, *&c. at* Simple Interest.

Given U, R, T; To find P?

Theorem 1. $\left\{\frac{TTR - TR + 2T}{2TR + 2}U = P\right.$

Given

Given P, R, T; To find U?

Theorem 2. $\left\{\dfrac{TR+1}{TTR-TR+2T}\right. 2P = U$

Given P, U, T; To find R?

Theorem 3. $\left\{\dfrac{2P-2TU}{TTU-TU-2PT}\right. = R$

Given U, P, R; To find T?

Theorem 4. $\sqrt{\dfrac{2P}{RU}+\dfrac{xx}{4}} \pm \tfrac{1}{2}x = T$

Question 1. What is 75 *l. Yearly Rent*, to continue 9 *Years*, *worth* in *ready Money*, at 6 *per Cent.* &c.

Here is given $\left\{\begin{array}{l} U = 75 \\ R = 0{,}06 \\ T = 9 \end{array}\right\}$ To find P, *per Theorem* 1.

Multiply —	0,06	$= R$
By —	9	$= T$
The Product	,54	$= TR$
Again by	9	$= T$
From which	4,86	$= TTR$
Take —	54	$= TR$
Remains	4,32	$= TTR - TR$
To which add	18,00	$= 2T$
Dividend =	22,32	$= TTR - TR + 2T$
Divisor =	3,08	$= 2TR + 2$

 Then

Then 3,08) 22,32 (7,24675
2156   75 = *U*
··760   3623375
616   5072725
1440   543,50625 = *P*
1232   = 543 *l.* 10 *s.* 1 $\frac{1}{2}$ *d.*
Answer.
·2080
1848
·2320
2156
·1640
1540
·100

Question 2. What *Annuity*, to continue 21 *Years*, will 192 *l.* 1 *s.* 5 $\frac{1}{2}$ *d.* purchase, at 5 *per Cent.*?

Here is given $\left\{\begin{array}{l} P = 192{,}0731 \\ T = 21 \\ R = 0{,}05 \end{array}\right\}$ To find *U*, *per The.* 2.

Multiply —	21	$= T$
By —	,05	$= R$
That Product	1,05	$= TR$
Again by —	21	$= T$
	105	=
	210	
The Product is	22,05	$= TTR$
From which take	1,05	$= TR$
There remains	21,00	$= TTR - TR$
To which add	42,00	$= 2T$
The Divisor =	63,00	$= TTR - TR + 2T$
Then to —	1,05	$= TR$
Add Unity	1,00	$= 1$
The Dividend =	2,05	$= TR + 1$

Then

Then 63) 205 (,03254
189

160
126

340
315

250
252

But — 384,1462 = 2*P*
Multiply by 45230,8 invert

115244
7683
1920
153
Product — 12,5 = *U*

The *Annuity* then is 12,5 *l.* = 12 *l.* 10 *s.* the Anſwer.

Note, This is a very *frequent* and *uſeful Queſtion* ; and ought to be work'd with *great Exactneſs* ; and therefore if a Perſon be not very ready at, nor *rightly underſtands* the Manner of *Contracted Multiplication and Diviſion*, 'twill be beſt to work the *common Way*. Which alſo is to be obſerved in all *Queſtions of Moment*.

Queſtion 3. At what *Rate* of *Simple Intereſt*, will 250,3 *l.* or 250 *l.* 6 *s.* 8 *d.* purchaſe an *Annuity* of 30 *l.* 10 *s. per Annum*, to continue 10 *Years* ?

Here is given $\left\{\begin{array}{l} P = 250,3 \\ U = 30,5 \\ T = 10 \end{array}\right\}$ To find *R*, *per Theorem* 3.

Multiply — $30,5 = U$
By — $10 = T$

The Product is — $305 = TU$
Which again mult. by $10 = T$

Produceth — $3050 = TTU$
From which take $305 = TU$

There remains $2745 = TTU - TU$.
Then — $500\dot{6},\dot{6} = 2PT$

The Difference the Divisor. $2261,\dot{6} = TTU - TU - 2PT$

And $500,\dot{6} = 2P$
Also $610,0 = 2TU$

Differ. $109,\dot{3} = 2P - 2TU$ the Dividend.

Then $2261,\dot{6})109,\dot{3}$ } See Division of Repetends.
$2261 \quad 109$

$2035,5)\ 8840\ (,04343 = R$
8142
698
610
88
81
7
6
1

l. *l.* *l.* *l.*
Then say, As 1 : 0,04343 : : 100 : 4,343 = 4 *l.* 6 *s.* 10 $\frac{1}{4}$ *d.* the Rate *per Cent.* sought.

Question 4. In what *Time* will 7 *l. per Annum* pay a *Debt* of 120 *l.* 8 *s.* at 6 *l. per Cent.* Or, For how long a Time may an *Annuity* of 7 *l. per Annum* be purchas'd or enjoy'd for 120 *l.* 8 *s.* at the aforesaid *Rate*?

Here is given $\left\{ \begin{array}{l} U = 7 \\ R = 0,06 \\ P = 120,4 \end{array} \right\}$ To find *T*, *per Theorem* 4.

First

First — $240,8 = 2P$
And — $34,4 = \frac{2P}{U}$
From which take $33,3 = \frac{2}{R}$
To the Remaind. $1,06 = \frac{2}{R} - \frac{2P}{U}$
Add Unity — $1 = +1$
The Sum is $2,06 = \frac{2}{R} - \frac{2P}{U} + 1 = x$ by Substitut.
Then — $1,03 = \frac{1}{2}x$
And — $1,067 = \frac{1}{4}xx$
Again — $0,42 = RU$
And — $573,3 = \frac{2P}{RU}$
Then — $574.4 = \frac{2P}{RU} + \frac{xx}{4}$
Sq. Root of that $23,96 = \sqrt{\frac{2P}{RU} + \frac{xx}{4}}$
To which add $1,03 = \frac{1}{2}x$
The Sum is $24,9 = 25 = T$ the Time sought.

$$\left\{\begin{array}{l} 1,03 = \frac{1}{2}x \\ \underline{103 = \frac{1}{2}x} \\ 9)309 = \\ \overline{344} \\ \underline{10333} \\ 1,067 = \frac{1}{4}xx \end{array}\right.$$

Having thus, in a most perspicuous Manner, shew'd the *great and invaluable Service of Decimals* in working Questions of *Simple Interest*, &c. I shall proceed to treat of the same things, in the *like* Manner, in *Compound Interest*.

Theorems *resolving all Questions of* Compound Interest.

Given P, R, t; To find A?
Theorem 1. $PR^t = A$?
Given A, R, t; To find P?
Theorem 2. $\left\{\frac{A}{R^t} = P.\right.$

Given $\left\{\begin{array}{l} P,\ A,\ R, \\ P,\ A,\ t \end{array}\right\}$ to find $\left\{\begin{array}{l} t? \\ R? \end{array}\right.$

Theorem 3. $\left\{\frac{A}{P} = R^t\right.$

Rates.

Rates.		*Logarithms.*	*Rates.*		*Logarithms.*
1,10	=	0,041392	1,055	=	0,023252
1,08	=	0,033423	1,045	=	0,019116
1,06	=	0,025305	1,04	=	0,017033
1,05	=	0,021189	1,03	=	0,012837

Question 1. What will 256 *l.* 10 *s. amount* to in 7 *Years*, at 6 *l. per Cent.* &c. *Compound Interest?*

Here is given $\left\{ \begin{array}{l} P = 256{,}5 \\ R = 1{,}06 \\ t = 7 \end{array} \right\}$ To find *A*, *per Theorem* 1.

Multiply the *Logarithm* of the Rate	1,06 =	0,025305
By the Index of its Power (viz. $t =$)		7
The Product the Logarit. of R^7 =	1,50363 =	0,177135
Multiply that by *P* — =	256,5 =	2,409087
The Product is the Amount *A* =	385,6811 =	2,586222

That is, 385 *l.* 13 *s.* 7 ½ *d.* the Answer required.

Question 2. What *Principal*, or *Sum* of *Money*, will raise a Stock of 20,6 in 5 *Years*, at 5 *l. per Cent. per Annum Compound Interest?*

Here is given $\left\{ \begin{array}{l} A = 20{,}6 \\ t = 5 \\ R = 1{,}05 \end{array} \right\}$ To find *P*, *per Theorem* 2.

Multiply the *Logarithm* of the Rate	1,05 =	0,021189
By the *Index* of its Power t =	—	5
The Product is the Logar. of R^t =	1,27628 =	0,105945
By which divide the *Amount A* =	20,6 =	1,313867
	l.	
The *Quotient* is the *Principal P* =	16,1407 =	1,207922

That is 16 *l.* 7 *s.* the Sum required.

Question 3. In what *Time* will 37 *l.* 15 *s. amount* to 76,65 *l.* (or 76 *l.* 13 *s.*) at 4 *l.* 10 *s. per Cent?*

Here is given $\left\{ \begin{array}{l} A = 76{,}65 \\ P = 37{,}75 \\ R = 1{,}045 \end{array} \right\}$ To find *t*, *per Theorem* 3.

Divide

Divide the Amount	—	$A = 76,65$	$= 1,884512$
By the Principal (or Sum)		$P = 37,75$	$= 1,576916$
The Quotient is	—	$\frac{A}{P} = R^t = 2,03046$	$= 0,307596$

Then 2,03046 divided by ($R =$) 1,045; and that Quotient again by 1,045; and thus continually dividing the Quotients by 1,045, 'till nothing remains, the *Number* of such *Divisions* will be equal to ($t =$) the *Time* sought. But this is sooner, and easier done by much, by *Logarithms*.

Thus, *Divide* the *Logarithm* of 2,03046 ($= R^t$) by the Logarithm of 1,045 ($= R$) and the Quotient is the Time.

0,019116) 0,307596 (16,091 $= t$ the *Time* sought.

```
19116
------
116436
114696
------
  1740
  1720
  ----
    20
    19
    --
     1
```

Viz. 16 Years, 1 Month, and 5 Days, the Ans.

Question 4. At what *Rate* of *Compound Interest*, will 51 *l.* 15 *s.* *amount* to 70 *l.* 18 *s.* in 5 *Years*?

Here is given $\left\{\begin{array}{l} P = 51,75 \\ A = 70,9 \\ t = 5 \end{array}\right\}$ To find *R*, *per Theorem* 3.

Divide the Amount	—	$A = 70,9$	$= 1.850646$
By the Principal or Sum	—	$P = 51,75$	$= 1,713910$
The Quotient will be $R^t = R^5$		$= 1,370048$	$= 0,136736$
The Sursolid Root of which is R		$= 1,065$	$= 0,027347$

l. *l.* *l.* *l.* *l.* *s.*

Then say, As 1 : 1,065 :: 100 : 106,5 = 106—10 = R the *Rate per Cent. per Annum sought.*

Note; R^5 being equal to 1,370048, of Consequence $R = \sqrt[5]{1,370048}$, which may be extracted by an *Algebraick converging Series*; the Manner of doing it, see in *Chap.* 11. of the *Use of Decimals* in *Algebra*.

Theo-

Theorems *resolving all Questions relating to* Annuities, *&c. in* Arrear, *calculated at* Compound Interest.

Given U, R, t; To find A?

Theorem 1. $\left\{ \dfrac{UR^t - U}{R - 1} = A \right.$

Given A, R, t; To find U?

Theorem 2. $\left\{ \dfrac{RA - A}{R^t - 1} = U \right.$

Given U, A, R; To find t?

Theorem 3. $\left\{ \dfrac{RA + U - A}{U} = R^t \right.$

Given A, U, t; To find R?

Theorem 4. $\left\{ \dfrac{A}{U} R - R^t = \dfrac{A - U}{U} \right.$

Question 1. If 30 *l. Yearly Rent* be forborn or unpaid 9 *Years*; What will it *amount* to at the *Rate* of 6 *l. per Cent.* &c. *Compound Interest*?

Here is given $\left\{ \begin{array}{l} U = 30 \\ t = 9 \\ R = 1{,}06 \end{array} \right\}$ To find A, *per Theorem* 1.

In the first Place, let $R =$ — 1,06 = 0,025305
Be involved to the 9 Power (*viz.* R^t) — 9

That will be — $R^9 = 1{,}689451 = 0{,}227745$
Multiply by — $U = 30 = 1{,}477121$

The Product is — $UR^t = 50{,}683530 = 1{,}704866$
From that Substract $U = 30$
The Remainder is the Divid. $= 20{,}68353 = UR^t - U$.

Divide therefore $UR^t - U = 20{,}68353 = 1{,}315626$
By — $R - 1 = 0{,}06 = 8{,}778151$

The Quotient is — $A = l.\ 344{,}7267 = 2{,}537475$
That is the Amount = 344 *l.* 14 *s.* 6 $\frac{1}{4}$ *d.* the Answer.

Question

Question 2. What *Annuity* 3 *l.* 10 *s. per Cent. Compound Interest*, will raise a *Stock* of 344 *l.* 5 *s.* being forborn 8 *Years*?

Here is given $\left\{\begin{array}{l} A = 344{,}25 \\ R = 1{,}035 \\ t = 8 \end{array}\right\}$ To find *U*, *per Theor.* 2.

Multiply the Amount	—	$A = 344{,}25$	$= 2{,}536874$
By the Rate	—	$R = 1{,}035$	$= 0{,}014940$
From that Product	—	$RA = 356{,}29875$	$= 2{,}551814$
Substract the Amount	—	$A = 344{,}25$	
The Remainder is		$RA - A = 12{,}04875$,	the Divid.
Then involve	—	$R = 1{,}035$	$= 0{,}014940$
To the 8th Power	—	viz. R^t —	8
That Power will be		$R^8 = 1{,}316803$	$= 0{,}119520$
The same less Unity is		$R^t - 1 = 0{,}316803$,	the Divisor.
Therefore divide		$RA - A = 12{,}04875$	$= 1{,}080908$
By	—	$R^t - 1 = 0{,}316803$	$= 9{,}500785$
The Quotient is	—	$U = 38{,}0297$	$= 1{,}580123$

The *Annuity* therefore which was sought, is found to be 38,0297 *l.* = 38 *l.* 0 *s.* 7 *d. per Annum*, Answer.

Question 3. In what *Time* will 38 *l.* 0 *s.* 7 *d.* raise a *Stock* of 344 *l.* 5 *s.* at 3 *l.* 10 *s*, *per Cent. per Annum, Compound Interest*?

Here is given $\left\{\begin{array}{l} U = 38{,}0297 \\ A = 344{,}25 \\ R = 1{,}035 \end{array}\right\}$ To find *t*, *per Theor.* 3.

First multiply the Amount		$A = 344{,}25$	$= 2{,}536874$
By the given Rate	—	$R = 1{,}035$	$= 0{,}014940$
To that Product		$RA = 356{,}29875$	$= 2{,}551814$
Add the Annuity	—	$U = 38{,}0297$	
From the Sum		$RA + U = 394{,}32845$	
Take the Amount		$A = 344{,}25$	
The Remainder is the Divi. which divide by		$RA + U - A = 50{,}07845$	$= 1{,}699651$
		$U = 38{,}0297$	$= 1{,}580123$
The Quotient will be		$R^t = 1{,}316803$	$= 0{,}119528$

Then Divide 1,316803 continually by the Rate 1,035 untill nothing remains, and the Number of those Divisions will be $8 = t =$ Time required.

But much better by Logarithms thus; Divide the Logarithm of the Power by the Logarithm of the Rate, the Quotient is $= t$ the Time fought.

Thus 0.014941) 0,119528 (8 $= t$ the Time, *viz.* 8 Years; Answer.
0,119528

......

Question 4. At what *Rate per Cent. Compound Interest*, will 30 *l. Yearly Rent*, being *forborne* or unpaid 9 *Years*, amount to 344 *l.* 14 *s.* 6 $\frac{1}{4}$ *d.*

Here is given $\left\{\begin{array}{l} U = 30 \\ A = 344{,}7267 \\ t = 9 \end{array}\right\}$ To find *R*, *per The.* 4.

First divide the Amount		$A = 344{,}7267$	$= 2{,}537475$
By the Annuity	—	$U = 30$	$= 1{,}477121$
The Quotient is	—	$\frac{A}{U} = 11{,}4909$	$= 1{,}060354$
Again the Amount less the Annuity is		$A - U = 314{,}7267$	$= 2{,}497933$
Which divide by	—	$U = 30$	$= 1{,}477121$
The Quotient is	—	$\frac{A - U}{U} = 10{,}49014$	$= 1{,}020812$

Now $R^t = R^9$. Therefore the Theorem affords this *Equation*, viz. $11{,}4909R - R^9 = 10{,}49014$

This

This *Equation* is easily resolv'd by a *Converging Series* (which see in the *Use of Decimals* in *Algebra.*)

Note, This Question may be very easily and expeditiously answer'd, by the Rule of *False Position* ; thus

Make *two Suppositions* of the *Rate*, which may include between them the *Rate you seek.*

Then find what the *Amounts* of the given *Annuity* would be at the *two supposed Rates* of *Interest, per Theorem* 1.

Lastly; Observe the *Errors* of *those Amount* from the *Amount here* given, then by those *Suppositions* and their *Errors*, find the true *Rate*, (*viz*, 1,06) as is there taught.

Theorems *resolving all Questions concerning the* Present Worth of Annuities, Pensions, *or* Leases in Reversion, *at* Compound Interest.

Given U, R, t; To find P?

Theorem 1. $$\left\{\frac{U - \frac{U}{R^t}}{R - 1} = P.\right.$$

Given P, R, t; To find U?

Theorem 2. $$\left\{\frac{PR^t \times R - PR^t}{R^t - 1} = U.\right.$$

Given U, R, P; To find t?

Theorem 3. $$\left\{\frac{U}{P + U - PR} = R^t.\right.$$

Given U, P, t; To find R?

Theorem 4. $$\left\{\frac{U}{P} = \frac{U}{P} R^t + R^t - R^{t+1}.\right.$$

Question 1. What is 30 *l. Yearly Rent, Worth* in *ready Money*, for its Continuance 7 *Years*, allowing 6 *l. per Cent. Compound Interest* to the Purchaser?

Here is given $\left\{\begin{array}{l} U = 30 \\ R = 1,06 \\ t = 7 \end{array}\right\}$ To find P, *per Theorem* 1.

 First

First, involve — $R = 1,06 = 0,025305$
To the 7th Power, (viz. R^7) — 7

That will be — $R^7 = 1,50061 = 0,177135$
Then divide — $U = 30 = 1,477121$

By R^t, there will remain $\frac{U}{R^t} = 19,9520 = 1,299986$

Then from the Annuity $U = 30$

Substract — $\frac{U}{R^t} = 19,952$

Remains the Dividend $U - \frac{U}{R^t} = 10,048 = 1,002079$

Which divide by the Rate less Unity $= R - 1$ — $= 0,06 = 8,778151$

The Quotient is the present Worth — P — $= 167,4716 = 2,223928$

The Present Worth, In ready Money is 167 *l.* 9 *s.* 5 *d.* the Answer.

N. B. Suppose this were an *Annuity* in *Reversion*, or not to be entered on till after 7 *Years* are past, and thenceto continue 7 *Years*; and you would know the *present Worth*; find by the second Theorem of *Compound Interest*, what *ready Money* will *amount* to 167 *l.* 9 *s.* 5 *d.* in 7 *Years*, at the *same Rate* of Interest; and that will be its *present Worth*; and so for any other *Annuity* in *Reversion*.

Question 2. What *Annuity*, to continue 7 *Years*, may be *purchased* for 120 *l.* 5 *s.* at 6 *per Cent. Compound Interest?*

Here is given $\left\{ \begin{array}{l} P = 120,25 \\ R = 1,06 \\ t = 7 \end{array} \right\}$ To find U, *per Theor.* 2.

Involve the Rate — $R = 1,06 = 0,025305$
To the Index of its Power (viz. $= t$) — 7

The Power of R will be $R^7 = 1,50361 = 0,177135$
Which mult. by the present Worth $P = 120,25 = 2,080084$

The Product is — $PR^t = 180,8087 = 2,257219$
Multiply that by the Rate — $R = 1,06 = 0,025305$

That Product is $PR^t \times R = 191,65722 = 2,282524$
From which substract $PR^t = 180,8087$
There Remains the Dividend $10,84852 = PR^t \times R - PR^t$.

Divide

Divide therefore $PR^t \times R - PR^t = 10,84852 = 1,035369$
By the Power of R less $1 = R^t - 1 = 0,50361 = 9,702113$

The Quotient is the Annity $U = 21,54057 = 1,333256$

The Annuity sought therefore is $21,54057\,l. = 21\,l.$ $10\,s.\ 9\frac{1}{2}\,d.$

Question 3. For what *Time* will $167\,l.\ 9\,s.\ 5\,d.$ purchase an *Annuity* of $30\,l.$ *per Annum*, at 6 *per Cent. Compound Interest?*

Here is given $\left\{\begin{array}{l} P = 167,4716 \\ U = 30 \\ R = 1,06 \end{array}\right\}$ To find t, *per Theor.* 3.

To the present Worth	$P =$	$167,4716$
Add the Annuity —	$U =$	30
The Sum is —	$P + U =$	$197,4716$

Then mult. the present Worth	$P =$	$167,4716$	$= 2,223928$
By the Rate —	$R =$	$1,06$	$= 0,025305$
The Product is —	$PR =$	$177,5199$	$= 2,249233$
Which subst. from Sum of	$P + U =$	$197,4716$	
The Remainder is	$P + U - PR =$	$19,9517$	the Divisor.
Then Divide the Annuity —	$U =$	30	$= 1,477121$
By the Divisor	$P + U - PR =$	$19,9517$	$= 1,299986$
The Quotient is —	$R^t =$	$1,50361$	$= 0,177135$

Lastly, *Divide* the *Logarithm* of R^t, by the *Logarithm* of the *Rate*; the *Quotient* will be the *Time* $= t$ sought.

Thus, $0,025305)\ 0,177135\ (7 = t$ the Time sought, *viz.* 7 Years. Answer.
$\qquad\qquad\qquad\ \ 0,177135$
$\qquad\qquad\qquad\ \ \ldots\ldots$

Question 4. Suppose I purchase an *Annuity* of $21\,l.\ 10\,s.$ $9\frac{1}{2}\,d.$ to continue 7 *Years*, for $120\,l.\ 5\,s.$ *ready Money*; at what *Rate per Cent. Compound Interest*, was the Purchase made?

Here is given $\left\{\begin{array}{l} P = 120,25 \\ U = 21,54057 \\ t = 7 \end{array}\right\}$ To find R, *per Th.* 4.

First,

First, divide the Annuity	$U =$	$21,54057 =$	$1,333256$
By the present Worth —	$P =$	$120,25 =$	$2,080084$
The Quotient is —	$\frac{U}{P} =$	$0,17915 =$	$9,253172$

Then multiply it into the *given Power* of the *Rate*, to which add the *Power*, &c. as *per Theorem*; and you have this *Equation*, viz. $0,17915R^7 + R^7 - R^8 = 0,17915$; whence a Person ready at *Algebraick Extractions*, may soon discover $R = 1,06$. Then say, as 1 *l.* : 0,06 *l.* : : 100 *l.* : 6 *l.* the Rate *per Cent.* requir'd.

Note; This (and all Questions of this Nature) may be answered by the *Rule* of *Position*, in the same manner as was directed in the fourth Question of *Annuities* in *Arrears*.

Theorems *resolving all Questions relating to the Purchasing of* Free-hold *or* Real Estates, *at* Compound Interest.

Given $P\,R$; To find U?

Theorem 1. $PR - P = U$.

Given $U\,R$; To find P?

Theorem 2. $\frac{U}{R-1} = P$.

Given P, U; To find R?

Theorem 3. $\frac{P+U}{P} = R$.

Question 1. Suppose a *Free-hold* Estate of 25 *l. per Annum* were to be sold; What is the *Worth*, allowing 5 *l.* 10 *s. per Cent.* &c. *Compound Interest* to the Buyer?

Here is given $\left\{ \begin{array}{l} U = 25 \\ R = 1,055 \end{array} \right\}$ To find P, *per Theor.* 2.

Divide the Annual Rent —	$U =$	$25 =$	$1,397940$
By the Rate less Unity	$R - 1 =$	$0,055 =$	$8,740362$
The Quotient is the Worth	$P =$	$454,5 =$	$2,657578$

The *Value* of that Estate therefore is 454 *l.* 10 *s.* 10¾ *d.* *Q. E. I.*

Question

Question 2. Suppose a Person would lay out 416*l.* 13 *s.* 4 *d.* on a *Free-hold Estate*, and so as to be allowed 6 *l. per Cent.* for his Money, *Compound Interest*; What must be the *Annual Rent* of such an Estate?

Here is given $\left\{ \begin{array}{l} P = 416{,}6 \\ R = 1{,}06 \end{array} \right\}$ To find U, *per Theor.* 1.

Multiply the present Worth —	$P = 416{,}6$	$= 2{,}619789$
By the Rate —	$R = 1{,}06$	$= 0{,}025305$
The Product is —	$PR = 441{,}6$	$= 2{,}645094$
From which substract the Worth	$P = 416{,}6$	
There remains the Annual Rent / The Answer	$U = 25$ *l. per Annum.*	

Question 3. Suppose one give 416 *l.* 13 *s.* 4 *d.* for a *Free-hold Estate* of 25 *l. per Annum*; What *Rate per Cent. Compound Interest*, has the *Purchaser* for his Money?

Here is given $\left\{ \begin{array}{l} P = 416{,}6 \\ U = 25 \end{array} \right\}$ To find R, *per Theor.* 3.

To the present Worth —	$P = 416{,}6$	
Add the Annual Rent —	$U = 25{,}0$	
Divide their Sum —	$P + U = 441{,}6$	$= 2{,}645094$
By the present Worth —	$P = 416{,}6$	$= 2{,}619789$
The Quotient is the Rate sought	$R = 1{,}06$	$= 0{,}025305$

Then say, As 1 *l.* : ,06 *l.* : : 100 *l.* : 6 *l. per Cent.* the Answer.

Rebate *or* Discount.

What this is I have already defin'd in the Beginning of this Chapter; The *Interest*, and *Discount*, of the *same Parcel* of Money, is very different, tho' *vulgarly* understood (and accordingly is reckoned) the *same* thing.

In order therefore to have a right Notion of *Discount*, and how it differs from *Interest*; we must consider, that *Interest* is the *Increase* of any *Principal*, or Sum of Money, according to *any Rate*, or *Proportion*, *agree'd on*; and in computing it, we have Regard only to the *bare Principal*; But what is properly call'd *Discount*, is the *Difference* between a Sum of Money *due any Time hence*, and such another Sum as, being *put to Interest*, would, with its *Increase* by *Interest*, become *equal* to the said Sum *hereafter due*.

Thus,

Thus, for *Example*, If I have 105 *l.* due to me 12 Months hence; the *Discount* for *prompt Payment* thereof at 5 *l. per Cent. Simple Interest* must be 5 *l.* and the *present Money*, or *Worth* of that 105 *l.* is 100 *l.*; Because if I put 100*l.* out at the aforesaid *Rate*, it would in *that Time* be *equal* or *amount* to 105 *l.* Wherefore the *Interest* of 105 *l* discounted (as is the common way) I should receive but 99 *l.* 15 *s.*; the *Interest* of the 105 *l.* being 5 *s.* more than the true *Discount*; and consequently the reckoning *Interest* for *Discount* is very disadvantagious to those who make *such Discounts.*

The Proportion for *Rebate* or *Discount* then is,

As 100 *l.* and the *Rate* : is to the *Rate* : : so is any other Sum : to its *true Discount* for the *same* Time.

The *Theorem* for finding at once both the *Discount* and *present Worth* of any Sum of Money, due any time hereafter, is the second *Theorem* of *Simple* and *Compound Interest*, as I there observ'd.

By the second *Theorem* of *Simple Interest*, it was found that the *present Worth* of 405 *l.* 6 *s.* 0 *d.* due 5 Years and 8 Months hence at 5 *per Cent.*

	l.	*s.*	*d.*
Must be — — —	316	9	5
Which substracted from the *Debt*, leaves the *true Discount* —	88	16	7
But the *Interest* of that *Sum* is for that *Time* — —	113	15	2
Which exceeds the *true Discount* by —	24	18	7

Above a fourth Part loss to the *Discounter of Interest* for such a Sum.

Note. This *Theorem* of Mr. *Ward*'s, is far more easy, concise, and elegant, than any other extant, for finding the *present Worth*, or *Discount* for *prompt Payment of any Debt.*

A TA-

A Table *of* Days *for any given Time.*

Days	January	February	March	April	May	June	July	August	September	October	November	December
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

The Use of the Table.

First; To know the *Number* of *Days* from the *Beginning* of the *Year*, to any given *Day* of any *Mouth*.

 This

This is obtain'd by Inspection only; Thus from *January* the 1st, to *September* the 7th, is 250 *Days*; To *November* the 27th are 331, *&c.*

Secondly, To know what is the *Number* of *Days* from any given *Day* of any *Month*, to the *End* of the *Year*.

Suppose *September* the seventh, then from	—	365
Substract the Number answering to *Sept.* 7	—	250
There remains the Number of Days sought, *viz.*		115 Days.

Thirdly, To find the *Number* of *Days* between the given *Day* of any one *Month*, and any given *Day* of any other *Month*, in the *same Year*.

For Instance, To know how many *Days* there are between *April* the 17th, and *October* 23.

Thus, From the Number answering to *October* 23	—	296
Substract that answering to *April* 17	—	107
The *Remainder* is the Number of Days sought	—	189

Fourthly, To find the *Number* of *Days*, from any given *Day* of any *Month* in one *Year*, to any given *Day* of any *Month* in the *next Year*.

How many Days is it from *September* the 7th, 1733, to *April* the 19th, 1734?

From the Days of a Whole Year	—	365
Substract the Number to *September* 7	—	250
Remains the Number to the *End* of the *Year*	—	115
To which *add* the Number to *April* 19	—	109
The *Sum* is the *Number* of *Days* required	—	224

And thus is the *Number of Days* readily found for any *Interval* of *Time* given, in the same *Year* compleatly; or which is part of one, or part of another Year.

How very *necessary* and *useful* a *Table* this is in all Parts of *Arithmetical* Science relating to *Time* is sufficiently evident to the *Skilful* therein; but because it is more *particularly* so in the whole Affair of *Interest*, I have therefore prefixed it to the other Tables.

Having then the *Number of Days*, 'tis easy to find what *Decimal* Part of the *Year*, they make; and having found that, you have the *T*, *t*, in the foregoing *Theorems* representing any *Part* of a *Year*.

An

An Example in *Simple* and *Compound Interest*, will make the whole Matter easy and conspicuous.

Example 1. What will 65 *l. amount* to, being lent from *March* the 7th to *November* the 3d, at 5 *l. per Cent. per Annum Simple Interest?*

From *March* the 7th to *November* the 3d are 241 Days; those make 8 *Months*, 2 *Weeks*, and 3 *Days*, = 660273 *Decimals* of a Year. Then by *Theorem* 1.

Multiply the *Time*	— —	$T = 0,660273$
By the *Ratio* of the *Rate*	—	$R = 0,05$
And that *Product*	—	$TR = 0,03301365$
Multiply by the *Principal*	—	$P = 65$
The Product is	—	$TRP = 2,14588725$
To which add the *Principal*		$P = 65$
The Sum is the *Amount* sought		$= 67,1458$ &c. *l.*

Example 2. What is the *Amount* thereof at *Compound Interest*, the *Rate* and *Time* being the *same?*

The Logarithm of the *Rate*	$R = 1,05 =$	$0,0211893$
Multiply by the *Time*	— $t =$	$,6603$
The Product is the Logar. of R^t	$= R^{0,6603} =$	$0,0139912$
To which add the Log. of the *Prin.*	$P = 63 =$	$1,8129133$
The Sum is the Log. of *Amount*	$A = 67,1281 =$	$1,8269045$

And thus the *Theorems* serve to answer *Questions*, when the *Time* is only *part* of a Year, as well as when *compleat* Years.

Proem *to the* Tables *of* Simple Interest, *concerning their* Nature, Construction, *and* Use.

The great Design of *Tables* of *Interest* (both *Simple* and *Compound*) is *Ease* and *Expedition* in practical *Calculations*. For, besides that the *Rules* expressed in *Words* for answering Questions of *Interest* are *tedious* and *intricate*, and the *Reason* no ways to be understood; the *Operations* themselves are, for the most part, very *laborious*; and consequently *Tables* which expedite and facilitate the *Practice* are indispensibly necessary.

This being undeniably evident, the Question occurs, Whether *these Tables* are to be made in *Decimal* or *mixed Numbers* (*i. e.* such as express the *Money* in its common Denominations of *Pounds, Shillings,* and *Pence*)? The Answer to this can admit of no *Demurr* amongst those who understand the Doctrine of *Decimal Numbers*; they all know the *Excellency* and superiour Use of the *first Sort*, viz. *Decimal Tables.* But *Interest Tables* expressed in *common Money* are indulged to those who understand not *Decimals*, as *Crutches* to the *Lame*, and *Spectacles* to the *Weak-sighted.*

The *Numbers* in the *first* of these *Tables* of *Simple Interest* for *Days*, and in the *Second* for *Years*, being in *Arithmetical Proportion*, makes them capable of that *Perfection*, which *no other Tables* can pretend to.

These Tables are *so* contrived, That the *Interest* of any *Principal Sum* is easily found for *any Number* of *Days* or *Years* at any *Rate* from *one Pound* to *Ten*, with the *Halves* and *Quarters*. Having followed herein the Rev. Mr. *Brown* in his *Arithmetica Infinita.*

The *Construction* of these *Tables* is easy from the *Theorems* themselves, (and indeed the *Reason* of their *Construction* can be no otherways so easily conceiv'd.) Thus by *Theorem* the *first* of *Simple Interest*, viz. $tRP + P = A$ is the *first* and *second Table* constructed. For since the *Amount* less the *Principal*, is equal to the *Interest*, therefore the *Theorem* will be $tRP =$ *Interest.* Now if $P = 1\ l.$ $t =$,002739 &c. (the *Decimal* of a *Year* for *one Day*), and $R =$ any *Ratio* of *Interest*, suppose 5 *per Cent.*; then the *Simple Interest* of *one Pound* for one Day, at 5 *per Cent.* is ,002739 &c. × ,05 × 1 = 00013698 &c. which being multiplied by the *nine Digits* severally constitute that part of the Table of Interest at 5 *per Cent.* and thus the whole *first Table* is made. The *second Table* for *Years* is only the various *Ratio's* of *Interest* multiplied by the said *Nine Digits*; for since $t = 1$ *Year*, and $P = 1\ l.$ it will be $tRP = R$ the *Interest* for the *first Year*, &c.

The *third Table* shews the *Rebate* or *Discount* to be made for one Pound, at the several *Rates per Cent.* for Days. The *Manner, Truth,* and *Reason* of its Construction is derived from *Theorem* 2. of *Simple Interest*, viz. $\frac{A}{tR - 1} = P.$ For since the *Principal* or *present Worth* subducted from the *Amount* gives the *Rebate* or *Discount* of that *Amount*; there-

fore

fore the *Discount* of any *Amount* for any *Time* at any *Rate* (without Regard of the *present* Value or *principal* Money) may be found by this *Theorem* $\frac{AtR}{tR+1} = D =$ *Discount.* Hence if we put $A = 1$ *l.* $t = ,002739$ &c. and $R =$ any *Ratio* of *Interest*, suppose 5 *per Cent.* then by this *last Theorem* we have the *Discount* of *one Pound* for *one Day* at the *Rate* of 5 *per Cent. per Annum*; For $AtR = 1 \times ,002739$ &c. $\times ,05 = ,00013698$ &c. And $tR + 1 = 1,00013698$ &c. then by *Division*; 1,00013698 &c.) ,00013698 &c. (= ,00013697 &c. the *Discount.* If $t = 1$ *Year*; then the *Annual Discount* of *one Pound* at 5 *per Cent.* will be found, by the above *Theorem*, thus; $AtR = ,05$ and $tR + 1 = 1,05$. Therefore by Division, 1,05) ,05 (= ,04761904 &c. the *Discount.* And thus is the *Discount* of any *Sum* at any *Rate* for any *Time* above one Year found at once by the above Theorem; and for any *Time* under a *Year* by the *Table* of *Discount* for *Days*, of which I have now taught the Construction in a ***new*** and ***more rational Method*** than any I have yet seen.

The Use of Table I, *and* II.

In order to understand how to make those two *Tables* universally usefull, the Reader is to observe, that if a Number consists of only one Digit with Cyphers affixed, as 10, 50, 700, 9000, 800000, &c. 'tis called a *pure Number*; but those Numbers which consist of more than one, or wholly of Digits, As 370, 568, 7569, &c. may be called *Mixed Numbers.* Now every *mixed* Number may be resolved into those *pure Numbers*, of which they are composed; thus the *mixed* Number 507, may be resolved into the Pure Numbers 500, 60, and 7; so also 15890 is resolved into 10000, 5000, 800, and 90.

Now then as to the *Use* of the Tables, observe these Rules;

I. If the Number of *Days*, *Years*, &c. proposed, be a *mixed Number*, let it be resolved into *pure* Numbers.

II With the *pure Numbers* severally enter the Tables, and take those *Decimal Numbers* which stand against the first Figure of each *pure Number*, in the Column marked *Numbers*

III. Remove the *Decimal Point* in each such *Decimal* Num-

Number, so many Places to the *Right-hand*, as there are *Cyphers* in the respective *pure* Numbers.

IV. Lastly, Add together all the *Decimal Numbers*, and find the *Value* thereof by the *Tables* for that purpose.

These things *premised*, the *Use* of the Tables will be obvious from the *Examples* of the following *Problems*.

Problem 1.

To find the *Interest* of any *Sum* of Money for a *Day*, or a *Year*, at any Rate *per Cent. per Annum*.

Example 1.

What is the Interest of 2746 *l.* at 5 *l.* 15 *s. per Cent.* for a Day?

			Decimals.
In *Table* 1. under the Rate $5\frac{3}{4}$. You find against the *pure* Numbers	2000	—	,31506
	700	—	,11027
	40	—	,00630
	6	—	,00094
The Answer is	—	—	,43257 = 8 *s.* $7\frac{3}{4}$ *d.*

Example 2.

What is the *Interest* of the same *Sum*, at the same *Rate* for a *Year*?

			Decimals.	
In *Table* 2. You find against	2000	—	150,00000	Under $5\frac{3}{4}$ *d. per Cent.*
	700	—	40,25000	
	40	—	2,30000	
	6	—	0,34500	

The Answer in *Decimals* *l.* 192,895
Which is in *Money* = 192 *l.* 17 *s.* $10\frac{3}{4}$ *d.*

Problem 2.

To find the *Interest* of any *Sum* of *Money* for any Number of *Days*.

Example.

What is the *Interest* of 265 *l.* for 149 Days, at the *Rate* of 3 *l.* 15 *s. per Cent.* &c.

Multi-

Multiply the *Principal Sum*	—	265 *l.*
By the given Number of *Days*	—	149
The Product is the *mixed Number*		39485, with which

resolved, enter the Table as before :

			Decimals.	
Thus in *Table* 1. You find against the *pure* Numbers	30000	—	3,08200	Under $3\frac{1}{4}$ *per Cent.*
	9000	—	0,92466	
	400	—	0,04109	
	80	—	0,00822	
	5	—	0,00051	
The Answer in *Decimals*		— *l.*	4,05648	

In *Money* 4 *l.* 1 *s.* $1\frac{1}{2}$ *d.*

The Method is the same for any *greater* Number of Days.

Problem. 3.

To find the *Interest* of any *Sum* forborne any Number of *Years* at any of the given *Rates per Cent.*

Example.

What is the *Interest* of 175 *l.* 15 *s.* forborne 13 *Years* at the *Rate* of 6 *per Cent.* &c.?

Multiply the *Principal* Sum	—	175,75
By the Number of *Years* given	—	13
The Product is the *mixed* Number	—	2284,75

Which resolved, as before, will stand thus,

In *Table* 2. You find against	2000,	—	120,000	Under 6 *per Cent.*
	200,	—	12,000	
	80,	—	4,800	
	4,	—	0,240	
	,7	—	0,042	
	,05	—	0,003	
The Answer in *Decimals*		*l.*	137,085	

The same in *Money* 137 *l.* 1 *s.* 8 *d* $\frac{1}{4}$.

N. B. The

N. B. The Reader must observe, in resolving a *mixed* Number wherein are *Decimals*, to remove the Point one Place more to the *left* than are the *Number* of *Cyphers* in the *Decimal pure* Number, as in the *last* Example.

The Use of Table III. *Of* Discount.

In seeking the *Discount* for any *Sum* due at the End of any Number of *Days*, if the Number of Days be a *mixed* one, resolve them into *pure* Numbers as before taught; and even with them in the Table take the *Discount* of 1 *l.* which add and multiply by the *Principal* Sum, the Product will be the *Discount* thereof.

Problem 4.

To find the *Discount* of any *Sum*, for any Number of *Days*, at any given *Rate in the Table.*

Example.

What is the *Discount* of 83 *Pounds*, 10 *Shillings*, for 235 *Days* at 4 *per Cent. per Annum?*

			Decimals.	
You find	200	—	,0214478	
In the *Table*	30	—	,0032769	Under 4 *per Cent.*
even with	5	—	,0005476	
The Sum is		—	,0252723	
Which multiplied by the Sum			835	*l. s. d.*
The Product is the Answer			2,110237 &c.	= 2—2—2¼

Problem 5.

To find the *Discount* of any *Sum* for a *Year.*

Example.

What is the *Discount* of 100 *l.* for one *Year*, at 5 *per Cent?*

In

In the Table under 5 *per Cent* and against 365 Days is } ,047619 &c.
Which mul. by the *Principal* Sum 100

		l.	*s.*	*d.*
The Product is the Answer	*l.* 4,7619 &c. =	4	15	$2\frac{1}{4}$
Now the Interest of 100 *l.* for one Year, at 5 *per Cent.* is — —	}	5	0	0
The Differ. therefore of *Discount* and *Interest*, is		0	4	$9\frac{1}{4}$

Whence 'tis evident, he who allows *Interest* for *Discount* wrongs himself considerably, which yet is very common among *Traders*; for so much Money ought to be paid, as, at *Interest*, would amount to the *Sum* due, in the *Time* proposed.

Example 2.

What is the *Discount* of 9342 *l.* at $4\frac{1}{2}$ *per Cent.* for a *Year?*

The *Discount* of 1 *l.* for 365 Days, at $4\frac{1}{2}$ *per Cent.* in the *Table*, is — } ,043062, &c.
Which multiplied by the *Principal* Sum 9342

The Product is the Answer — *l.* 402,2852 &c.

In *Money* 402 *l.* 5 *s.* $8\frac{1}{2}$. And thus proceed for other *annual Discounts.*

I must acknowledge this Table of *Discount* gives not the precise Truth, and yet differs but little from it; being sufficiently *exact* for any Use. None but a Table of the *Discount* for *every Day*, can be *perfect*; because every *Day's Discount* differs, being still less as the Number of *Days* increase.

This Table is *perfectly true* for all the Days express'd therein, and, as I said, may be used without much Errour for any other.

		l.	*s.*	*d.*
Example in *Prob.* 4.	The true *Discount* is —	2	1	11
	The *Discount* by this Table	2	2	$2\frac{1}{4}$
	The *Int.* for the Time and Rate	2	3	0

TABLES *of* Simple Interest.

Table I. *The Interest of one Pound for* Days.

Table II. *The Interest of one Pound for* Years.

Both at any Rate per Cent. *from one to ten Pounds with Halves and Quarters.*

Table I. *The Interest of one Pound* per Diem.

Numb.	1 *per Cen.*	1 $\frac{1}{4}$ *per C.*	1 $\frac{1}{2}$ *per C.*	1 $\frac{3}{4}$ *per C.*
1	,00002740	,00003425	,00004110	,00004794
2	,00005480	,00006850	,00008220	,00009589
3	,00008220	,00010274	,00012329	,00014383
4	,00010959	,00013699	,00016438	,00019178
5	,00013698	,00017123	,00020548	,00023972
6	,00016438	,00020548	,00024657	,00028767
7	,00019178	,00023973	,00028767	,00033562
8	,00021918	,00027398	,00032877	,00038356
9	,00024657	,00030822	,00036986	,00043151
Month.		,00104166	,00125000	,00145833

Table II. *The Interest of one Pound* per Annum.

Numb.	1 *per Cent.*	1 $\frac{1}{4}$ *per C.*	1 $\frac{1}{2}$ *per C.*	1 $\frac{3}{4}$ *per C.*
1	0,01000000	0,01250000	0,01500000	0,01750000
2	0,02000000	0,02500000	0,03000000	0,03500000
3	0,03000000	0,03750000	0,04500000	0,05250000
4	0,04000000	0,05000000	0,06000000	0,07000000
5	0,05000000	0,06250000	0,07500000	0,08750000
6	0,06000000	0,07500000	0,09000000	0,10500000
7	0,07000000	0,08750000	0,10500000	0,12250000
8	0,08000000	0,10000000	0,12000000	0,14000000
9	0,09000000	0,11250000	0,13500000	0,15750000

Table

Table I. *The Interest of one Pound* per Diem.

Numb.	2 per C.	2 ¼ per C.	2 ½ per C.	2 ¾ per C.
1	,00005480	,00006164	,00006849	,00007534
2	,00010959	,00012329	,00013699	,00015068
3	,00016438	,00018493	,00020547	,00022602
4	,00021918	,00024657	,00027397	,00030137
5	,00027397	,00030822	,00034246	,00037671
6	,00032876	,00036986	,00041095	,00045205
7	,00038356	,00043151	,00047945	,00052739
8	,00043835	,00049315	,00054794	,00060274
9	,00049315	,00055479	,00061644	,00067808
Month	,00166666	,00187500	,00208333	,00229166

Table II. *The Interest of one Pound* per Annum.

Numb.	2 per Cent.	2 ¼ per C.	2 ½ per C.	2 ¾ per C.
1	0,02000000	0,02250000	0,02500000	0,02750000
2	0,04000000	0,04500000	0,05000000	0,05500000
3	0,06000000	0,06750000	0,07500000	0,08250000
4	0,08000000	0,09000000	0,10000000	0,11000000
5	0,10000000	0,11250000	0,12500000	0,13750000
6	0,12000000	0,13500000	0,15000000	0,16500000
7	0,14000000	0,15750000	0,17500000	0,19250000
8	0,16000000	0,18000000	0,20000000	0,22000000
9	0,18000000	0,20250000	0,22500000	0,24750000

Table I. *The Interest of one Pound* per Diem.

Numb.	3 per Cent.	3 ¼ per C.	3 ½ per C.	3 ¾ per C.
1	,00008220	,00008904	,00009589	,00010274
2	,00016438	,00017808	,00019178	,00020548
3	,00024657	,00026712	,00028767	,00030822
4	,00032877	,00035616	,00038356	,00041096
5	,00041096	,00044520	,00047945	,00051363
6	,00049315	,00053424	,00057534	,00061644
7	,00057534	,00062328	,00067123	,00071917
8	,00065753	,00071232	,00076712	,00082192
9	,00073972	,00080137	,00086301	,00092465
Month.	,00250000	,00270833	,00291666	,00312500

Table II. *The Interest of one Pound* per Annum.

Numb.	3 per Cent.	$3\frac{1}{4}$ per C.	$3\frac{1}{2}$ per C.	$3\frac{3}{4}$ per C.
1	0,03000000	0,03250000	0,03500000	0,03750000
2	0,06000000	0,06500000	0,07000000	0,07500000
3	0,09000000	0,09750000	0,10500000	0,11250000
4	0,12000000	0,13000000	0,14000000	0,15000000
5	0,15000000	01,6250000	0,17500000	0,18750000
6	0,18000000	0,19500000	0,21000000	0,22500000
7	0,21000000	0,22750000	0,24500000	0,26250000
8	0,24000000	0,26000000	0,28000000	0,30000000
9	0,27000000	0,29000000	0,31500000	0,33750000

Table I. *The Interest of one Pound* per Diem.

Days.	4 per Cent.	$4\frac{1}{4}$ per C.	$4\frac{1}{2}$ per C.	$4\frac{3}{4}$ per C.
1	,00010959	,00011644	,00012329	,00013014
2	,00021918	,00023288	,00024657	,00026027
3	,00032877	,00034931	,00036986	,00039041
4	,00043836	,00046575	,00049315	,00052055
5	,00054794	,00058219	,00061643	,00065068
6	,00065753	,00069863	,00073973	,00078082
7	,00076712	,00081507	,00086301	,00091096
8	,00087671	,00093151	,00098630	,00104109
9	,00098630	,00104794	,00110959	,00117123
Month.	,00333332	,00354166	,0037500	,0039582

Table II. *The Interest of one Pound* per Annum.

Years.	4 per Cent.	$4\frac{1}{4}$ per C.	$4\frac{1}{2}$ per C.	$4\frac{3}{4}$ per C.
1	0,04000000	0,04250000	0,04500000	0,04750000
2	0,08000000	0,08500000	0,09000000	0,09500000
3	0,12000000	0,12750000	0,13500000	0,14250000
4	0,16000000	0,17000000	0,18000000	0,19000000
5	0,20000000	0,21250000	0,22500000	0,23750000
6	0,24000000	0,25500000	0,27000000	0,28500000
7	0,28000000	0,29750000	0,31500000	0,33250000
8	0,32000000	0,34000000	0,36000000	0,38000000
9	0,36000000	0,38250000	0,40500000	0,42750000

Table

Table I. *The Interest of one Pound* per Diem.

Days.	5 *per Cent.*	5 ¼ *per C.*	5 ½ *per C.*	5 ¾ *per C.*
1	,00013698	,00014383	,00015068	,00015753
2	,00027397	,00028767	,00030137	,00031507
3	,00041096	,00043151	,00045205	,00047260
4	,00054794	,00057534	,00060274	,00063014
5	,00068493	,00071918	,00075342	,00078767
6	,00082192	,00086301	,00090411	,00094520
7	,00095890	,00100685	,00105479	,00110274
8	,00109589	,00115068	,00120548	,00126027
9	,00123288	,00129452	,00135616	,00141781
Month.	,00416666	,00437500	,00458333	,00479166

Table II. *The Interest of one Pound* per Annum.

Years.	5 *per Cent.*	5 ¼ *per C.*	5 ½ *per C.*	5 ¾ *per C.*
1	0,05000000	0,05250000	0,05500000	0,05750000
2	0,10000000	0,10500000	0,11000000	0,11500000
3	0,15000000	0,15750000	0,16500000	0,17250000
4	0,20000000	0,21000000	0,22000000	0,23000000
5	0,25000000	0,26250000	0,27500000	0,28750000
6	0,30000000	0,31500000	0,33000000	0,34500000
7	0,35000000	0,36750000	0,38500000	0,40250000
8	0,40000000	0,42000000	0,44000000	0,46000000
9	0,45000000	0,47250000	0,49500000	0,51750000

Table I. *The Interest of one Pound* per Diem.

Days.	6 *per Cent.*	6 ¼ *per C.*	6 ½ *per C.*	6 ¾ *per C.*
1	,00016438	,00017123	00017808	,00018493
2	,00032876	,00034246	00035616	,00036986
3	,00049315	,00051370	00053424	,00055479
4	,00065753	,00068493	00071232	,00073972
5	,00082192	,00085616	00089041	,00092466
6	,00098630	,00102740	00106849	,00110959
7	,00115068	,00119863	00124657	,00129452
8	,00131507	,00136986	00142465	,00147945
9	,00147945	,00154109	00160274	,00166438
Month.	,00500000	,00520833	00541666	,00562500

Table II. *The Interest of one Pound* per Annum.

Years.	*6 per Cent.*	*6 ¼ per C.*	*6 ½ per C.*	*6 ¾ per C.*
1	0,06000000	0,06250000	0,06500000	0,06750000
2	0,12000000	0,12500000	0,13000000	0,13500000
3	0,18000000	0,18750000	0,19500000	0,20250000
4	0,24000000	0,25000000	0,26000000	0,27000000
5	0,30000000	0,31250000	0,32500000	0,33750000
6	0,36000000	0,37500000	0,39000000	0,40500000
7	0,42000000	0,43750000	0,45500000	0,47250000
8	0,48000000	0,50000000	0,52000000	0,54000000
9	0,54000000	0,56250000	0,58500000	0,60750000

Table I. *The Interest of one Pound* per Diem.

Days.	*7 per Cent.*	*7 ¼ per C.*	*7 ½ per C.*	*7 ¾ per C.*
1	,00019178	,00019863	,00020548	,00021233
2	,00038356	,00039726	,00041096	,00042466
3	,00057534	,00059589	,00061644	,00063699
4	,00076712	,00079452	,00082192	,00084932
5	,00095890	,00099315	,00102739	,00106164
6	,00115068	,00119178	,00123288	,00127397
7	,00134246	,00139041	,00143836	,00148630
8	,00153425	,00158904	,00164384	,00169863
9	,00172603	,00178767	,00184932	,00191096
Month.	,00583333	,00604166	,00625000	,00645833

Table II. *The Interest of one Pound* per Annum.

Years.	*7 per Cent.*	*7 ¼ per C.*	*7 ½ per C.*	*7 ¾ per C.*
1	0,07000000	0,07250000	0,07500000	0,07750000
2	0,14000000	0,14500000	0,15000000	0,15500000
3	0,21000000	0,21750000	0,22500000	0,23250000
4	0,28000000	0,29000000	0,30000000	0,31000000
5	0,35000000	0,36250000	0,37500000	0,38750000
6	0,42000000	0,43500000	0,45000000	0,46500000
7	0,49000000	0,50750000	0,52500000	0,54250000
8	0,56000000	0,58000000	0,60000000	0,62000000
9	0,63000000	0,65250000	0,67500000	0,69750000

Table I. *The Interest of one Pound* per Diem.

Days.	8 *per C.*	8 ¼ *per C.*	8 ½ *per C.*	8 ¾ *per C.*
1	,00021918	,00022603	,00023287	.00023973
2	,00043835	,00045205	,00046575	,00047945
3	,00065753	,00067808	,00069863	,00071918
4	,00087671	,00090411	,00093150	,00095890
5	,00109589	,00113014	,00116438	,00119863
6	,00131507	,00135616	,00139726	,00143835
7	,00153425	,00158219	,00163013	,00167808
8	,00175342	,00180822	,00186301	,00191781
9	,00197260	,00203424	,00209589	,00215753
Month.	,00666666	,00687500	,00708333	,00729166

Table II. *The Interest of one Pound* per Annum.

Years.	8 *per Cent.*	8 ¼ *per C.*	8 ½ *per C.*	8 ¾ *per C.*
1	0,08000000	0,08250000	0,08500000	0,08750000
2	0,16000000	0,16500000	0,17000000	0,17500000
3	0,24000000	0,24750000	0,25500000	0,26250000
4	0,32000000	0,33000000	0,34000000	0,35000000
5	0,40000000	0,41250000	0,42500000	0,43750000
6	0,48000000	0,49500000	0,51000000	0,52500000
7	0,56000000	0,57750000	0,59500000	0,61250000
8	0,64000000	0,66000000	0,68000000	0,70000000
9	0,72000000	0,74250000	0,76500000	0,73750000

Table I. *The Interest of one Pound* per Diem.

Days.	9 *per Cent.*	9 ¼ *per C.*	9 ½ *per C.*	9 ¾ *per C.*
1	,00024657	,00025342	,00026028	00026712
2	,00049315	,00050684	,00052055	00052424
3	,00073972	,00076027	,00078082	00080137
4	,00098630	,00101370	,00104109	00106849
5	,00123287	,00126712	,00130137	00133561
6	,00147945	,d0152055	,00156164	00160274
7	,00172602	,00177397	,00182192	00186986
8	,00197260	,00202739	,00208219	00213699
9	,00221918	,00228082	,00234246	00240410
Month.	,00750000	,00770833	,00791666	00812500

Table

Table II. *The Interest of one Pound* per Annum.

Years.	*9 per Cent.*	*9 ¼ per C.*	*9 ½ per C.*	*9 ¾ per C.*
1	0,09000000	0,09250000	0,09500000	0,09750000
2	0,18000000	0,18500000	0,19000000	0,19500000
3	0,27000000	0,27750000	0,28500000	0,29250000
4	0,36000000	0,37000000	0,38000000	0,39000000
5	0,45000000	0,46250000	0,47500000	0,48750000
6	0,54000000	0,55500000	0,57000000	0,58500000
7	0,63000000	0,64750000	0,66500000	0,68250000
8	0,72000000	0,74000000	0,76000000	0,78000000
9	0,81000000	0,83250000	0,85500000	0,87750000

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TABLE III.

Of Simple Interest.

The Rebate *or* Discount *of one Pound for* Days, *at the Rates of* 2 ; 2 ½ ; 3 ; 3 ½ ; 4 ; 4 ½ ; 5 ; 6 ; per Cent. per Annum.

Days.	2 *per Cent.*	2 ½ *per C.*	3 *per Cent.*	3 ½ *per C.*
1	,0000548	,0000685	,0000822	,0000959
2	,0001096	,0001370	,0001644	,0001917
3	,0001644	,0002054	,0002465	,0002876
4	,0002191	,0002739	,0003287	,0003834
5	,0002739	,0003424	,0004108	,0004792
6	,0003287	,0004108	,0004929	,0005750
7	,0003834	,0004792	,0005750	,0006708
8	,0004382	,0005477	,0006571	,0007666
9	,0004929	,0006161	,0007392	,0008623
10	,0005477	,0006845	,0008212	,0009580
20	,0010947	,0013680	,0016411	,0019141
30	,0016411	,0020506	,0024597	,0028685
40	,0021870	,0027322	,0032769	,0038210
50	,0027322	,0034139	,0040928	,0047716
60	,0032769	,0040928	,c049073	,0057205
70	,0038210	,0047716	,0057205	,0066676
80	,0043644	,0054496	,0065324	,0076128
90	,0049073	,0061266	,0073429	,0085563
100	,0054496	,0068027	,0081522	,0094980
110	,0059913	,0074779	,0089601	,0104379
120	,0065324	,0081522	,0097667	,0113760
130	,0070729	,0088255	,0105720	,0123123
140	,0076128	,0094980	,0113760	,0132468
150	,0081522	,0101695	,0121786	,0141796
160	,0086909	,0108401	,0129780	,0151106

TABLE III.

The Discount of one Pound for Days.

Days.	4 *per Cent.*	4 ½ *per C.*	5 *per Cent.*	6 *per Cent.*
1	,0001096	,0001233	,0001370	,0001644
2	,0002191	,0002465	,0002739	,0003287
3	,0003287	,0003697	,0004108	,0004929
4	,0004382	,0004929	,0005477	,0006571
5	,0005477	,0006161	,0006845	,0008212
6	,0006571	,0007392	,0008212	,0009853
7	,0007665	,0008623	,0009580	,0011494
8	,0008759	,0009853	,0010947	,0013133
9	,0009853	,0011084	,0012314	,0014773
10	,0010947	,0012314	,0013680	,0016411
20	,0021870	,0024597	,0027322	,0032769
30	,0032769	,0036850	,0040928	,0049073
40	,0043644	,0049073	,0054496	,0065324
50	,0054496	,0061266	,0068027	,0081522
60	,0065234	,0073429	,0081522	,0097667
70	,0076128	,0085563	,0094980	,0113760
80	,0086909	,0097667	,0108401	,0129780
90	,0097667	,0109741	,0121786	,0145788
100	,0108401	,0121786	,0135135	,0161725
110	,0119112	,0133802	,0148448	,0177610
120	,0129800	,0145788	,0161725	,0193444
130	,0140465	,0157746	,0174966	,0209228
140	,0151006	,0169674	,0188172	,0224960
150	,0161725	,0181574	,0201342	,0240642
160	,0172321	,0193444	,0214477	,0256273

TABLE III.

The Discount of one Pound for Days.

Days.	2 per Cent.	2 ½ per C.	3 per Cent.	3 ½ per C.
170	,0092291	,0115098	,0137801	,0160399
180	,0097667	,0121786	,0145788	,0169674
190	,0103037	,0128465	,0153763	,0178932
200	,0108401	,0135135	,0161725	,0188172
210	,0113759	,0141796	,0169674	,0197395
220	,0119112	,0148448	,0177610	,0206601
230	,0124459	,0155091	,0185534	,0215789
240	,0129800	,0161725	,0193444	,0224959
250	,0135135	,0168350	,0201342	,0234114
260	,0140465	,0174966	,0209227	,0243251
270	,0145788	,0181574	,0217100	,0252370
280	,0151106	,0188172	,0224960	,0261473
290	,0156418	,0194762	,0232807	,0270558
300	,0161725	,0201342	,0240642	,0279627
310	,0167026	,0207914	,0248464	,0288679
320	,0172321	,0214477	,0256273	,0297714
330	,0177610	,0221031	,0264070	,0306732
340	,0182894	,0227577	,0271855	,0315734
350	,0188172	,0234114	,0279627	,0324718
360	,0193444	,0240642	,0287387	,0333686
361	,0193971	,0241294	,0288162	0,334582
362	,0194498	,0241946	,0288937	0,335478
363	,0195025	,0242598	,0289712	0,336374
364	,0195552	,0243251	,0290487	0,337269
365	,0196078	,0243902	,0291262	0,338164

TABLE III.

The Discount of one Pound for Days.

Days.	4 per Cent.	4 ½ per C.	5 per Cent.	6 per Cent.
170	,0182894	,0205286	,0227577	,0271855
180	,0193444	,0217100	,0240642	,0287387
190	,0203972	,0228885	,0253672	,0302869
200	,0214477	,0240642	,0266667	,0318302
210	,0224960	,0252370	,0279627	,0333686
220	,0235420	,0264070	,0292553	,0349022
230	,0245858	,0275743	,0305445	,0364309
240	,0256273	,0287387	,0318302	,0379547
250	,0266667	,0299003	,0331126	,0394737
260	,0277038	,0310592	,0343915	,0409879
270	,0287387	,0322153	,0356671	,0424974
280	,0297714	,0333686	,0369393	,0440021
290	,0308019	,0345192	,0382082	,0455021
300	,0318302	,0356671	,0394737	,0469974
310	,0328564	,0368122	,0407352	,0484880
320	,0338804	,0379547	,0419948	,0499740
330	,0349022	,0390444	,0432503	,0514553
340	,0359218	,0402314	,0445026	,0529320
350	,0369393	,0413657	,0457516	,0544041
360	,0379547	,0424974	,0469974	,0558717
361	,0380561	,0426104	,0471218	,0560182
362	,0381575	,0427234	,0472462	,0561647
363	,0382588	,0428364	,0473705	,0563111
364	,0383602	,0429493	,0474948	,0564575
365	,0384615	,0430622	,0476191	,0566028

The

The Nature, Conſtruction, *and* Uſe *of the* Decimal Tables *of* Compound Intereſt.

What *Compound Intereſt* is, I have already ſhewn in the *Theoretical* Part of this Doctrine; and from the ſaid *Theory* it alſo appears that Tables of *Compound* Intereſt are abſolutely neceſſary for thoſe who underſtand not *Logarithms* or *Algebra*; and therefore (though I have taught the Uſe of *Logarithms* after the *beſt Manner* in this Book) yet I have ſupplied the Reader with a *Set of Six Tables* for the Purpoſes of *Compound* Intereſt; I have framed them from the moſt compleat and approved *Calculations* of Mr. *John Smart*; his Book (which is *wholly* on *Tables of Intereſt*) having the beſt *Character* for Exactneſs, and the *Errata*'s of the Preſs, no more than *four*.

As I intend *nothing* ſhall be *wanting* in any Part of this *Syſtem*, to make it *compleat*; ſo I have contrived theſe Tables to anſwer *any Queſtion* of *Compound Intereſt*, for the *Rates* contained therein: For though they are not ſo large as the *Largeſt*, yet are they *larger* and more *univerſal* than any others, in any *mixed Pieces* of *Arithmetick* I have yet ſeen; I have choſen all the moſt *uſual* and *neceſſary Rates* of *Intereſt*; and continued each *Annual Table* to 50 *Years* which is farther than is generally needful; and ſhall ſhew how they may be uſed for any *indefinite* Number of Years required; but firſt of their *Conſtruction*, which is thus in the moſt *demonſtrative* Manner deduced from the *Theorems* aforegoing, whence not only the *Manner*, but the *Reaſon* of their *Conſtruction* (a Thing very *neceſſary*, though I know not where elſe to be met with) will be exceeding apparent.

The *Conſtruction* of the *Firſt* Table which ſhews the *Amount* of *one Pound* for *Days*, as alſo of the *Second* Table, which ſhews the ſame for *Years*, is made from *Theorem* 1. of *Compound Intereſt*, which is $PR^t = A$. Now if we put $P = 1\,l.$ then is the *Theorem* reduced to $R^t = A$. Conſequently, if $R = 1{,}05\,l.$ *per Cent.* (or any other *Ratio*) and $t = 1, 2, 3, 4,$ &c. Years;

Amount

Amounts.

Then it will be $R \overset{t=1}{=} A = 1{,}05$ the *first* Year.
Which mult. by $R = 1{,}05$

The Product is $R^2 = A = 1{,}1025$ the *second* Year.
And again by $R = 1{,}05$

The Product is $R^3 = A = 1{,}157625$ the *third* Year.
And again by $R = 1{,}05$

The Product is $R^4 = A = 1{,}21550625$ the *fourth* Year.

And thus for the other Years subsequent in the Table.

Thus also if $R = 1{,}00013368$ the *Ratio* for 1 Day.
Mult. as before $R = 1{,}00013368$

The Prod. is $R^2 = A = 1{,}00026738$ the *Amount* for 2 D.
And again by $R = 1{,}00013368$

The Prod. is $R^3 = A = 1{,}00040110$ the *Amount* for 3 D.

Thus is found the *Amounts* for all the subsequent Days in the Table.

Here it may be proper to observe, that the *Amount* or *Interest* of any Sum, at the same *Rate*, is more at *Compound* Interest than at *Simple*, for any time *above a Year*; *equal* for *one Year*; but *less* for any time *less* than *a Year*. Though this seems *strange* in the last Assertion; yet the *Reason* is evident to any who understands and considers that *Simple Interest* is grounded on *Arithmetical*, but *Compound Interest* on *Geometrical Progression*.

The *Construction* of *Table* 3. is by *Theorem* 2. viz. $\frac{A}{R^t} = P$ the *present Worth* or *Value* of one Pound, which is here to be considered as the *Amount*; therefore if $A = 1$ *l*. $R = 1{,}05$ and $t = 1, 2, 3, 4, 5$, and *Years*, as before; 'tis evident that *Unity*, or 1, being divided by the Numbers in the *second* Table (designed by R^t) will give the Numbers in this *third* Table, or the *present Values* of 1 *l*. for the *Tabular Years*, and 5 *per Cent*. and so for any other *Rate* of *Interest*.

Example

Example at 5 per Cent.

Unity or 1 divided by		The Quot. is		The Present Value of 1 *l.* for	
	1,05		,952381		1
	1,1025		,907029		2
	1,157625		,863837		3
	1,21550625		,822702		4
	1,276281		,783526		5 Years, &c.

The *Construction* of *Table* 4. is from *Theorem* 1. of *Annuities*, &c. in *Arrears*, viz. $\frac{UR^t - U}{R - 1} = A$. Now as it is $U = 1\,l.$ and $R = 1{,}05$ as before, then the *Theorem* will be brought to $\frac{R^t - 1}{,05} = A$, the *Amount* of 1 *l. Annuity* for the Number of Years design'd by *t*. That is, from (R^t) the Numbers in the *second Table*, substract *Unity*, or 1. The *Remainder* divided by ,05 (or $R - 1$) gives the Numbers in the *fourth* Table.

Example at 5 *per Cent.*

The Numbers in the 1st Table less Unity or 1. are		Which divided by ,05 the Quotients are		The Amounts of 1 *l.* Annuity for		Years &c.
	,05		1,00000		1	
	,1025		2,05		2	
	,157625		3,1525		3	
	,21550625		4,310125		4	
	,27628156		5,525631		5	

And thus you proceed for any *other Rate* of Interest.

The *Construction* of *Table* 5. is contained in *Theorem* 1. of the *present Worth* or Value of *Annuities*, which see; Now therein $U = 1\,l.$ and putting $R = 1{,}05$, and $t = 1, 2, 3, 4, 5,$ &c. Years; that *Theorem* immediately becomes $\frac{R^t - 1}{,05R^t} = P$, the *present Worth* sought. But in Construction of *Table* 4. 'twas shewn that $\frac{R^t - 1}{,05}$ constituted the Numbers of that Table. Therefore 'tis manifest, if the Numbers in *Table* 4. be divided by R^t (that is, by $1{,}05^1$, $1{,}05^2$, $1{,}05^3$, $1{,}05^4$, &c.) the *Quotients* will be the *Numbers* of the *fifth Table*, at 5 *per Cent.* Which, in short, is only this;

Divide

Divide the Numbers of the fourth Table, by the Numbers of the first Table, the Quotients make the fifth Table.

Table 1.	*Table* 4.	*Table* 5.	
1,05)	1,00000	(=0,95238, &c.	for the 1st Year.
1,1025)	2,05000	(=1,85941, &c.	for the 2d Year.
1,157625)	3,15250	(=2,72324, &c.	for the 3d Year.
1,21550625)	4,310125	(=3,54595, &c.	for the 4th Year.
1,27628156)	5,525631	(=4,32947, &c.	for the 5th Year, &c.

And thus for any other *Rate* of Interest.

The *Construction* of *Table* 6. is to be deduced from *Theorem* 2. of the *present Worth* of Annuities, &c. which see. Now since in this Case P is $= 1\,l.$ therefore that Theorem will be reduced to this form, $RR^t - R^t = UR^t - U$; whence (at 5 *per Cent.*) 'twill be $,05\,R^t = UR^t - U$; consequently $\frac{,05R^t}{R^t - 1} = U$ the Annuity required; but this being just the Reverse of $\frac{R^t - 1}{,05\,R^t}$, which make the Numbers of *Table* 5. 'tis plain, these two Theorems which constitute the Numbers of *Table* 5. and 6. multiplied together can make but 1. that is $\frac{,05\,R^t}{R^t - 1} \times \frac{R^t - 1}{,05\,R^t} = 1.$

Hence then if the Numbers of *Table* 5. be made Divisors, and Unity or 1. the constant Dividend, the Quotients shall be the Numbers which constitute the sixth Table, at 5 *per Cent.* and after the same Manner for any other Rate of Interest.

Example at 5 *per Cent.*

The Numbers of *Table* 5 at 5 *per C.* are		By which divide Unity, the Quotients will be		The Annuities 1 *l.* will purchase, for		
	,9523809		1,05		1	Years, &c.
	1,8594103		,5378049		2	
	2,7232480		,3637734		3	
	3,5459505		,2787437		4	
	4,3294767		,2277916		5	

In like manner, when necessary, may other Tables be constructed from the Theorems; Here are as many Tables as any Book (that I have seen) contains, and more than are in most. My Aim in the Construction of these Tables is more to shew the

the young Artiſt the Rationale or Reaſon thereof, than the Manner how only; ſince the latter has been often done, the former not at all that I know of; at leaſt, not in the natural Method by Deduction from the Theory it ſelf, as I have here done it.

Quomodo factum eſt? Is a Queſtion proper to *Mechanicks*; *Cur ita ſit faciendum?* Beſeems an Artiſt to enquire.

The Uſe of the following Tables.

The *Uſe* of all theſe Tables depends on this one obvious and eaſy *General Rule*,

Multiply the Tabular Number, which ſtands againſt the given Number of Days or Years, and under the given Rate of Intereſt, by the given Principal Sum; and the Product will ſatisfy the Queſtion.

Example *of* 246 *l.* *at* 5 per Cent. *for* 30 Days, *or* Years.

In *Table* I. againſt 30 Days under 5 *per C.* ſtands	1,0040182
Which multiplied by the *Principal* Sum —	246
The Product is the *Amount* required; *viz.* *l.*	247,0684772
In *Table* II. Againſt 30 *Years*, at 5 *per Cent.* is	4,3219424
Which multiplied by — — —	246
The Product is the *Amount* required; *viz.* *l.*	1063,1978 &c.
In *Table* III. Againſt the ſame *Time* and *Rate*, is	0,2313775
Which multiplied by — —	246
The Product is the *preſent Worth* required; *l.*	56,9189 &c.
In *Table* IV. For the given *Time* and *Rate*, is	66,4388475
Which multiplied by — —	246
The Prod. is the *A.* of ſuch an *Annuity*; *l.*	16343,9565 &c.
In *Table* V. For the given *Time* and *Rate*, is	15,372451
Which multiplied by — —	246
The Prod. is the *preſent W.* of that *Ann.* *l.*	3781,6229 &c.

In *Table* VI. For the given *Time* and *Rate*, is		,0650514
Which multiplied by	— —	246
The Product is the purchased *Annuity* ;		*l.* 16,0026 &c.

Therefore by the Tables we immediately know, that 246 *l.* forborne 30 Days, at 5 *per Cent. per Annum Compound Interest* will *amount* to 247 *l.* 1 *s.* 4 $\frac{1}{4}$ *d.*

That 246 *l.* forborne 30 *Years*, at 5 *per Cent.* &c. will *Amount* to 1063 *l.* 3 *s.* 11 $\frac{1}{4}$ *d.*

That the *present Worth* of 246 *l.* due 30 *Years* hence, at the *Rate* of 5 *per Cent.* &c. is 56 *l.* 18 *s.* 3 $\frac{1}{4}$ *d.*

That the *Amount* of an *Annuity* of 246 *l. per Annum*, forborne or unpaid 30 Years, at 5 *per Cent.* &c. is 16343 *l.* 19 *s.* 1 $\frac{1}{4}$ *d.*

That the *present Worth* of an *Annuity* of 246 *l.* to continue 30 *Years*, at 5 *per Cent. per Annum*, is 3781 *l.* 12 *s.* 5 $\frac{1}{2}$ *d.*

That the *Annuity* which 246 *l.* will purchase, to continue 30 *Years*, reckoning 5 *per Cent. Interest*, is 16 *l. per Annum.*

If the *Amount* of any *Sum* be sought, for a Number of Days which are not in the *first Table*, and *Years* which are not in the *second*, observe this

Rule ; Divide the given Number of *Days*, or *Years*, into two such Numbers as are in the Table, then multiply the *Amounts* pertaining to each, into each other ; then shall the Product be the *Amount* for the *Time* required.

Example 1.

What will 523 *l. amount* to, in 194 *Days*, at 5 *per Cent. per Annum ?* The two Parts of this Number in the *Table*, are 190, and 4 ; therefore,

In *Table* I. Against 190 Days, under 5 *per* C. is		1,0257228
And against 4 Days, at the same *Rate*, is		1,0005348
The Prod. is the *Am.* of 1 *l.* for 194 Days, *viz.*		1,0262714
Which multiply by the *Principal* Sum, *viz.*		523
This Product is the Answer	—	*l.* 536,7399840
In *Money*, 536 *l.* 14 *s.* 9 $\frac{1}{2}$ *d.*		

Example

Example 2.

What is the *Amount* of 150 *l.* in 91 *Years*, at 5 *per Cent?*

In *Table* II. Against 50 *Years*, under 5 *per C.* is	11,4674000
And against 41 *Years*, at 5 *per C.* is	7,3919881
The Prod. is the *Am.* of 1 *l.* for 91 Years, *viz.*	84,7668833
Which multiply by the *Principal* Sum, *viz.*	150
The Product is the Answer —	*l.* 12715,032495

In *Money* 12715 *l.* 0 *s.* 7 $\frac{1}{2}$ *d.*

Example 3.

What will 523 *l.* *amount* to in 5 *Years* and 194 *Days*, at 5 *per Cent?*

In *Table* II. against 5 *Years*, at 5 *per Cent.* is	1,2762816
And the *Am.* of 1 *l.* in 194 *Days*, as above, is	1,0262714
The Pr. is the *Am.* of 1 *l.* in 5 *Years*, and 194 *D.*	1,3098113
Which multiplied by the *Principal* Sum —	523
The Product is the Answer, *viz.* —	*l.* 685,0313413

In *Money* 685 *l.* 0 *s.* 7 $\frac{1}{2}$ *d.*

N. B. The other Tables of *Compound Interest*, as they cannot in this Manner be extended, so they seldom require it.

I shall now present the Reader with a few Questions of a more *complex Nature*, and which frequently happen, in order to shew the more *extensive Use* of the Tables.

Question 1.

Suppose I have 790 *l.* to be paid me within 7 Years, in this Manner; at the End of the first Year 90 *l.* of two Years, 100 *l.* of four Years 200 *l.* and of seven Years 400 *l.* Quere what the *present Worth* of those several Payments is in *ready Money*, allowing 4 $\frac{1}{2}$ *per Cent. Compound Interest?*

In *Table* III. the *present Worth* of 1 *l.* at 4 ½ *per Cent.*

Due at the End of 1 *Year*, is	—	0,9569378
Which multiply by the *Principal*	—	90
The Product is the *present Worth* of 90 *l.*	=	86,124402
Thus the present Worth of 100 *l.* due at the End of two *Years*, is found	=	91,57299
Also, if 200 *l.* at the End of 4 *Years*	=	167,71226
And of 400 *l.* at the End of 7 *Years*	=	293,93140
The *Sum* of all these is	—	*l.* 639,341052

Which answers the Question, *viz.* 639 *l.* 6 *s.* 9¼ *d.*

Question 2.

A owes to B 455 *l.* to be *paid* in 14 Years, *viz.* at the End of every 2 Years 65 *l.* But he would agree to pay him in 7 *Years*, by *equal Payments* each Year; which B agrees to, and at the *Rate* of 6 *per Cent. Compound Interest.* Quere what the *Annual Payment* must be?

1. Find the *present Worth* (by *Table* III.) of the 7 Payments which were at first to be made, as *per Quest.* 1. which you will find to be 293 *l.* 5 *s.* 2 *d.*

2. Then find (by *Table* VI.) what *Annuity*, to continue 7 Years at the given *Rate*, 293 *l.* 5 *s.* 2 *d.* will purchase; which you will find to be 52 *l.* 10 *s.* 8 *d.* and is the Answer to the *Question.*

Question 3.

A has a Term of 7 Years in an *Estate* of 35 *l. per Annum.* *B* has a Term of 14 Years in the same Estate in *Reversion* after the 7 Years; and *C* has a farther Term of 20 Years in *Reversion* after the 21 Years. Quere the *present Values* of the *several Terms*, at the *Rate* of 5 *per Cent. per Annum?*

By *Table* V. the *present Value* of 35 *l. per Annum*, may be

		l.	*s.*	*d.*
found, for 41 Years, to be	— —	605	6	0½
for 21 Years, to be	—	448	14	9½
for 7 Years, to be	— —	202	10	5½

Which

Which substract from each other, it will appear,

		l.	*s.*	*d.*
That the *present Value* of *A*'s Term is		202 —	10 —	$5\frac{1}{2}$
of *B*'s Term		246 —	4 —	4
of *C*'s Term		156 —	11 —	3
For these *Values* answer the Question	*l.*	605 —	6 —	$0\frac{1}{2}$

Question 4.

Which is most advantagious a *Term* of 15 Years in an Estate of 100 *l. per Annum*, or the *Reversion* of such an *Estate* for ever after the *Expiration* of the said 15 Years; computing at the *Rate* of 5 *per Cent. per Annum Compound Interest*?

An Estate of 100 *l. per Annum*, in *Fee Simple* at 5 *per Cent.* is *Worth* —	*l.* 2000
In *Table* V. the *present Value* of the same *Estate*, at the same *Rate*, for 15 Years, is	*l.* 1037,9658
The Difference is	*l.* 962,0342

Now this *Difference* being the *Value of the Reversion*, it appears that the first Term of 15 Years is better than the *Reversion* for ever afterwards by 75,9316 *l.* = 75 *l.* 18 *s.* $7\frac{1}{2}$ *d.* Answer.

Question 5.

A Person having 12 Years to come, in a *Lease* of an Estate of 60 *l. per Annum* for 40 Years, would know what *present Money* he must pay in order to *renew* or *compleat* the *Lease* by adding 28 Years thereto, computing at 6 *per Cent. Compound Interest*?

By *Table* V. the *present Value* of 1 *l. per Annum*, at 6 *per Cent.* for 40 Years, is	*l.* 15,046297
By the same Table the *Value* of 1 *l. per An.* at that *Rate*, for 12 Years to come, is	*l.* 8,383844
The Difference is	*l.* 6,662453
Which multiplied by	60
The Product is the Answer, *viz.* —	*l.* 399,747180

In *Money*, 399 *l.* 14 *s.* 11 *d.*

Question

Question 6.

A gives 1550 *l.* for an *Annuity* of 100 *l.* *per Annum* for 50 Years. *B* puts 1550 *l.* out at *Interest*. It is required to know which will *amount* to the *greatest Sum* at the End of the 50 Years, at the *Rate* of 6 *l.* *per Cent.* &c. *Compound Interest* ?

By *Table* IV. the *Amount* of 100 *l.* *Annuity*, in 50 Years, at 6 *per Cent.* may be found to be — — } *l.* 29033,59046

By *Table* II. it may be found, that the *Amount* of 1550 *l.* for that *Time* and *Rate* will be — — } *l.* 28551,23885

Hence *A's Annuity* is more than *B*'s 1550 *l.* by *l.* 482,35161

at the End of 50 Years. The *present Value* of which Difference is found, by *Table* III. to be 26 *l.* 3 *s.* 8 ½ *d.* and so much was *A*'s Case *better* than *B*'s.

Question 7.

What *Annuity* to continue 14 Years, may be purchased with 1000 *l.* due at the end of 5 Years; the *Annuity* to commence presently, at 5 *l.* *per Cent* ?

By *Table* III. the *present Worth* of 1000 *l.* due 5 Years hence at 5 *per Cent.* may be found — — } = *l.* 783,5262

By *Table* VI. it may be found, that the *Annuity* which 783,5262 *l.* will purchase for 14 Years, at the *Rate* of 5 *per Cent.* is } = *l.* 79,1518

In *Money*, 79 *l.* 3 *s.* 0 ¼ *d.* *per Annum*, the Answer.

Question 8.

For a *Lease* of certain *Profits* for 7 Years, *A*, makes *two* Offers, either to pay 150 *l.* as a *Fine*, and 300 *l.* *per Annum*; or 1700 *l.* *Fine*, without any *Rent*. *B*, bids 650 *l.* *Fine*, and 200 *l.* *per Annum*. And *C*, offers 200 *l.* *Fine*, and 405 *l.* *per Annum*. Quere which is the *best* Offer, and what the *Difference*, computing at 5 *l.* *per Cent.* &c. *Compound Interest* ?

1. By

1. By *Table* II. the *Amount* of 150*l.* in 7 Years, at 5 *per Cent.* may be found to be	*l.* 211,0659
By *Table* IV. the *Amount* of 300 *l. per Annum* in 7 Years at the given Rate may be found — — —	*l.* 242,6025
Therefore *A*'s Offer, at the End of 7 Years would be — — —	*l.* 2453,6684
2. By *Table* II. the *Amount* of 1700*l.* in 7 Years (*A*'s second Offer) at the said *Rate*, is found to be — — —	*l.* 2392,0802
3. By *Table* II. the *Amount* of 650*l.* in 7 Years, at the given *Rate*, will be found to be — — —	*l.* 914,6189
By *Table* IV. the *Amount* of 200*l. per Annum* in 7 Years, at that *Rate*, will be found to be — —	*l.* 1628,4016
Therefore B's Offer will, in 7 Years, amount to — — —	*l.* 2543,0205
4. By *Table* II. the *Amount* of 200 *l.* in 7 Years, at the given *Rate*, will be found to be — — —	*l.* 281,4212
By *Table* IV. the *Amount* of 405 *l. per Annum* for the given *Time* and *Rate*, will be found to be — —	*l.* 3297,5132
So that *C*'s Offer, in 7 Years, will amount to	*l.* 3578,9344

The *Amounts* therefore of the said Offers, at the End of the said Term, being thus known; the *Present Worth* of the several *Amounts* may be found by *Table* III. which are as follow.

	l.	*s.*	*d.*
The *present Worth* of *A*'s first Offer will be	1885	18	03
A's second Offer —	1700	00	00
B's Offer —	1807	05	06
C's Offer —	2543	09	08

Therefore the *present Worth* of what *C* offers is more

	l.	*s.*	*d.*
than — — *A*'s first Offer, by	657	11	5
A's second Offer, by	843	9	8
B's Offer, by —	736	4	2

Which fully answers the Question.

N. B. This

N. B. This Question might be more readily answered by finding the *present Worths* of the several offer'd *Annuities* (as *per Table* V.) and adding to them the several *Fines*; as the Reader may try at his *Leisure*.

Question 9.

What *Annuity* is sufficient to pay off a *Debt* of 50 *Millions* in 30 Years at 4 *l. per Cent. Compound Interest?*

In *Table* IV. against 30 Years, under 4 *per* C. is	0578301
Which multiply by the Debt —	50000000
The Product is the *Annuity sought*, viz. *per Annum.*	*l.* 2891505

So that supposing the *National Debt* to be 50 *Millions*, and the Interest paid to be 2 Millions *per Annum*, or 4 *l. per Cent.* then will a *Sinking Fund* of 891505 *l*, *per Annum*, clear the whole *Debt* in 30 Years.

N. B. By this Example appears the Necessity of continuing the *Tabular Numbers* to *so many* Places of *Decimals*.

Question 10.

Suppose *one Farthing* had been lent at *Compound Interest* at 5 *per Cent.* in the *first Year* of the *Christian Æra*, or Birth of *Christ*, and so continued to this *present Year* thereof 1734; Quere the *Amount* thereof?

N. B. Though this Question might be answered by *Table* II. as I have before shewn, yet I shall here use *Logarithms*, as most expeditious in this Case. For having said enough about the Use of the Tables, I here intend only to give the Reader a hint of the *surprising Nature* of Numbers in *Geometrical Proportion*.

	l.
Therefore, The *Logarithm* of the *Rate*	1,05=0,0211893
Multiplied by the *Time* — —	1734
The Product is — — —	36,7422462
To which add the *Logarithm* of 1 *Farthing*, or the ,0010416 Part of a Pound, viz. — — —	= 7,0177288
The Sum is the *Log.* of the *Amount sought* =	33,7599750

Now

Now the *Index* of this *Logarithm* being 33, shews the *Number* of Figures, of which the *Amount* of *one Farthing* in the given *Time* doth consist, to be 34; of which let it be sufficient to express the 4 first in Figures; the Rest in *Cyphers*; then will the said *Amount* be

5754000000000000000000000000000000 *l.*

Now the *Value* of a *solid Body, perfectly Spherical*, whose *Diameter* is 8000 *English Miles*, (which is somewhat *bigger* than the *Diameter* of the *Globe* of our *Earth.*) I say such a solid Body of *fine Gold* would be in *Value* about

2386600000000000000000000000 *l.*

Now if from each of these *great Numbers*, be cut off 23 Cyphers, the *remaining Figures* will be 57540000000 in the *Amount* of the *Farthing*; and 23866 in the *Value* of the *Globe* of *Gold*. But 23866) 57540000000 (= 2400000 nearly.

Hence it appears, That one *single Farthing* put out to *Usury* in the Manner aforesaid would *amount* to more in Value than *two Millions* and *four hundred Thousand Globes of fine solid Gold*, each *bigger* than the *Globe of the Earth!* A *strange* and *surprising*, but no less *certain Truth!* And this *immense Amount* would be greatly increased by inlarging the *Rate of Interest*.

I shall now conclude this Part, by presenting the *Reader* with a *small Table* concerning the *present Worth* or *Value* of *Estates* upon *Lives*, with its Use; This *Table* was at first composed by the Great and Learned Dr. *Halley*, for every *Fifth Year* of *Age* to the 70th, as follows.

Age	*Year's Purchase*	*Age*	*Year's Purchase*	*Age*	*Year's Purchase*
1	10,28	25	12,27	50	9,21
5	13,40	30	11,72	55	8,51
10	13,44	35	11,12	60	7,60
15	13,33	40	10,57	65	6,54
20	12,78	45	9,91	70	5,32

The Use of the TABLE.

Suppose a Person of 50 Years of *Age* offers to sell his Life in an Estate of 46*l. per Annum*, what is the *Value* thereof in *Ready Money*.

The Age of 50, is Years Purchase —	9,21
Which multiply by the *Annuity* — —	46
The Product is the Answer 423*l.* 13*s.* $2\frac{1}{4}$*d.* =	423,66

If it happen that a Life and a *Reversion* for so many Years after, be offered; 'twill be necessary to reduce the Year's Purchase into Years certain, or Years of a *Lease*, by the *Fifth Table*; thus, Suppose I would find what Number of certain Years corresponds to 10,57 Years Purchase, and 6 *per Cent.* I look in *Table* V. under the *given Rate*, and I find the next *nearest Value* of *Annuity* of 1 *l. per Annum* to 10,57, to be 10,4772597, opposite to which is 17 Years, which are to be added to the Years in *Reversion*, and then the Case is thus easily solved. *Example.*

What is the *present Worth* of an *Estate* of 78*l. per Annum* clear Rent for 20 Years after the *Death* of a Person 40 Years of *Age*, at 6 *per Cent*?

The Age of 40 Years is 10,57 Years purchase, which in *Table* V. gives 17 Years certain to come at 6 *per Cent.*

Then 20 + 17 = 37 Years.

	Years.	*l.*
Therefore the *present Worth* of 1 *l.* per *Annum* for the given *Rate* —	for 37 =	14,7367804
	for 17 =	10,4772597
The *present Worth* of 1 *l. per Annum*	for 20 =	4,2595207
Which multiply by the *Annuity*	—	78
The Prod. is the Answer 232*l.* 4 *s.* 11 *d.* =	*l.*	232,2426146

TABLES

TABLES *of* COMPOUND INTEREST.

TABLE I.

The Amount *of one Pound for* Days; *at the Rates of* 2; 2 ½; 3; 3 ½; 4; 4 ½; 5, *and* 6 per Cent. per Annum.

Days.	2 *per C.*	2 ½ *per C.*	3 *per C.*	3 ½ *per C.*
1	1,0000542	1,0000676	1,0000809	1,0000942
2	1,0001085	1,0001353	1,0001619	1,0001885
3	1,0001627	1,0002029	1,0002429	1,0002827
4	1,0002170	1,0002706	1,0003240	1,0003770
5	1,0002713	1,0003383	1,0004050	1,0004713
6	1,0003255	1,0004059	1,0004860	1,0005656
7	1,0003798	1,0004736	1,0005670	1,0006600
8	1,0004341	1,0005412	1,0006480	1,0007542
9	1,0004884	1,0006090	1,0007291	1,0008486
10	1,0005426	1,0006767	1,0008101	1,0009429
20	1,0010856	1,0013539	1,0016209	1,0018867
30	1,0016289	1,0020315	1,0024324	1,0028315
40	1,0021725	1,0027097	1,0032445	1,0037771
50	1,0027163	1,0033882	1,0040573	1,0047236
60	1,0032605	1,0040673	1,0048708	1,0056710
70	1,0038049	1,0047468	1,0056849	1,0066193
80	1,0043497	1,0054267	1,0064996	1,0075685
90	1,0048947	1,0061071	1,0073151	1,0085186
100	1,0054401	1,0067880	1,0081311	1,0094696
110	1,0059857	1,0074693	1,0089479	1,0104214
120	1,0065316	1,0081511	1,0097653	1,0113742
130	1,0070779	1,0088334	1,0105834	1,0123279
140	1,0076244	1,0095161	1,0114021	1,0132825
150	1,0081712	1,0101993	1,0122215	1,0142379
160	1,0087183	1,0108829	1,0130415	1,0151943

TABLE I.

The Amount of one Pound, Compound Interest.

Days.	4 *per C.*	4 ½ *per C.*	5 *per C.*	6 *per C.*
1	1,0001074	1,0001206	1,0001336	1,0001596
2	1,0002149	1,0002412	1,0002673	1,0003193
3	1,0003224	1,0003618	1,0004011	1,0004790
4	1,0004299	1,0004824	1,0005348	1,0006387
5	1,0005374	1,0006031	1,0006685	1,0007985
6	1,0006449	1,0007238	1,0008023	1,0009583
7	1,0007524	1,0008445	1,0009361	1,0011181
8	1,0008600	1,0009652	1,0010699	1,0012779
9	1,0009675	1,0010859	1,0012037	1,0014378
10	1,0010751	1,0012066	1,0013376	1,0015976
20	1,0021513	1,0024148	1,0026770	1,0031979
30	1,0032288	1,0036243	1,0040182	1,0048007
40	1,0043074	1,0048354	1,0053611	1,0064060
50	1,0053871	1,0060479	1,0067059	1,0080139
60	1,0064680	1,0072618	1,0080525	1,0096244
70	1,0075501	1,0084773	1,0094009	1,0112375
80	1,0086333	1,0096942	1,0107511	1,0128531
90	1,0097177	1,0109125	1,0121031	1,0144713
100	1,0108033	1,0121324	1,0134569	1,0160921
110	1,0118900	1,0133537	1,0148125	1,0177155
120	1,0129779	1,0145765	1,0161699	1,0193415
130	1,0140670	1,0158007	1,0175291	1,0209701
140	1,0151572	1,0170265	1,0188902	1,0226013
150	1,0162487	1,0182537	1,0202531	1,0242351
160	1,0173412	1,0194824	1,0216178	1,0258715

TABLE I.

The Amount of one Pound, Compound Interest.

Days.	2 *per C.*	2 ½ *per C.*	3 *per C.*	3 ½ *per C.*
170	1,0092658	1,0115670	1,0138623	1,0161516
180	1,0098135	1,0122516	1,0146837	1,0171098
190	1,0103615	1,0129366	1,0155057	1,0180689
200	1,0109098	1,0136221	1,0163284	1,0190288
210	1,0114584	1,0143081	1,0171518	1,0199897
220	1,0120073	1,0149945	1,0179759	1,0209515
230	1,0125565	1,0156814	1,0188006	1,0219142
240	1,0131060	1,0163687	1,0196260	1,0228778
250	1,0136558	1,0170565	1,0204520	1,0238424
260	1,0142059	1,0177448	1,0212788	1,0248078
270	1,0147563	1,0184336	1,0221062	1,0257741
280	1,0153070	1,0191228	1,0229342	1,0267414
290	1,0158580	1,0198125	1,0237630	1,0277096
300	1,0164093	1,0205026	1,0245924	1,0286786
310	1,0169609	1,0211932	1,0254225	1,0296486
320	1,0175127	1,0218843	1,0262532	1,0306195
330	1,0180649	1,0225758	1,0270847	1,0315914
340	1,0186174	1,0232679	1,0279168	1,0325641
350	1,0191702	1,0239603	1,0287495	1,0335378
360	1,0197233	1,0246533	1,0295830	1,0345123
361	1,0197786	1,0247226	1,0296664	1,0346098
362	1,0198340	1,0247919	1,0297497	1,0347073
363	1,0198893	1,0248613	1,0298331	1,0348049
364	1,0199446	1,0249306	1,0299165	1,0349024
365	1,0200000	1,0250000	1,0300000	1,0350000

TABLE I.

The Amount of one Pound, Compound Interest.

Days.	4 per C.	4 ½ per C.	5 per C.	6 per C.
170	1,0184350	1,0207126	1,0229843	1,0275105
180	1,0195299	1,0219442	1,0243527	1,0291522
190	1,0206261	1,0231774	1,0257228	1,0307964
200	1,0217233	1,0244120	1,0270949	1,0324433
210	1,0228218	1,0256481	1,0284687	1,0340928
220	1,0239215	1,0268858	1,0298444	1,0357450
230	1,0250233	1,0281249	1,0312219	1,0373998
240	1,0261243	1,0293655	1,0326013	1,0390572
250	1,0272275	1,0306076	1,0339825	1,0407173
260	1,0283319	1,0318512	1,0353656	1,0423800
270	1,0294375	1,0330963	1,0367505	1,0440454
280	1,0305443	1,0343429	1,0381373	1,0457135
290	1,0316522	1,0355910	1,0395259	1,0473842
300	1,0327614	1,0368406	1,0409164	1,0490576
310	1,0338717	1,0383917	1,0423087	1,0507336
320	1,0349832	1,0393444	1,0437029	1,0524124
330	1,0360960	1,0405985	1,0450990	1,0540938
340	1,0372099	1,0418542	1,0464969	1,0557779
350	1,0383250	1,0431114	1,0478967	1,0574647
360	1,0394413	1,0443700	1,0492984	1,0591542
361	1,0395530	1,0444960	1,0494387	1,0593233
362	1,0396648	1,0446220	1,0495790	1,0594924
363	1,0397765	1,0447479	1,0497193	1,0596616
364	1,0398882	1,0448739	1,0498596	1,0598308
365	1,0400000	1,0450000	1,0500000	1,0600000

TABLE

TABLE II.

Of COMPOUND INTEREST.

The Amount *of one Pound for* Years, *at the Rates of* 2 ; 2 ½ ; 3 ; 3 ½ ; 4 ; 4 ½ ; 5, *and* 6 ; per Cent. per Annum.

Years.	2 *per C.*	2 ½ *per C.*	3 *per C.*	3 ½ *per C.*
1	1,0200000	1,0250000	1,0300000	1,0350000
2	1,0404000	1,0506250	1,0609000	1,0712250
3	1,0612080	1,0768906	1,0927270	1,1087178
4	1,0824321	1,1038128	1,1255088	1,1475230
5	1,1040808	1,1314082	1,1592740	1,1876863
6	1,1261624	1,1596934	1,1948523	1,2292553
7	1,1486856	1,1886857	1,2298733	1,2722792
8	1,1716593	1,2184029	1,2667700	1,3168098
9	1,1950925	1,2488629	1,3047731	1,3628973
10	1,2189944	1,2800845	1,3439163	1,4105987
11	1,2433743	1,3120866	1,3842338	1,4599697
12	1,2682417	1,3448888	1,4257608	1,5110686
13	1,2936066	1,3785110	1,4685337	1,5639560
14	1,3194787	1,4129738	1,5125897	1,6186945
15	1,3458683	1,4482981	1,5579674	1,6753488
16	1,3727857	1,4845056	1,6047064	1,7339860
17	1,4002414	1,5216182	1,6528476	1,7946755
18	1,4282462	1,5596587	1,7024330	1,8574892
19	1,4568111	1,5986501	1,7535060	1,9225013
20	1,4859474	1,6386164	1,8061112	1,9897888
21	1,5156663	1,6795818	1,8602945	2,0594314
22	1,5459796	1,7215714	1,9161034	2,1315115
23	1,5768992	1,7646106	1,9735865	2,2061144
24	1,6084372	1,8087259	2,0327941	2,2833284
25	1,6406059	1,8539441	2,0937779	2,3632449

TABLE

TABLE II.

The Amount of one Pound, Compound Interest.

Years.	4 per C.	4 ½ per C.	5 per C.	6 per C.
1	1,0400000	1,0450000	1,0500000	1,0600000
2	1,0816000	1,0920250	1,1025000	1,1236000
3	1,1248640	1,1411661	1,1576250	1,1910160
4	1,1698586	1,1925186	1,2155063	1,2624769
5	1,2166529	1,2461819	1,2762816	1,3382256
6	1,2653190	1,3022601	1,3400956	1,4185191
7	1,3159318	1,3608618	1,4071064	1,5036303
8	1,3685691	1,4221006	1,4774554	1,5938481
9	1,4233118	1,4860951	1,5513282	1,6894790
10	1,4802443	1,5529694	1,6288946	1,7908477
11	1,5394541	1,6228530	1,7103393	1,8982980
12	1,6010322	1,6958814	1,7958563	2,0121965
13	1,6650735	1,7721961	1,8156491	2,1329283
14	1,7316764	1,8519449	1,9799316	2,2609039
15	1.8009435	1,9352824	2,0789282	2,3965582
16	1,8729812	2,0223701	2,1828746	2,5402517
17	1,9479005	2,1133768	2,2920183	2,6927728
18	2,0258165	2,2084787	2,4066192	2,8543392
19	2,1068492	2,3078603	2,5269502	3,0255995
20	2,1911231	2,4117140	2,6532977	3,2071355
21	2,2787681	2,5202111	2,7859626	3,3995636
22	2,3699188	2,6336520	2,9252607	3,6035374
23	2,4647155	2,7521663	3,0715238	3,8197497
24	2,5633042	2,8760138	3,2251000	4,0489346
25	2,6658363	3,0054344	3,3863549	4,2918707

TABLE

TABLE II.

The Amount of one Pound, Compound Interest.

Years.	2 *per Cent.*	2 ½ *per C.*	3 *per Cent.*	3 ½ *per C.*
26	1,6734181	1,9002927	2,1565912	2,4459585
27	1,7068864	1,9478000	2,2212890	2,5315871
28	1,7410242	1,9964950	2,2879276	2,6201719
29	1,7758446	2,0464073	2,3565655	2,7118779
30	1,8113615	2,0975675	2,4272624	2,8067937
31	1,8475888	2,1500067	2,5000803	2,9050314
32	1,8845405	2,2037569	2,5750827	3,0067075
33	1,9222314	2,2588508	2,6523352	3,1119423
34	1,9606760	2,3153221	2,7319053	3,2208603
35	1,9998895	2,3732051	2,8138624	3,3335904
36	2,0398873	2,4325353	2,8982783	3,4502661
37	2,0806850	2,4933487	2,9852266	3,5710254
38	2,1222987	2,5556824	3,0747834	3,6960113
39	2,1647447	2,6195744	3,1670269	3,8253717
40	2,2080396	2,6850638	3,2620377	3,9592597
41	2,2522004	2,7521904	3,3598989	4,0978338
42	2,2979444	2,8209952	3,4606958	4,2412579
43	2,3431893	2,8915500	3,5645167	4,3897020
44	2,3900531	2,9638080	3,6714522	4,5433416
45	2,4378542	3,0379032	3,7815958	4,7023585
46	2,4866112	3,1138508	3,8950437	4,8669411
47	2,5363435	3,1916971	4,0118950	5,0372840
48	2,5870703	3,2714895	4,1322518	5,2135889
49	2,6388117	3,3532768	4,2562194	5,3960645
50	2,6915880	3,4371087	4,3839060	5,5849268

TABLE II.

The Amount of one Pound, Compound Interest.

Years.	4 per Cent.	4 ½ per C.	5 per C.	6 per Cent.
26	2,7724697	3,1406790	3,5556727	4,5493829
27	2,8833685	3,2820095	3,7334563	4,8223459
28	2,9987033	3,4296999	3,9201291	5,1116866
29	3,1186514	3,5840364	4,1161356	5,4183878
30	3,2433975	3,7453181	4,3219424	5,7434911
31	3,3731334	3,9138574	4,5380395	6,0881006
32	3,5080587	4,0899810	4,7649415	6,4533866
33	3,6483811	4,2740301	5,0031885	6,8405898
34	3,7943163	4,4663615	5,2533480	7,2510252
35	3,9460889	4,6673478	5,5160154	7,6860867
36	4,1039325	4,8773784	5,7918161	8,1472519
37	4,2680898	5,0968604	6,0814069	8,6360870
38	4,4388134	5,3262192	6,3854773	9,1542523
39	4,6163659	5,5658990	6,7047511	9,7035074
40	4,8010206	5,8163645	7,0399887	10,2857178
41	4,9930614	6,0781009	7,3919881	10,9028609
42	5,1927839	6,3516154	7,7615875	11,5570326
43	5,4004952	6,6374381	8,1496669	12,2504545
44	5,6165150	6,9361229	8,5571503	12,9854818
45	5,8411756	7,2482484	8,9850078	13,7646107
46	6,0748227	7,5744196	9,4342582	14,5904873
47	6,3178156	7,9152684	9,9059711	15,4659166
48	6,5705282	8,2714555	10,4012696	16,3938716
49	6,8333493	8,6436710	10,9213331	17,3775039
50	7,1066833	9,0326362	11,4674000	18,4201541

TABLE

TABLE III.

Of COMPOUND INTEREST.

The Present Worth *of one Pound for* Years, *at the Rates of* 2; 2 ½; 3; 3 ½; 4; 4 ½; 5, *and* 6; per Cent. per Annum.

Years.	2 *per C.*	2 ½ *per C.*	3 *per C.*	3 ½ *per C.*
1	,9803921	,9756097	,9708738	,9661836
2	,9611687	,9518144	,9425959	,9335107
3	,9423223	,9285994	,9151417	,9019427
4	,9238454	,9059506	,8884870	,8714422
5	,9057308	,8838542	,8626088	,8419732
6	,8879713	,8622968	,8374843	,8135006
7	,8705601	,8412654	,8130915	,7859910
8	,8534903	,8207465	,7894092	,7594116
9	,8367552	,8007283	,7664167	,7337710
10	,8203483	,7811984	,7440939	,7089188
11	,8042630	,7621447	,7224213	,6849457
12	,7884931	,7435558	,7013799	,6617833
13	,7730325	,7254203	,6809513	,6394041
14	,7578750	,7077272	,6611178	,6177818
15	,7430147	,6904655	,6418619	,5968906
16	,7284458	,6736249	,6231669	,5767059
17	,7141625	,6571950	,6050164	,5572038
18	,7001593	,6411659	,5873946	,5383611
19	,6864307	,6255277	,5702860	,5201557
20	,6729713	,6102709	,5536758	,5025659
21	,6597758	,5953862	,5375493	,4855709
22	,6468390	,5808646	,5218925	,4691506
23	,6341559	,5666972	,5066917	,4532856
24	,6217214	,5528753	,4919337	,4379571
25	,6095308	,5393905	,4776056	,4231470

TABLE III.

The Present Worth of one Pound, Comp. Interest.

Years.	4 per C.	4 ½ per C.	5 per C.	6 per C.
1	,9615385	,9569378	,9523810	,9433962
2	,9245562	,9157299	,9070295	,8899964
3	,8889964	,8762966	,8638376	,8396193
4	,8548042	,8385613	,8227025	,7920937
5	,8219271	,8024511	,7835262	,7472582
6	,7903145	,7618957	,7462154	,7049605
7	,7599178	,7348285	,7106813	,6650571
8	,7306902	,7031851	,6768394	,6274124
9	,7025867	,6729044	,6446089	,5918985
10	,6755642	,6439277	,6139133	,5583948
11	,6495809	,6161987	,5846793	,5267875
12	,6245971	,5896639	,5568374	,4969694
13	,6005741	,5642716	,5303214	,4688390
14	,5774751	,5399729	,5050679	,4423010
15	,5552645	,5167204	,4810171	,4172651
16	,5339082	,4944693	,4581115	,3936463
17	,5133733	,4731764	,4362967	,3713644
18	,4936281	,4528004	,4155207	,3503438
19	,4746424	,4333018	,3957340	,3305130
20	,4563870	,4146429	,3768895	,3118047
21	,4388336	,3967874	,3589424	,2941554
22	,4219554	,3797009	,3418499	,2775051
23	,4057263	,3633501	,3255713	,2617973
24	,3901215	,3477035	,3100679	,2469786
25	,3751168	,3327306	,2953028	,2329986

TABLE III.

The Present Worth of one Pound, Comp. Interest.

Years.	2 per C.	2 ½ per C.	3 per C.	3 ½ per C.
26	,5975793	,5262347	,4636947	,4088378
27	,5858620	,5133997	,4501891	,3950123
28	,5743746	,5008778	,4370768	,3816543
29	,5631123	,4886613	,4243464	,3687482
30	,5520709	,4767427	,4119868	,3562784
31	,5412460	,4651148	,3999871	,3442304
32	,5306333	,4537706	,3883370	,3325897
33	,5202287	,4427030	,3770263	,3213427
34	,5100282	,4319053	,3660449	,3104761
35	,5000276	,4213711	,3553834	,2999769
36	,4902232	,4110937	,3450324	,2898327
37	,4806109	,4010671	,3349829	,2800316
38	,4711872	,3912849	,3252262	,2705619
39	,4619482	,3817414	,3157536	,2614125
40	,4528904	,3724306	,3065568	,2525725
41	,4440102	,3633470	,2976280	,2440314
42	,4353041	,3544848	,2889592	,2357791
43	,4267688	,3458389	,2805429	,2278059
44	,4184008	,3374038	,2723718	,2201023
45	,4101968	,3291744	,2644386	,2126594
46	,4021537	,3211458	,2567365	,2054679
47	,3942684	,3133129	,2492588	,1985197
48	,3865376	,3056712	,2419988	,1918065
49	,3789584	,2982158	,2349503	,1853202
50	,3715279	,2909422	,2281071	,1790534

TABLE III.

The Present Worth of one Pound, Comp. Interest.

Years.	4 *per C.*	4 ½ *per C.*	5 *per C.*	6 *per C.*
26	,3606892	,3184025	,2812407	,2198100
27	,3468166	,3046914	,2678483	,2073680
28	,3334775	,2915707	,2550936	,1956301
29	,3206514	,2790150	,2429463	,1845567
30	,3083187	,2670000	,2313775	,1741101
31	,2964603	,2555024	,2203595	,1642548
32	,2850579	,2444999	,2098662	,1549574
33	,2740942	,2339712	,1998762	,1461862
34	,2635521	,2238959	,1903548	,1379115
35	,2534155	,2142544	,1812903	,1301052
36	,2436687	,2050282	,1726574	,1227408
37	,2342969	,1961992	,1644356	,1157932
38	,2252854	,1877504	,1566054	,1092389
39	,2166206	,1796655	,1491479	,1030555
40	,2082890	,1719287	,1420457	,0972222
41	,2002779	,1645251	,1352816	,0917191
42	,1925749	,1574403	,1288396	,0865274
43	,1851682	,1506605	,1227044	,0816296
44	,1180464	,1441728	,1168613	,0770091
45	,1711984	,1379644	,1112965	,0726501
46	,1646139	,1320233	,1059967	,0685378
47	,1582826	,1263381	,1009492	,0646583
48	,1521948	,1208977	,0961421	,0609984
49	,1463411	,1156916	,0915639	,0575457
50	,1407126	,1107097	,0872037	,0542884

TABLE

TABLE IV.

Of COMPOUND INTEREST.

The Amount *of one Pound* per Annum, *or* Annuity, *for* Years; *at the Rates of* 2; 2 $\frac{1}{2}$; 3; 3 $\frac{1}{2}$; 4; 4 $\frac{1}{2}$; 5, *and* 6 per Cent. per Annum.

Years.	2 *per Cent.*	2 $\frac{1}{2}$ *per C.*	3 *per Cent.*	3 $\frac{1}{2}$ *per C.*
1	1,0000000	1,0000000	1,0000000	1,0000000
2	2,0200000	2,0250000	2,0300000	2,0350000
3	3,0604000	3,0756230	3,0909000	3,1062250
4	4,1216080	4,1525156	4,1836270	4,2149429
5	5,2040402	5,2563285	5,3091358	5,3624659
6	6,3081210	6,3877367	6,4684099	6,5501522
7	7,4342834	7,5474302	7,6624622	7,7794075
8	8,5829691	8,7361159	8,8923360	9,0516866
9	9,7546284	9,9545188	10,1591061	10,3684958
10	10,9497210	11,2033818	11,4638793	11,7313931
11	12,1687154	12,4834663	12,8077957	13,1419919
12	13,4120897	13,7955580	14,1920296	14,6019616
13	14,6803315	15,1404418	15,6177904	16,1130303
14	15,9739381	16,5189528	17,0863242	17,6769864
15	17,2934169	17,9319267	18,5989139	19,2956809
16	18,6392853	19,3802248	20,1568813	20,9710297
17	20,0120719	20,8647304	21,7615877	22,7050158
18	21,4123124	22,3863487	23,4144354	24,4996913
19	22,8405586	23,9460074	25,1168684	26,3571805
20	24,2973698	25,5446576	26,8703745	28,2796818
21	25,7833172	27,1832740	28,6764857	30,2694707
22	27,2989835	28,8628559	30,5367803	32,3289022
23	28,8449632	30,5844273	32,4528837	34,4604137
24	30,4218625	32,3490379	34,4264702	36,6665282
25	32,0302997	34,1577639	36,4592643	38,9498567

TABLE IV.

The Amount of 1 l. Annuity, Compound Interest.

Years.	2 per C.	2 ½ per C.	3 per C.	3½ per C.
1	1,0000000	1,0000000	1,0000000	1,0000000
2	2,0400000	2,0450000	2,0500000	2,0600000
3	3,1216000	3,1370250	3,1525000	3,1836000
4	4,2464640	4,2781911	4,3101250	4,3746016
5	5,4163226	5,4707097	5,5256312	5,6370930
6	6,6329755	6,7168917	6,8019128	6,9753187
7	7,8982945	8,0191518	8,1420084	8,3938378
8	9,2142263	9,3800136	9,5491089	9,8974681
9	10,5827953	10,8021142	11,0265643	11,4913162
10	12,0061071	12,2882094	12,5778925	13,1807958
11	13,4863514	13,8411788	14,2067871	14,9716435
12	15,0258055	15,4640318	15,9171265	16,8699420
13	16,6268377	17,1599133	17,7129828	18,8821385
14	18,2919112	18,9321094	19,5986320	21,0150667
15	20,0235876	20,7840543	21,5785636	23,2759707
16	21,8245311	22,7193367	23,6574918	25,6725289
17	23,6975124	24,7417069	25,8403664	28,2128806
18	25,6454129	26,8550837	28,1323847	30,9056534
19	27,6712294	29,0635625	30,5390039	33,7599925
20	29,7780786	31,3714228	33,0659541	36,7855920
21	31,9692017	33,7831368	35,7192518	39,9927275
22	34,2479698	36,3033779	38,5052144	43,3922911
23	36,6178886	38,9370299	41,4304751	46,9958285
24	39,0826041	41,6891963	44,5019989	50,8155782
25	41,6459083	44,5652101	47,7270988	54,8645128

TABLE IV.

The Amount of 1 l. Annuity, Compound Interest.

Years.	2 per Cent.	2 ½ per C.	3 per Cent.	3 ½ per C.
26	33,6709057	36,0117080	38,5530422	41,3131017
27	35,3443238	37,9120007	40,7096335	43,7590602
28	37,0512103	39,8598008	42,9309225	46,2906273
29	38,7922345	41,8562958	45,2188502	48,9107993
30	40,5680792	43,9027032	47,5754157	51,6226773
31	42,3794408	46,0002707	50,0026782	54,4294719
32	44,2270296	48,1502775	52,5027585	57,3345025
33	46,1115702	50,3540345	55,0778413	60,3412101
34	48,0338016	52,6128653	57,7301765	63,4531524
35	49,9944776	54,9282074	60,4620818	66,6740127
36	51,9943672	57,3014126	63,2759443	70,0076032
37	54,0342545	59,7339479	66,1742226	73,4578693
38	56,1149396	62,2272966	69,1594493	77,0288947
39	58,2372384	64,7829791	72,2342327	80,7249060
40	60,4019832	67,4025535	75,4012597	84,5502778
41	62,6100228	70,0876174	78,6632975	88,5095375
42	64,6822233	72,8398078	82,0231964	92,6073713
43	67,1594678	75,6608030	85,4838923	96,8486293
44	69,5026511	78,5523231	89,0484191	101,2383313
45	71,8927103	81,5161312	92,7198614	105,7816729
46	74,3305645	84,5540344	96,5014172	110,4840315
47	76,8171758	87,6678853	100,3965009	115,3509726
48	79,3535193	90,8595824	104,4083960	120,3882566
49	81,9405697	94,1310729	108,5406479	125,6018456
50	84,5794015	97,4843488	112,7968673	130,9979102

TABLE IV.

The Amount of 1 *l. Annuity, Compound Interest.*

Years.	4 *per Cent.*	4 ½ *per Cent.*	5 *per Cent.*	6 *per Cent.*
26	44,3117446	47,5706446	51,1134538	59,1563827
27	47,0842144	50,7113236	54,6691265	63,7057657
28	49,9675830	53,9933332	58,4025828	68,5281116
29	52,9662863	57,4230332	62,3227119	73,6397983
30	56,0849377	61,0070698	66,4388475	69,0581862
31	59,3283352	64,7523878	70,7607899	84,8016774
32	62,7014687	68,6662452	75,2988294	90,8897780
33	66,2095274	72,7562263	80,0637708	97,3431647
34	69,8579045	77,0302565	85,0669594	104,1837546
35	73,6522248	81,4966180	90,3203073	111,4347799
36	77,5983138	86,1639658	95,8363227	119,1208667
37	81,7022464	91,0413443	101,6281388	127,2681187
38	85,9703362	96,1382048	107,7095458	135,9042058
39	90,4091497	101,4644249	114,0950231	145,0584581
40	95,0255157	107,0303231	120,7997742	154,7619656
41	99,8265363	112,8466876	127,8397829	165,0476836
42	104,8195978	118,9247885	135,2317511	175,9505446
43	110,0123817	125,2764040	142,9933386	187,5075772
44	115,4128169	131,9138422	151,1430056	199,7580319
45	121,0293920	138,8499651	159,7001559	212,7435138
46	126,8705677	146,0982135	168,6851637	226,5081246
47	132,9453904	153,6726331	178,1194218	241,0986121
48	139,2632060	161,5879016	188,0253929	256,5645288
49	145,8337342	169,8593572	198,4266626	272,9584006
50	152,6670836	178,5030282	209,3479957	290,3359046

TABLE

TABLE V.

Of COMPOUND INTEREST.

The Present Worth *of one Pound* per Annum, *or Annuity for* Years, *at the Rates of* 2; 2 ½; 3; 3 ½; 4; 4 ½; 5, *and* 6; per Cent. per Annum.

Years.	2 *per Cent.*	2 ½ *per C.*	3 *per Cent.*	3 ½ *per C.*
1	0,9803922	0,9756098	0,9708738	0,9661836
2	1,9415609	1,9274242	1,9134697	1,8996943
3	2,8838833	2,8560236	2,8286114	2,8016379
4	3,8077287	3,7619742	3,7170984	3,6730792
5	4,7134595	4,6458285	4,5797072	4,5150524
6	5,6014309	5,5081254	5,4171914	5,3285530
7	6,4719911	6,3493906	6,2302829	6,1145439
8	7,3254814	7,1701372	7,0196922	6,8739555
9	8,1622367	7,9708655	7,7861089	7,6076865
10	8,9825850	8,7520639	8,5302028	8,3166053
11	9,7868480	9,5142087	9,5256241	9,0015510
12	10,5753412	10,2577646	9,9540040	9,6633343
13	11,3483737	10,9831839	10,6349553	10,3027385
14	12,1062487	11,6909122	11,2960731	10,9205203
15	12,8492635	12,3813777	11,9379351	11,5174109
16	13,5777093	13,0550027	12,5611020	12,0941168
17	14,2918719	13,7121977	13,1661185	12,6513206
18	14,9920313	14,3533636	13,7535131	13,1896812
19	15,6784620	14,9788913	14,3237991	13,7098374
20	16,3514333	15,5891623	14,8774748	14,2124033
21	17,0112092	16,1845486	15,4150241	14,6979742
22	17,6580482	16,7654132	15,9369166	15,1671248
23	18,2922041	17,3321105	16,4436084	15,6204105
24	18,9139256	17,8849858	16,9355421	16,0583676
25	19,5234565	18,4243764	17,4131477	16,4815146

TABLE V.

The Present Worth of 1l. Annuity, Comp. Interest.

Years.	4 *per Cent.*	4 ½ *per C.*	5 *per Cent.*	6 *per Cent.*
1	0,9615385	0,9569378	0,9523809	0,9433962
2	1,8860947	1,8726678	1,8594103	1,8333926
3	2,7750910	2,7489644	2,7232480	2,6730119
4	3,6298952	3,5875257	3,5459505	3,4651056
5	4,4518223	4,3899767	4,3294767	4,2123638
6	5,2421369	5,1578725	5,0756921	4,9173244
7	6,0020547	5,8927009	5,7863734	5,5823815
8	6,7327448	6,5958861	6,4632128	6,2097939
9	7,4353314	7,2687905	7,1078217	6,8016923
10	8,1108955	7,9127182	7,7217349	7,3600871
11	8,7604763	8,5289169	8,3064142	7,8868747
12	9,3850733	9,1185808	8,8632516	8,3838440
13	9,9856473	9,6828524	9,3925730	8,8526831
14	10,5631223	10,2228253	9,8986409	9,2949840
15	11,1183868	10,7395457	10,3796500	9,7122491
16	11,6522949	11,2340151	10,8377695	10,1058953
17	12,1656680	11,7071914	11,2740662	10,4772597
18	12,6592961	12,1599918	11,6895869	10,8276035
19	13,1339385	12,5932936	12,0853208	11,1581165
20	13,5903253	13,0079365	12,4622103	11,4699213
21	14,0291589	13,4047239	12,8211527	11,7640767
22	14,4511142	13,7844248	13,1630026	12,0415818
23	14,8568405	14,1477749	13,4885739	12,3033790
24	15,2469619	14,4954784	13,7986418	12,5503576
25	15,6220787	14,8282089	14,0939445	12,7833562

TABLE V.

The Present Worth of 1l. Annuity, Comp. Interest.

Years.	2 per C.	2 ½ per C.	3 per Cent.	3 ½ per C.
26	20,1210358	18,9506111	17,8768420	16,8903523
27	20,7068978	19,4640109	18,3270315	17,2853645
28	21,2812724	19,9648887	18,7641082	17,6670188
29	21,8443847	20,4535499	19,1884546	18,0357670
30	22,3964556	20,9302926	19,6004413	18,3920454
31	22,9377015	21,3954074	20,0004285	18,7362758
32	23,4683348	21,8491780	20,3887655	19,0688656
33	23,9885636	22,2918809	20,7657918	19,3902082
34	24,4985917	22,7237863	21,1318367	19,7006842
35	24,9986193	23,1451573	21,4872200	20,0006612
36	25,4888425	23,5562511	21,8322525	20,2904938
37	25,9694534	23,9573181	22,1672354	20,5705254
38	26,4406406	24,3486030	22,4924616	20,8410874
39	26,9025888	24,7303444	22,8082151	21,1024999
40	27,3554792	25,1027751	23,1147719	21,3550723
41	27,7994895	25,4661220	23,4123999	21,5991037
42	28,2347936	25,8206068	23,7013592	21,8348828
43	28,6615623	26,1664457	23,9819021	22,0626887
44	29,0799631	26,5038495	24,2542739	22,2827910
45	29,4901599	26,8330239	24,5187125	22,4954503
46	29,8923136	27,1541696	24,7754490	22,7009181
47	30,2865820	27,4674826	25,0247078	22,8994378
48	30,6731196	27,7731537	25,2667066	23,0912443
49	31,0520780	28,0713695	25,5016569	23,2765645
50	31,4236059	28,3623117	25,7297640	23,4556179

TABLE

TABLE V.

The Present Worth of 1 l. Annuity, Comp. Inter.

Years.	4 *per Cent.*	4 ½ *per C.*	5 *per Cent.*	6 *per Cent.*
26	15,9827678	15,1466115	14,3751853	13,0031663
27	16,3295844	15,4513028	14,6430336	13,2105342
28	16,6630618	15,7428735	14,8981272	13,4061644
29	16,9837132	16,0218885	15,1410735	13,5907211
30	17,2920318	16,2888885	15,3724510	13,7648312
31	17,5884921	16,5443909	15,5928104	13,9290861
32	17,8735500	16,7888909	15,8026766	14,0840435
33	18,1476441	17,0228621	16,0025491	14,2302297
34	18,4111962	17,2467580	16,1929039	14,3681412
35	18,6646116	17,4610124	16,3741942	14,4982465
36	18,9082803	17,6660406	16,5468516	14,6209872
37	19,1425771	17,8622398	16,7112872	14,7367804
38	19,3678625	18,0499902	16,8678926	14,8460192
39	19,5844831	18,2296557	17,0170406	14,9490747
40	19,7927721	18,4015844	17,1590862	15,0462969
41	19,9930500	18,5661095	17,2943678	15,1380160
42	20,1856250	18,7235498	17,4232074	15,2245434
43	20,3707931	18,8742103	17,5459118	15,3061730
44	20,5488395	19,0183831	17,6627732	15,3831821
45	20,7200378	19,1563474	17,7740697	15,4558321
46	20,8846517	19,2883707	17,8800663	15,5243699
47	21,0429342	19,4147088	17,9610155	15,5890282
48	21,1951289	19,5356066	18,0771576	15,6500266
49	21,3414700	19,6512981	18,1687215	15,7075723
50	21,4821826	19,7620078	18,2559253	15,7618610

TABLE

TABLE VI.

Of COMPOUND INTEREST.

The Annuity which one Pound will purchase for any Number of Years; *at the Rates of* 2; 2 ½; 3; 3 ½; 4; 4 ½; 5, *and* 6 per Cent. per Annum.

Years.	2 *per Cent.*	2 ½ *per C.*	3 *per Cent.*	3 ½ *per C.*
1	1,0200000	1,0250000	1,0300000	1,0350000
2	,5150495	,5188272	,5226108	,5264005
3	,3467547	,3501372	,3535304	,3569342
4	,2626238	,2658179	,2690271	,2722511
5	,2121584	,2152469	,2183546	,2214814
6	,1785258	,1815499	,1845975	,1876682
7	,1545120	,1574954	,1605064	,1635445
8	,1365098	,1394674	,1424564	,1454767
9	,1225154	,1254569	,1284339	,1314460
10	,1113265	,1142588	,1172305	,1202414
11	,1021779	,1051060	,1080775	,1110920
12	,0945596	,0974871	,1004621	,1034840
13	,0881183	,0910483	,0940295	,0970616
14	,0826020	,0855365	,0885263	,0915707
15	,0778255	,0807665	,0837666	,0868251
16	,0736501	,0765990	,0796109	,0826848
17	,0699698	,0729278	,0759525	,0790431
18	,0667021	,0696701	,0727087	,0758168
19	,0637818	,0667606	,0698139	,0729403
20	,0611567	,0641471	,0672157	,0703610
21	,0587847	,0617873	,0648718	,0680366
22	,0566314	,0596466	,0627474	,0659321
23	,0546681	,0576964	,0608139	,0640188
24	,0528511	,0559128	,0590474	,0622728
25	,0512204	,0542759	,0574279	,0606740

TABLE

TABLE VI.

The Annuity which one Pound will purchase, Compound Interest.

Years.	4 per Cent.	4 ½ per C.	5 per C.	6 per Cent.
1	1,0400000	1,0450000	1,0500000	1,0600000
2	,5301961	,5339976	,5378049	,5454369
3	,3603485	,3637734	,3672086	,3741098
4	,2754901	,2787437	,2820118	,2885915
5	,2246271	,2277916	,2309748	,2373964
6	,1907619	,1938784	,1970157	,2033626
7	,1666096	,1097015	,1728198	,1791350
8	,1485279	,1516097	,1547218	,1610359
9	,1344930	,1375745	,1406901	,1470222
10	,1232909	,1263788	,1295046	,1358680
11	,1141490	,1172482	,1203890	,1267929
12	,1065522	,1096662	,1128254	,1192770
13	,1001437	,1032754	,1064558	,1129601
14	,0946690	,0978203	,1010240	,1275849
15	,0899411	,0931138	,0963423	,1029628
16	,0858200	0,890154	,0922699	,0989521
17	,0821985	0,854176	,0886991	,0954448
18	,0789933	0,822369	,0855462	,0923565
19	,0761386	0,794073	,0827450	,0896209
20	,0735818	0,768761	,0802426	,0871846
21	,0712801	,0746006	,0779961	,0850046
22	,0691988	,0725457	,0759705	,0830456
23	,0673091	,0706825	,0741368	,0812785
24	,0655868	,0689870	,0724709	,0796790
25	,0640121	,0674390	,0709545	,0782267

TABLE VI.

The Annuity, which one Pound will purchase, Compound Interest.

Years.	*2 per Cent.*	*2 ½ per C.*	*3 per Cent.*	*3 ½ per C.*
26	,0496992	,0527688	,0559383	,0592054
27	,0482931	,0513769	,0545642	,0578524
28	,0469897	,0500879	,0532932	,0566027
29	,0457784	,0488913	,0521147	,0554454
30	,0446499	,0477776	,0510193	,0543713
31	,0435964	,0467390	,0499989	,0533724
32	,0426106	,0457683	,0490466	,0524415
33	,0416865	,0448594	,0481561	,0515724
34	,0408187	,0440068	,0473220	,0507597
35	,0400022	,0432056	,0465393	,0499984
36	,0392329	,0424516	,0458038	,0492842
37	,0385068	,0417409	,0451116	,0486133
38	,0378206	,0410701	,0444593	,0479821
39	,0371711	,0404362	,0438439	,0473878
40	,0365558	,0398362	,0432624	,0468273
41	,0359719	,0392679	,0427124	,0462982
42	,0354173	,0387288	,0421917	,0457983
43	,0348899	,0382169	,0416981	,0453254
44	,0343879	,0377304	,0412299	,0448777
45	,0339096	,0372675	,0407852	,0444534
46	,0334534	,0368268	,0403625	,0440511
47	,0330179	,0364067	,0399605	,0436692
48	,0326018	,0360060	,0395778	,0433065
49	,0322040	,0356235	,0392131	,0429617
50	,0318032	,0352581	,0388655	,0426337

TABLE VI.

The Annuity which one Pound will purchase, Compound Interest.

Years.	4 per Cent.	4 ½ per C.	5 per Cent.	6 per Cent.
26	,0625674	,0660214	,0695643	,0769044
27	,0612385	,0647195	,0682919	,0756972
28	,0600130	,0635208	,0671225	,0745926
29	,0588799	,0624146	,0660455	,0735796
30	,0578301	,0613915	,0650514	,0726489
31	,0568[illegible]54	,0604435	,0641321	,0717522
32	,0559486	,0595632	,0632804	,0710023
33	,0551036	,0587445	,0624900	,0702729
34	,0543148	,0579819	,0617554	,0695984
35	,0535773	,0572705	,0610717	,0689739
36	,0528869	,0566058	,0604345	,0683948
37	,0522396	,0559840	,0598398	,0678574
38	,0516319	,0554017	,0592842	,0673581
39	,0510608	,0548557	,0587646	,0668938
40	,0505235	,0543431	,0582782	,0664915
41	,0500174	0,538616	,0578223	,0660589
42	,0495402	0,534087	,0573947	,0656834
43	,0490899	0,529824	,0569933	,0653331
44	,0486645	0,525807	,0566163	,0650061
45	,0482625	0,522020	,0562617	,0647005
46	,0478821	,0518447	,0559282	,0644149
47	,0475219	,0515073	,0556142	,0641477
48	,0471807	,0511886	,0553184	,0638977
49	,0468571	,0508872	,0550397	,0636636
50	,0465502	,0506021	,0547767	,0634443

CHAP.

CHAP. X.

The Use of DECIMALS *in Vulgar, Duodecimal, and Sexagesimal Fractions.*

Vulgar Fractions in Decimals.

I HAVE already ſhewn the Method of finding the Decimal equivalent to any *Vulgar Fraction*, in *Reduction*; What I propoſe here, is to ſhew with how much greater Eaſe and Pleaſure any of the Operations of *Vulgar Fractions* are wrought by *Decimal Numbers*. I ſhall exemplify the Matter in the *common Rules* as follows.

Addition.

Example 1. What is the Sum of $\frac{7}{9}$ and $\frac{3}{7}$ of a Pound?

Add $\left\{\begin{array}{l}\text{The } Decimal \text{ of } \frac{7}{9} = ,7777 \\ \text{The } Decimal \text{ of } \frac{3}{7} = ,4285\end{array}\right\}$ By the general Decimal Table, *P.*

The Sum of both is $= l.\ 1,2062 = 1 l.\ 4 s.\ 1\frac{1}{2} d.$ Anſ.

Example 2. What is the Value of $\frac{1}{16}$ and $\frac{5}{6}$ of a *Shilling*?

Add $\left\{\begin{array}{l}\text{The } Decimal \text{ of } \frac{1}{16} = ,0625 \\ \text{To the } Dec. \text{ of } \frac{5}{6} = ,83333\end{array}\right\}$ By the ſaid Table.

Their Sum is — $0,89583 = 10\frac{3}{4} d.$ Anſ.

Example 3. What is $\frac{2}{13}$, $\frac{3}{13}$ and $\frac{7}{13}$ of a *Yard*?

$$\text{Add}\left\{\begin{array}{l}\text{The } \textit{Decimal} \text{ of } \frac{2}{13} = ,1538 \\ \text{And the } \textit{Dec.} \text{ of } \frac{3}{13} = ,2307 \\ \text{And the } \textit{Dec.} \text{ of } \frac{7}{13} = ,5384\end{array}\right\}\text{By the Tab.}$$

The Sum, of Course, is $\frac{12}{13}$ = 0,9229 = 2 *F.* 9 *In.* An.

Example 4. What is the Value of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{2}{9}$, and $\frac{3}{4}$ of $\frac{5}{6}$ of a *Hundred Weight Averdupois*?

$$\text{Add}\left\{\begin{array}{l}\text{The } \textit{Decimal} \text{ of } - \frac{2}{3} = ,6666 \\ \text{The } \textit{Decimal} \text{ of } - \frac{4}{5} = ,8000 \\ \text{The } \textit{Decimal} \text{ of } - \frac{2}{9} = ,2222 \\ \text{The } \textit{Dec.} \text{ of } \frac{3}{4} \text{ of } \frac{5}{6} = \frac{5}{8} = ,6250\end{array}\right\}\begin{array}{l}\text{By the} \\ \text{Table.}\end{array}$$

The Sum of all is — *C.* 2,3138 = 2 : 1 : 7 : 2 (*C. Q. lb. oz.*)

Example 5. What Number of *Years*, do $476\frac{7}{11}$, $36\frac{9}{16}$, $21\frac{10}{11}$, $7\frac{13}{14}$, $1\frac{14}{17}$ and $\frac{17}{19}$ of a Year, make?

		Years.
	$476\frac{7}{11}$ =	476,6363
	$36\frac{9}{16}$ =	36,5625
	$21\frac{10}{11}$ =	21,9090
Those *mixed Fractions* being set down in order, and the *Decimals* of the *Fractional Parts* being found in the *General Table* and set down opposite thereto; add them, and their	$7\frac{13}{14}$ =	7,9285
	$1\frac{14}{17}$ =	1,8234
	$\frac{17}{19}$ =	0,8944
Sum will be	— —	545,7541 *Years.*

That is, 545 *Years*, 9 *Months*, 3 *Weeks*, and 2 *Days* nearly.

Substraction.

Substraction.

Example 1. What is $\frac{5}{6}$ less $\frac{3}{8}$ of a *Pound Sterling*?

From — — $\frac{5}{6} = ,8333$ } By the general Decimal Table.

Substract — — $\frac{3}{8} = ,375$

There remains — — $0,4583 = 9s.\ 2d.$ Ans.

Example 2. What is the Value of $\frac{2}{3}$ of $\frac{7}{8}$ less $\frac{3}{4}$ of $\frac{5}{9}$ of a *Rod*?

From — $\frac{2}{3}$ of $\frac{7}{8} = \frac{7}{12} = ,583$ } By the Table.

Substract $\frac{3}{4}$ of $\frac{5}{9} = \frac{5}{12} = ,416$

There remains — $0,167 = 0$ *Y.* 2 *F.* 9 *In.*

Example 3. What is $14\frac{2}{9}$, less $\frac{5}{7}$ of a *Pound Troy*?

lb.

From — $14\frac{2}{9} = 14,2222$ } By the Table.

Take — $\frac{5}{7} = 0,7142$

lb. oz. pw.

There remains — $13,5080 = 13 : 6 : 2$

Example 4. What is $170\frac{17}{19}$, less $159\frac{9}{19}$ *C. Weight, Averdupois*?

C.

From — $170\frac{17}{19} = 170,8947$ } By the Table.

Take — $159\frac{9}{19} = 159,4736$

There remains the Answer 11,4211 *C.*

Multi-

Multiplication.

Example 1. What is $\frac{4}{7}$ of a *Yard* multiplied by $\frac{2}{9}$ of a *Yard*?

Multiply — $\frac{4}{7}$ = ,5714

By — — — $\frac{2}{9}$ = ,2

9) ,11428

The Product is 1 *Fq.* 20,3 *Inq.* = ,12697

Example 2. Suppose a Piece of *Timber* $14\frac{4}{7}$ Feet in Length, $\frac{8}{11}$ of a Foot Wide, and $\frac{1}{2}$ of $\frac{5}{8}$ of a Foot thick, What is the *Solid Content* of that Piece?

Multiply the Length — $14\frac{4}{7}$ = 14,4285

By the Width — $\frac{8}{11}$ = ,72

The Product will be found to be — 10,49342

That multiplied by the Thickness, which is $\frac{1}{2}$ of $\frac{5}{8}$ = $\frac{5}{16}$ } = ,3125

The Product is the *Content* sought = 3,27818, &c.

That is, 3 *Feet*, 480 *Inches*, $46\frac{1}{2}$ *Quarters*, solid Content the Answer.

Example 3. There is a *Cistern* $10\frac{1}{6}$ Feet in Length, $4\frac{5}{9}$ Deep, and $5\frac{4}{11}$ Wide; Quere how many *Corn Gallons* it will hold?

Multi-

Multiply the Length — $10\frac{1}{6} = 10,16$

By the Width — — $5\frac{4}{11} = 5,36$

That Product will be — — $54,52630$

Multiply that by the Depth — $4\frac{5}{9} = 4,5$

The Product is the Solid Content in Feet and Parts — } $= 248,3976$ &c.

Multiply that by the Gallons in a Solid Foot; *viz.* $6\frac{13}{32}$ } $= 6,40625$

The Product is the Number of Corn Gallons sought, *viz.* } $= 1591,2969$ &c.

Note; When there is given any Number of pure Fractions to be multiplied into one another; You may multiply the *Numerators* and *Denominators* into one another, and the Products will be a Fraction, whose Value in *Decimals* you may find as before taught,

Thus,
Multiply $\frac{1}{9}$, $\frac{2}{7}$, $\frac{5}{11}$, $\frac{9}{17}$ all into one another.
Then $1\times2=2$, $2\times5=10$, $10\times9=90$
And $9\times7=63$, $\times11=693$, $\times17=11781$ } $= \frac{10}{1309}$
But $\frac{10}{1309} = 0,00764$ The Answer

Division.

Example 1. Divide $\frac{5}{7}$ by $\frac{1}{17}$.

Thus

Thus $\frac{1}{17}=0{,}0588)\,{,}714285=\frac{5}{7}\,(12{,}14754$ = The Answer.

588
1262
1176
··868
588
2805
2352
·4437
4116
·3211
2940
·2714
2352
·362

Example 2. What is $\frac{16}{19}$ divided by $\frac{2}{5}$ of $\frac{8}{15}$?

Thus $\frac{2}{5}$ of $\frac{8}{15}=\frac{16}{75}={,}213),842105263=\frac{16}{19}$

21   842105263
,192   ,7578947368 (3,947368
576
1818
1728
··909
768
1414
1344
··707
576
1313
1152
1616
1536
··808

There is somewhat remarkable and uncommon in the Work of this Question, *viz.* First, that the 1st, 3d, 5th, *&c.* Remainders, with the Numbers taken down, consist of 2 Pair of the same Figures; Secondly, That the 2d, 4th, *&c.* Remainders, with the Figures taken down, are just half the others; Thirdly, That where the Remainders of Pairs are prime Numbers, the next is a Remainder is a Pair of even Numbers, and then goes on as before, as at the 5th and 6th.

Example 3. What is $2\frac{7}{18}$ of a *Pound, divided* by $\frac{10}{11}$?

Thus $\frac{10}{11}$ = ,90) 2,388 = $2\frac{7}{18}$

,00   23

,9) 2,365 (2,627 = 2 *l.* 12 *s.* 6 $\frac{1}{2}$ *d.*
18
·56
54
25
18
·70
63
7

(The Answer.

Example 4. What is it *per C. Weight,* when $21\frac{7}{11}$ *C.* costs $67\frac{7}{16}$ *Pounds*?

Thus $21\frac{7}{11}$ = 21,63) 67,4375 = $67\frac{7}{16}$

21   674375

21,42) 66,763125 (3,11685 *l.*
6426
2503
2142
·3611
2142
14692
12852
18405
17136
12690
10710
1980

That is 3,11685 *l.* = 3 *l.* 2 *s.* 4 *d. per C.* the Answer.

Extraction of Roots.

Example 1. What is the *Square Root* of $\frac{25}{36}$?

The Decimal of $\frac{25}{36}$ = ,694 (,833 = $\frac{5}{6}$ = the Root (sought.

```
        64
      ------
163) ·544
      489
      --------
1663) ·5544
       4989
      -------
        555
```

Example 2. Required the *Square Root* of the *Surd Fraction* $\frac{13}{17}$?

The Decimal of $\frac{13}{17}$ = ,7647058 &c. the Root of which extracted will be ,8744726 &c. the Answer.

Example 3. Extract the *Square Root* of $58\frac{7}{9}$

The *Decimal* of the *mixed Fraction* $58\frac{7}{9}$ = 58,7

Then extract thus; 58,7 (7,66 = $7\frac{2}{3}$ = the Root (sought.

```
        49
      ------
146) ·977
      876
      ---------
1526) 19177
       9156
      ---------
       1021
```

Exam-

Example 4. Extract the *Cube* Root of $\frac{3}{16}$

This is best (as being vastly soonest and easiest) done by *Logarithms*, thus,

From the *Logarithm* of the *Numerator* 3 = 0,4771212
Substract the *Logar.* of the *Denominator* 16 = 1,2041200

There remains the *Lagar.* of the *Dec.* ,1875 = 9,2730012
Add to the *Index* of that *Logarithm* 20 — ,29,2730012
One Third of which *Logorithm* is the *Logarithm* of the *Root* sought } ,572358 = ,9,7576670

The Cube Root then of $\frac{3}{16}$ is ,572358 which was to be found.

Example 5. What is the *Cube Root* of $512\frac{13}{15}$?

First, From the *Logarithm.* of the *Numerator* of the Fractional Part } — 13 = 1,113943
Substract the *Logarith.* of the *Denominator* 15 = 1,176091

There remains the *Logarithm* of the *Dec.* ,86 = ,9,937852
To which add the *integral* Part 512

The Sum is the equivalent *mixed Dec.* 512,86 = 2,710004
A Third of whose *Logarithm* is the *Logarithm* of the *Cube Root* sought — — — } 8,0045 = 0,903334

The *Cube Root* therefore of $512\frac{13}{15}$ is 8,0045

Note. The Use of *Decimals* is not only very obvious in all Parts of the Doctrine of *Vulgar Fractions*, but absolutely necessary in *Extraction of Roots*; which sometimes else cannot be done.

The Use of DECIMALS *in* Duodecimal Arithmetick.

Duodecimals are a Sort of *Fractions* made use of in *Mensuration*; Where one Foot is the *Integer*; The *Foot* is divided into twelve Parts or *Inches*; one of these, into twelve others, and so on dividing by twelve. Whence as ten is the *Common Denominator* in *Decimals*, so twelve is the *Common Denominator* in *Duodecimal* Fractions.

The Notation and Reading of *Duodecimals* is

Thus $\begin{cases} \textit{Feet, Primes, Seconds, Thirds, Fourths, \&c.} \\ 15 \; : \; 09' \; : \; 10'' \; : \; 06''' \; : \; 11'''' \; \&c. \end{cases}$

Now becauſe this kind of *Arithmetic* is uſeful to Perſons concern'd in *Building*, *Meaſuring*, &c. and the moſt uſeful Parts, viz. *Multiplication*, *Diviſion*, and *Extraction*, being by far the moſt difficult; I thought it very proper (and hope it will be very acceptable) to ſhew how thoſe Operations may be moſt eaſily and ſpeedily perform'd by *Decimal Arithmetick*.

To that End I have made the *following Table* for the ready converting any *Duodecimals* into *Decimals*, and the contrary. The Uſe of which, to thoſe who underſtand any Decimal Tables at all, is very obvious and eaſy.

The Duodecimal Table.

Duodecimals.	*The Decimal Parts.*			
	Primes. ′	*Seconds* ″	*Thirds.* ‴	*Fourths.* ⁗
1	,083333	,006944	,000578	,000048
2	,166666	,013888	,001157	,000096
3	,25	,020833	,001736	,000144
4	,333333	,027777	,002314	,000192
5	,416666	,034722	,002893	,000241
6	,5	,041666	,003472	,000289
7	,583333	,048611	,004051	,000337
8	,666666	,055555	,004629	,000386
9	,75	,0625	,005208	,000435
10	,833333	,069444	,005787	,000482
11	,916666	,076388	,006365	,000530

This Table, as I ſaid, being ſo eaſy, needs no Inſtructions for its Uſe; nor ſhall I pretend to ſay *Decimals* are of any Service in the *Rules* of *Addition* and *Subſtraction* of *Duodecimals*.

But

But their extream Utility in the aforesaid Operations of *Multiplication*, *Division*, and *Extraction* of Roots, of *Duodecimals*, will be undeniably evident by the ensuing Examples.

Multiplication.

Example 1. What is 9 *F*, 10' multiplied by 8 *F*. 08' ?

Multiply the *Decimal* of *F.* 9 : 10 = $9,8\dot{3}$
By the *Decimal* of 8 : 08 = $8,\dot{6}$

9) 5900

6555

The Product is the } *Feet* ' " 78666

Answer, *viz.* } 85 : 2 : 8 = $85,\dot{2}22$ *Feet.*

Example 2. What is the Product of 40 *F.* 09' : 10" : by 11' : 09" ?

Multiply the *Decimal* of *Feet* 40 : ' 09 : " 10 = $40,81\dot{9}4$
By the *Decimal* of — 11 : 09 = $,97\dot{9}1\dot{6}$

In such Cases where the *Decimals* run far, and *terminate* in *Repetends*, 'tis best to multiply by the *contracted*, or *inverted* Way, heretofore taught, thus;

$$
\begin{array}{r}
40,8194\dot{4} \\
\dot{6}1979,0 \\
\hline
3673750 \\
285736 \\
36737 \\
468 \\
272 \\
\hline
\end{array}
$$

Answer. 39 F. 11' : 07" : 06''' : 06 = 39,96903 *Feet.*

Example

Example 3. What is 175 *Feet*, 00′ : 04″ : by 08⁗?

```
                              Feet     '    ''    '''   ''''
Multiply the Decimal of        175 :  00    04 :   00 :  00 = 175,027
By the Decimal of                      —     —          08  = ,000386
                                                             ---------
                                                              1050162
                                                             1400222
                                                             525083
                                                             ---------
The Product is   F      '      ''      '''    ''''
the Answer, viz. 0 :   00 :    09 :    08 :   07 = ,067560721
```

```
                       Feet    '     ''    '''
Example 4. Maltiply     17  :  09 :  02 :  06 = 17,76736
           By            —            —     6 =        6
                                              ----------
The Product is the Answer 106 F. 07′ : 03″ = 106,60416
```

Example 5. What is the *Square* of 12 *Feet*, 09′ : 07″ : 10‴?

```
             '      ''      '''
Thus   12 ; 09  :  07  ;  10   = 12,804390
       12 : 09  :  07  :  10   = 93408,21
                  Inverted
                                -----------
                                 12804390
                                  2560878
                                  1024351
                                     5121
                                      384
                                      115
                                -----------
Answer 163 Feet, 11′ 05″ 01‴ 09⁗ = 163,95239
```

Division.

Division.

Example 1. Divide 12 *Feet*, 10′ : 07″ : by 3.

Thus, the *Decimal* of 12 *Feet*, 10′ : 07″ is 12,88194 *Feet*

Then 3) 12,88194 (4,2939814 = 4 : 03′ : 06‴ : 04″

```
12
——
··8
  6
———
  28
  27
————
   11
    9
————
   29
   27
————
    24  )
    24  |
————    |
   ··4  |
     3  }  Ad Infinitum.
————    |
    14  |
    12  |
————    |
     24 )
```

Example 2. Divide 14 *Feet*, 07′ : 09″ : by 06′ : 04″

The *Decimal* of { 14 : 07′ : 09″ = 14,64583 ; 06 : 04 = ,527 }

```
Then ,527) 14,64583
        52   146458
     ————    ————————
     ,475)  13,18125 (27,75 = 27 Feet, 09′
              950
             ————
             ·3681
              3325
             —————
              ·3562
               3325
              —————
                2375
                2375
              ——————
                ····
```

Exam-

Example 3. Divide 5 *Feet*, by 1 *Foot*, 02' : 03'' : 11'''

The *Decimal* of 1 *Foot*, 02' : 03'' : 11''' = 1,193865

```
Then   1,193865) 5,000000  (4,188079 =
                 4775460   (4 F. 02' : 03'' : 01'''
                 -------
                  224540
                  119386
                  ------
                  105154
                   95509
                   -----
                    9645
                    9550
                    ----
                      94
                      83
                      --
                      11
                      10
                      --
                       1
```

```
                       Feet    '    ''   '''
Example 4. { Divide     32 : 10 : 11 : 06 = 32,913194
           { By          8 : 01 : 10 : 11 =  8,159142
```

```
Thus   8,159142) 32,913194  (4,033904 =
                 32636568   (4 F. 00' : 04'' : 10'''
                 --------
                   276626
                   244774
                   ------
                    31852
                    24477
                    -----
                     7375
                     7343
                     ----
                       32
                       32
                       --
                       ..
```

Extraction

Extraction of Roots.

Example 1. What is the Side of the Square of

$\overset{F.}{163} : \overset{,}{11} : \overset{''}{05} : \overset{'''}{01} : \overset{''''}{09}$?

The *Decimal* of those *Duodecimals* is 163,95239, the *Square Root* of which extracted either by *Logarithms* or in the common way, gives the Side $\overset{F.}{12} : \overset{,}{09} : \overset{''}{07} : \overset{'''}{10}$ for Answer.

Example 2. What is the Side of the Cube, whose Solid Content is $\overset{F.}{1} : \overset{,}{10} : \overset{''}{07} : \overset{'''}{06} : \overset{''''}{10}$?

By *Logarithms*

Thus, the Decimal $\overset{F.}{1} : \overset{,}{10} : \overset{''}{07} : \overset{'''}{06} : \overset{''''}{10} = 1{,}885898$
Then the *Logarithm* of — 1,885898 = 0,2755182
One Third of that is — 1,23549 = 0,0918394

But 1,23549 is the Decimal of $\overset{F.}{1} : \overset{,}{02} : \overset{''}{09} : \overset{'''}{10} : \overset{''''}{11}$
Which is the Length of the Side of the Cube proposed.

As in Vulgar, so in *Duodecimal Fractions*, the *Extraction* of Roots, can be performed, no way so well as by Decimal Parts; and the other Laborious Operations are hereby rendered easy and concise.

The Use of Decimals in Sexagesimal Arithmetick.

Sexagesimals are those *Fractions* which have 60 for their *common Denominator*; and are chiefly used in Computations of *Motion* and *Time*.

Hence this Kind of *Arithmetick* is proper to *Astronomy*, which, as it is a Science of *Motion*, and *Time*, makes use thereof in all its Calculations: Hereby it is the *Astronomer* calculates the *Motion*, *Place*, *Magnitude*, *Distance*, *Time*, *Aspects*, and other *Phænomena* of the Heavenly Bodies; the *Sun*, *Moon*, *Planets*, *Comets*, and *Stars*.

The *Notation* and Reading of *Sexagesimals* is in this Manner following;

Viz.	*Signs*,		*Degrees*,		*Minutes*,		*Seconds*,		*Thirds*,		&c.
			°		′		″		‴		
	05	:	26	:	57	:	53	:	47		&c.

And as 60 is one *Degree* of *Motion*, so it is one *Hour* of *Time*; hence *Sexagesimals* properly so called, begin only at *Minutes*, and go to *Seconds*, *Thirds*, &c. forwards, in both *Motion*, and *Time*; though in common, it comprehends *any Division* of either.

But as all *Astronomical* Calculations are made from *Sexagesimal* Numbers, already computed and disposed into Tables of various Sorts; if I would shew or demonstrate the Use of *Decimals*, and their Preference to *Sexagesimal Numbers* in these Kind of Computations; I must first suppose those *Sexagesimal* Tables, made into *Decimal* ones; and if such a thing were once done, I believe 'twould be no very hard Task to make good the Proposition asserted.

The *Reader* need only judge of this by the following Example of *Addition* in both *Species*.

	Sexagesimally.								*Decimally.*
	S		°		′		″		*S*
	03	:	21	:	57	:	49	=	3,73211
	11	:	29	:	47	:	58	=	11,99331
	10	:	18	:	59	:	37	=	10,63311
	09	:	25	:	17	:	43	=	9,84317
	08	:	29	:	59	:	57	=	8,99996
	11	:	18	:	43	:	49	=	11,62433
Sum	08	:	24	:	46	:	53	=	8,82601

In this *Specimen* I think 'tis easy to observe how concise, simple, natural and easy the Operation by *Decimals* is if compared with the *Sexagesimal* Process; which therefore I think must needs prove the Preference and Excellency of those Tables in Decimals.

But since none as yet have said any thing about this Affair, nor have we any Astronomical Tables in *Decimals*, I shall give a *Specimen* thereof in the *Mean Motions* of all the *Planets* for one whole *Year*, *Day*, *Hour*, and *Minute*, in the Table subjoin'd.

Planets

Planets.	*A Year.*	*A Day.*	*An Hour.*	*A Minute.*
	S.	S.	S.	S.
Sun	11,99025	0,03285	0,001367	0,000018
Moon	4,31271	0,43921	0,018292	0,000294
Saturn	0,40709	0,00111	0,000046	0,000000
Jupiter	1,01096	0,00276	0,000110	0,000000
Mars	6,37574	0,01749	0,000728	0,000009
Venus	7,49255	0,05340	0,00222	0,000036
Mercury.	1,79005	0,13639	0,005683	0,000092

Such then is the Form, and ſuch would be the Difference of *Decimal* and *Sexageſimal Tables*; The Numbers here are *homogeneous*, all of one Sort; in them, they are *heterogeneous*, or conſiſt of diverſe Sorts; here they are *Uniform* and to be wrote as *integral* Numbers, there they are ranged in a different Form and in diverſe *Claſſes*, as all mixed Numbers are; beſides the great Eaſe and Facility of Working *Decimal* in Compariſon of *Sexageſimal* Numbers, as I before obſerved. Upon all theſe Accompts, and ſeveral others I might mention, A Set of *Aſtronomical Tables* in *Decimal Numbers* muſt certainly be much more Uſeful, and every way preferable to the preſent *Sexageſimal Tables*.

After having turn'd your *Sexageſimal Numbers* into *Decimals*, they are to be worked in the ſame Manner as *Duodecimals* through all the *Rules*, as is there taught; and therefore needs not be here again repeated. Only, I would here obſerve, that the Rules of *Multiplication* and *Diviſion*, which are here often neceſſary, cannot be perform'd without a great deal of Difficulty, or a long and tedious Proceſs, whereas by *Decimals* 'tis done with the utmoſt Facility and Expedition.

To this End, I have taken Care that the Reader ſhould not want large and ſufficient Tables for the expeditious turning of his *Sexageſimals* into Decimals, and the contrary; the like of which are not to be found elſewhere, that I know of.

CHAP. XI.

The Use and Management of DECIMALS (*after a new Manner*) *by* Logarithms.

THERE may chance to happen to the Reader a *double* Advantage in this Chapter; for first, he may here perceive, not only the common, but an *entire new* Management of *Decimals* by *Logarithms*; and *secondly*, he may here as well as any where learn the whole noble and excellent *Art* of *Logarithmical Arithmetick*, if he has not learnt it already; for *Decimals* and *Integers* having the same essential Properties, the *Logarithms* of both are the same, and differ only in their *Indexes*.

But that the *young Student* may the better understand how to vary and adjust the *Index* of the *Logarithm*, I have in the following Table given *all Variety of Cases* that can happen to a Number, its Logarithm, and Index, under the various Conditions, and Denominations of *Whole Number*, *Mixed Number*, *Pure Decimal*, *Repeating Decimals*, *Decimals* with *Cyphers* prefixed, &c. as follow.

			Index Logar.
Whole Numbers	—	5243	= 3,7195799
Mixed Numbers	—	524,3	= 2,7195799
		52,43	= 1,7195799
		5,243	= 0,7195799
A *Perfect Decimal*	—	,5243	= 9,7195799
Decimals with Cyphers prefixed		,05243	= 8,7195799
		,005243	= 7,7195799
		,0005243	= 6,7195799
A *Single* Repetend	—	$\dot{3}$,	= 0,5228787
Mixed Single Repetends	—	$43,\dot{3}$ or $4\dot{3}$	= 1,6368221
		$243,\dot{3}$ or $24\dot{3}$	= 2,3862016
		$5243,\dot{3}$ or $524\dot{3}$	= 3,7196075
Compound Repetends	—	$\dot{4},\dot{3}$	= 0,6378333
		$\dot{2},4\dot{3}$	= 0,3860408
		$\dot{5},24\dot{3}$	= 0,7196234

Com-

		Index	Logar.
Compound Repetends with Cyphers — —	,0000$\dot{3}$	=	5,5228787
	,000$\dot{4}\dot{3}$	=	6,6378333
	,00$\dot{2}\dot{4}\dot{3}$	=	7,3860408
	,0$\dot{5}$2$\dot{4}\dot{3}$	=	8,7196234
Mix'd Compound Repetends	2$\dot{4}\dot{3}$	=	2,3863818
	5$\dot{2}$,$\dot{4}\dot{3}$	=	1,7196000
The same with Cyphers	,0002$\dot{4}\dot{3}$	=	6,3863818
	,005$\dot{2}\dot{4}\dot{3}$	=	7,7196000

From this general *Scheme*, the following Obſervations may be made relating to the *Logarithm*, and determining its *Index* for any Kind of Number.

Obſervation 1. That the *Index* of the *Logarithm* of any whole Number, is always one leſs than the Number of Places of Figures in the whole Number.

Obſervation 2. That the *Logarithm* of any Number, whether *Integral*, *Mix'd*, or wholly *Decimal*, is the very ſame; only the *Index* differs and muſt be adjuſted ſolely in regard of the *integral* Part of the Number; as *per Obſerv.* 1.

Obſervation 3. That if there be no *integral* Part but the Number is entirely *Decimal*, and the firſt left-hand Figure be one of the *nine Digits*, the *Index* is (0).

Obſervation 4. That if the Number be entirely *Decimal*, and have any Number of Cyphers prefixed, the Index (being in this Caſe dotted on both ſides) muſt be ſuch as, when ſubſtracted from 9, the Remainder may expreſs the Number of Cyphers prefixed.

Obſervation 5. That any *Repetend*, or Set of *Circulating* Numbers, whether *Whole*, or *Decimal*, obſerve all the Rules of *terminate* Numbers aforementioned, relating to the *Index*; but the *Logarithm* is different.

Obſervation 6. That the *Logarithm* varies, according as the ſame Figures are either *terminate* or *repetends*; and again as thoſe *Repetends* make either a *Part*, or the *Whole Number*; or thus, the *Logarithm* is bigger or leſs as the firſt Figure of the *Repetend* is ſo.

As to what concerns the *Adding* and *Subſtracting* of *Indexes*, that may be throughly underſtood by the following Table of all the *Varieties* that can happen in that Affair.

Addi-

Addition.			*Substraction.*		
1. To	—	2,5132176	1. From	—	4,3193976
Add	—	1,8061800	Substract		1,8061800
Sum	=	4,3193976	Rem.	=	2,5132176
2. To	—	,3,3916407	2. From	—	,18,5957607
Add	—	,5,2041200	Take	—	,3,3916407
Sum	=	,18,5957607	Rem.	=	,5,2041200
3. To	—	2,2671717	3. From	—	0,4086215
Add	—	,8,1414498	Take	—	,8,1414498
Sum	=	0,4086215	Sum	=	2,2671717
4. To	—	,8,5132176	4. From	—	0,4086215
Add	—	3,8061800	Take	—	2,2671717
Sum	=	2,3193976	Sum	=	,8,1414498
5. Add	{	,9,2671717 ,5,5132176 ,8,8061800	5. From	—	,7,4086215
			Take	—	,9,2671717
Sum	=	,3,5865693	Sum	=	,8,1414498

To understand the better what concerns the Ordering and Adjusting the *Indices*, in the foregoing Examples, I have subjoin'd the following *Scheme* of the Number of Cyphers, and their corresponding *Indices*.

Numb. of *Cy.* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, *&c.*
Their *Indic.* 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, 19, 18, 17, 16, 15, *&c.*

Hence observe, In *adding Indices*, 1. If both be *Affirmative*, their Sum is *Affirmative*. 2. If both be *Negative*, and the *Sum* be under 10, add 10 thereto; but if above 10, or just 10, cast away 10; the Remainder is *negative*. 3. If one *Index* be *affirmative*, and the other *negative*; the *Sum* if under 10 is *negative*: if *just* 10, or above 10, cast 10 away; the Remainder is *affirmative*.

In

In *Subſtracting Indices*, obſerve, 1. If they are both *Affirmative*, and the Higher be the *Greater*, the Remainder is *Affirmative* ; if the Lower be the *Greater*, the Remainder is *Negative*, (10 being added to the *higher.)* 2. If one or both be *Negative*, and the *higher ſmaller* than the *lower*, add 10 to it ; than if the *higher* be of *greater Value*, the Remains are *Affirmative* ; if not, they are *Negative.*

In order to underſtand the *Art* of Logarithms, and the dexterous Management of Numbers (more particularly *circulating Decimal Numbers)* thereby, 'twill be abſolutely neceſſary to underſtand, and that perfectly well, the following *Logarithmetical Problems.*

Problem 1. To find the *Arithmetical Complement* of any given *Logarithm.*

Rule. Begin at the Left-hand to ſubſtract *(mentally)* each Figure from 9, and the laſt of all from 10.

Exam. What is the *Arithm. Compl.* of the Log. 3,8649262
Anſwer *(per Rule)* is = 6,1350738

Problem 2. To find the *Logarithm* of any *terminate Number* under 10000000.

Rule. Take the *Logarithm* out of the Tables to the *four firſt Figures* of any given Number of above *four Places*, and alſo the next *greater Logarithm* ; then take the Difference of thoſe two *Logarithms*, and multiply it by the remaining Figures of the given Number ; from the Product cut off ſo many Places of Figures to the Right-hand, as were the remaining Figures above *four* ; then add the other Part of the Product to the Logarithm of the *four Figures* firſt taken out of the *Canon* ; that Sum is the *Logarithm* ſought.

Example. Required the *Logarithm* of 1012659 ?

The Logarithm of —	1012 =	3,0051805
	1013 =	3,0056094
Their Differerce — —		4289
Multiply by the remaining Figures		659
The Product (with 3 Places cut off)		2826,451
Which add to the Logarithm of 1012=		0051805
The Sum is the Logarithm —		6,0054631 ſought.

Problem 3. A *Logarithm* being given, to find the Number belonging to the ſame.

Rule. Seek the next leſs Logarithm to the given one, in the Tables, and its four Figures are the firſt four of the Number required. Then take the Differences of the given Logarithm and the *next leſs*, and alſo the *next greater* and *next leſs*; Add to the *firſt* Difference, ſo many Cyphers as you ſeek Figures more than four. Divide That by the ſecond Difference, and the Quotient annexed to the four Figures firſt found compleats the Number required.

Example. Required the Number of the given *Logarithm* 6,0054631?

The Logarithm *next leſs* is	—	1012	=	3,0051805
The Logarithm *next greater* is	—	1013	=	3,0056094
The Diff. of the given Log. and *next leſs* is			=	2826
The Dif. of the *next leſs*, and *next greater* is			=	4289

Then ſay, As 4289 : 2826 : : 1000 : 659, which annex to the firſt four Figures 1012, they compleat 1012659 the Number ſought for the given *Logarithm*.

Problem 4. To find the *Logarithm* of any *terminate Decimal* Number.

Rule. Seek the *Logarithm* for it as though it were a *whole Number*, and then adapt the *Index* as before taught.

Thus the Logarithm of 1012,659 is 3,0054631, and of ,1012659 = ,9,0054631, &c.

Problem 5. To find the *Logarithm* of a *ſingle Repetend*, or *circulating Digit*.

Rule. To the *tabular Logarithm* of the Digit, add the *Arithmetical Complement* of the Logarithm of 9, the Sum is the *Logarithm* ſought.

Example. Required the *Logarithm* of 6?

To the *Tabular Logarithm* of 6	=	0,7781512
Add the *Arith. Complement* of the Log. 9	=	0,0457575
The Sum is the *Logarithm* ſought of 6	=	0,8239087

In

In this Manner I have calculated the *Logarithms* of all the Nine *Digits perpetually circulating*, and disposed them ready for Use in the annexed *Table*.

Rep. Digits.		*Logarithms.*
$\dot{1}$	=	0,0457575
$\dot{2}$	=	0,3467875
$\dot{3}$	=	0,5228787
$\dot{4}$	=	0,6478175
$\dot{5}$	=	0,7447275
$\dot{6}$	=	0,8239087
$\dot{7}$	=	0,8908555
$\dot{8}$	=	0,9488475
$\dot{9}$	=	1,0000000

Problem 6. To find the *Logarithm* of any pure *Compound Repetend.*

Rule. To the *Tabular Logarithm* of the Number (as terminate,) add the *Arithmetical Complement* of the Logarithms of so many 9's, as are Places of the *Repetend*; the Sum is the Logarithm of the given Repetend.

Example 1. Required the *Logarithm* of the *Compound Repetend* $\dot{2}\dot{4}$?

To the *Tabular Logarithm* of —	24 =	1,3802112
Add the *Arithmetical Complement* of	99 =	0,0043648
The Sum is the *Logarithm* of —	$\dot{2}\dot{4}$ =	1,3845760

Example 2. Required the *Logarithm* of $\dot{3}6,\dot{5}$?

To the *Tabular Logarithm* of —	36,5 =	1,5622929
Add the *Arithmetical Complement* of	999 =	0,0004345
The Sum is the *Logarithm* of —	$\dot{3}6,\dot{5}$ =	1,5627274

Example 3. Required the *Logarithm* of $\dot{3}74\dot{6}$?

To the *Tabular Logarithm* of —	3746 =	3,5735678
Add the *Arithmetical Complement* of	9999 =	0,0000434
The Sum is the *Logarithm* of —	$\dot{3}74\dot{6}$ =	3,5736112

Example 4. Requir'd the *Logarithm* of $\dot{2}00,6\dot{0}$?

To the *Tabular Logarithm* of —	200,60 =	2,3023309
Add the *Arith. Comp.* of the Log. of	99999 =	0,0000043
The Sum is the *Logarithm* of	$\dot{2}00,6\dot{0}$ =	2,3023352

Note. In all the foregoing *Examples* (and in those which follow) the *Indexes* of the *Arithmetical Complements* are omitted; and this must be observed by the Learner in all Operations of this kind.

Problem 7. To find the *Logarithm* of any *mixed Repetend*, either *Single* or *Compound*.

Rule. From the given mix'd *Repetend*, substract its *terminate* Part; Then to the *Logarithm* of the Remainder add the *Arithmetical Complement* of the Logarithm of so many *Nines*, as there are Figures in the *Repetend*, the Sum will be the *Logarithm* sought.

Example 1. Requir'd the *Logarithm* of 2,$\dot{6}$?

From the given *Repetend* — 2,$\dot{6}$
Substract the *terminate* Part — 2

Then to the *Logarithm* of — 2,4 = 0,3802112
Add the *Arithm. Comp.* of the Logar. of 9 = 0,0457575

The Sum is the *Logarithm* of — 2,$\dot{6}$ = 0,4259687

Example 2. Requir'd the *Logarithm* of 57,2$\dot{3}$?

From the given *Repetend* — 57,2$\dot{3}$
Substract the *terminate* Part — 572

Then to the *Logarithm* of — 51,51 = 1,7118915
Add the *Arith. Compl.* of the Log. of 9 = 0,0457575

The Sum is the *Logarithm* of 57,2$\dot{3}$ = 1,7576490

Example 3. Requir'd the *Logarithm* of 2,7$\dot{5}\dot{3}$?

From the given *Repetend* — 2,7$\dot{5}\dot{3}$
Subduct the *terminate* Part — 27

Then to the *Logarithm* of — 2,726 = 0,4355258
Add the *Arith. Compl.* of the Log. of 99 = 0,0043648
The Sum is the *Logarithm* of — 2,7$\dot{5}\dot{3}$ = 0,4398906

Example

Example 4. Requir'd the *Logarithm* of 725,$\dot{6}$?

From	—	—	725,$\dot{6}$	
Subduct	—	—	7	
To the Logarithm of		—	724,9	= 2,8602781
Add the *Arit. Compl.* of the Log. of			999	= 0,0004345
The Sum is the *Logarithm* of		—	725,$\dot{6}$	= 2,8607126

Example 5. Requir'd the *Logarithm* of 26892$\dot{7}$?

From	—	—	26892,$\dot{7}$	
Substract	—	—	26	
To the Logarithm of		—	26890,1	= 4,4295924
Add the *Arit. Comp.* of the Log. of			9999	= 0,0000434
The Sum is the *Logarithm* of		—	26892,$\dot{7}$	= 4,4296358

In the like Manner may the *Logarithm* of any other *Mix'd Repetend* be found, so far as the *Canon* of *Logarithms* (you use) will permit.

Problem 8. Between *two Numbers* given, to find any Number of *mean Proportionals* required.

Rule. Substract the *Logarithm* of the *lesser Number* from the *Logarithm* of the *greater*; *divide* the Remainder by a Number *greater by one* than the *Number* of *Means sought*; this Quotient add to the *Logarithm* of the *lesser Number*; the *Sum* is the *Logarithm* of the *first Mean*; to which the said *Quotient* is to be added again for the *Logarithm* of the *second Mean*; and thus proceed for as many *Means* as you please.

Example. Between 8 and 56 to find *four Mean Proportional* Numbers.

The Logarithm of 56 is	—	1,7481880
The Logarithm of 8 is	— —	0,9030900
The Remainder or Difference is	—	0,8450980
One fifth Part (for four *Means*) is	—	0,1690196
To which add the Logarithm of 8	—	0,9030900
The Sum is the Log. of the *first Mean*	11,809 =	1,0721096
To which add again	— —	0,1690196
The Logarithm of the *second Mean*	27,42 =	1,2411292
Add again	— —	0,1690196
The Logarithm of the *third Mean* is	25,71 =	1,4101488
Add again	— —	0,1690196
Logarithm of the *fourth* and *last Mean*	37,94 =	1,5791684

This Problem I have chiefly inserted for their Sakes who would hereby learn to calculate *Tables* of *Compound Interest*; The Numbers in the Table of Amounts of 1 *l.* being only Mean Proportionals between the Logarithm of Rate and the last Year's Amount in the Table.

Multiplication of all Kinds of Decimals by Logarithms.

Rule. { To the *Logarithm* of the *Multiplicand*,
Add the *Logarithm* of the *Multiplier*;
The Sum is the *Logarithm* of the Product.

Example 1.	Multiply	—	12,4 =	1,0934217
	By	—	3,6 =	0,5563025
	The Product	—	44,64 =	1,6497242
Example 2.	Multiply	—	36,5 =	1,5622929
	By	—	,00019 =	,6,2787536
	Product	—	,006935 =	,7,8410465

Example

Example 3.	Multiply	—	,762	= ,9,8819550
	By	—	570	= 2,7558748
	Product	—	434,34	= 2,6378298
Example 4.	Multiply	—	,0097	= ,7,9867717
	By	—	,00021	= ,6,3222193
	Product	—	,000002037	= ,4,3089910
Example 5.	Multiply	—	26,4	= 1,4216039
	By	— —	6	= 0,8239087
	Product	—	176	= 2,2455126
Example 6.	Multiply	—	2,73	= 0,4361626
	By	— —	2,6	= 0,4259687
	Product	—	7,28	= 0,8621313
Example 7.	Multiply	—	,473	= ,9,6748611
	By	—	63	= 1,8037053
	Product	—	30,1	= 1,4785664
Example 8.	Multiply	—	57,24	= 1,7576996
	By	—	2,753	= 0,4398906
	Product	—	177,61236	= 2,1975902
Example 9.	Multiply	—	6,	= 0,8239087
	By	— —	,5	= ,9,7447275
	Product	—	3,70	= 0,5686362
Example 10.	Multiply	—	36,5	= 1,5627274
	By	—	2,4	= 0,3845760
	Product		88,5734218 &c.	= 1,9473034
Example 11.	Multiply	—	21,23	= 1,3265407
	By	—	42,0	= 1,6234581
	Product	—	900,718	= 2,9499988

Example

Example 12.	Multiply —	,00420	=	,7,6234581
	By —	,00008	=	,5,9488475
	Product	,0000003735 &c.	=	,3,5723056

Division of all Kinds of Decimals by Logarithms.

Rule { From the *Logarithm* of the *Dividend*,
Substract the *Logarithm* of the *Divisor*;
The *Remainder* is the *Logarithm* of the *Quotient*.

Example 1.	Divide —	44,64	=	1,6497242
	By —	12,4	=	1,0934217
	The Quotient —	36	=	0,5563025
Example 2.	Divide —	310	=	2,4913617
	By —	4,275	=	0,6309361
	Quotient —	72,51457	=	1,8604256
Example 3.	Divide —	434,34	=	2,6378298
	By —	,762	=	,9,8819550
	Quotient —	570	=	2,7558748
Example 4.	Divide —	,006935	=	,7,8410465
	By —	36,5	=	1,5622929
	Quotient —	,00019	=	,6,2787536
Example 5.	Divide —	,000002073	=	,4,3089910
	By —	,00021	=	,6,3222193
	Quotient —	,0097	=	,7,9867717
Example 6.	Divide —	176	=	2,2455126
	By —	6,	=	0,8239087
	Quotient —	26,4	=	1,4216039
Example 7.	Divide —	7,28	=	0,8621314
	By —	2,6	=	0,4259687
	Quotient —	2,73	=	0,436162

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Example 8.	Divide	—	30,1	=	1,4785664
	By	—	63,	=	1,8037053
	Quotient	—	,473	=	,9,6748611
Example 9.	Divide	—	177,61236	=	2,1975902
	By	—	2,753	=	0,4398906
	Quotient	—	57,24	=	1,7576996
Example 10.	Divide	—	3,70	=	0,5686362
	By	— —	,5	=	,9,7447275
	Quotient	—	6,	=	0,8239087
Example 11.	Divide	—	900,718	=	2,9499988
	By	—	42,0	=	1,6234581
	Quotient	—	21,23	=	1,3265407
Example 12.	Divide		,000000243308	=	,3,3861506
	By	—	,0002172	=	,6,3370052
	Quotient		,00111,98	=	,7,0491454

The Golden Rule in Decimals by Logarithms.

Example 1. Direct *Proportion.*

If 2 *C.* 3 *qr.* 21 *lb.* of *Sugar*	—	2,9375	=	0,4679778
Cost 6 *l.* 1 *s.* 8 *d.*	—	6,083	=	0,7841316
What costs 12 *C.* 2 *qrs.*	—	12,5	=	1,0969100
				1,8810416
Answer, 25 *l.* 17 *s.* $8\frac{3}{4}$ *d.*	—	25,8864583	=	1,4130638

Example 2. Inverse *Proportion.*

If *Wheat* be 6 *s.* 4 *d.* *per Bushel,*	—	6,3	=	0,8016325
And the *Pen. white Loaf* weigh $7\frac{3}{4}$ *oz.*		7,75	=	0,8893017
What must it weigh, when				1,6909342
Wheat is 3 *s.* 10 *d.* *per Bushel?*	—	3,83	=	0,5835766
Answer, 12 *oz.* 16 *pwt.* 2 *gr.* =		12,8043 &c.	=	1,1073576

Note,

Note, Either of those Questions may (and that most conveniently) be wrought at once, *viz.* by *One Addition* of the two Logarithms now added, and the *Arithmetical Complement* of that *Logarithm* that is substracted; for to *substract* a *Logarithm*, or *add* its *Arithmetical Complement*, produces the same Effect, or is the same Thing.

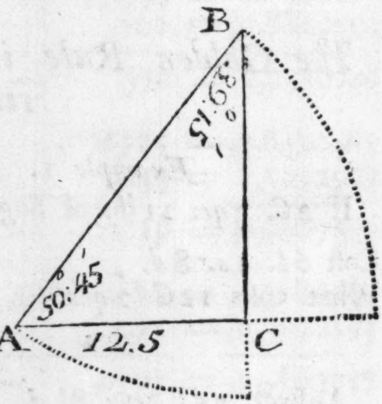
Example 3. Direct *Proportion at one Operation.*

If ½ *C.* of *Tobacco*	— —	,5	= 0,3010299
Cost 4*l.* 12*s.* 8*d.*	—	4,63	= 0,6658935
What cost 7 Pounds?	—	,0625	= ,8,7958800
Answer, 00*l.* 11 *s.* 7 *d.*	—	,57916	= ,9,7628034

Example 4. Inverse *Proportion at one Operation.*

If 8 *Rods* in *Width,*	—	8	= 0,9030900
Require 20 in *Length,* to make an *Acre*;		20	= 1,3010300
What Length does 12,5 *Rods* in *Width* require for an *Acre?*		12,5	= 8,9030899
Answer, 12,8 Rods	—	12,8	= 1,1072099

This Method with working at once with Arithmetical Complement is to be advised to the expert *Geometrician* in his *Trigonometrical Calculations,* as much the best.



Example 5.

As the Sine of the Angle *A B C*	$39^\circ : 15'$	= 0,1987985
Is to the Side given *A C* —	12,5	= 1,0969100
So is the Sine of the Angle *B A C*	$50^\circ : 45'$	= 9,8889612
To the Side sought *B C* —	15,3	= 1,1846697

Extraction

Extraction of Roots in Decimals by Logarithms.

To extract the Root of any Number, do thus;

If it be the	*Square* Root	Divide the Logarithm of the given Number by —	2
	Cube Root		3
	Biquadrate Root		4
	Surſolid Root		5

Then the ſecond, third. fourth, *&c.* Part of the Logarithm thus divided, ſhall be the Logarithm of the Root ſought.

Example 1. What is the *Square* Root of the Number 2830,24?

The Logarithm thereof is — 3,4518232
Half of which, is the Logarithm of 53,2 = 1,7259116
The *Root* ſought.

Example 2. Requir'd the *Square Root* of 13,2?

The Logarithm of the given Number 13,2 = 1,1205739
Half, is the Logar. of the Root 3,6331 *&c.* = 0,5602869

Example 3. Requir'd the *Square Root* of 14,6?

The *Logarithm* of — 14,6 = 1,1663314
Half, is the *Logar.* of the Root 3,8297 *&c.* = 0,5831657

Example 4. What is the *Cube Root* of 1,728?

The *Logarithm* of — — 1,728 = 0,2375437
One Third, is the Log. of the *Cube* Root 1,2 = 0,0791812

Example 5. What is the *Biquadrate* Root of 175,63?

The *Logarithm* of — 175,63 = 2,2445526
One 4th, is the Log. of the R. 3,64019 *&c.* = 0,5611381

Example 6. Quere the *Surſolid* Root of 31,25?

The *Logarithm* of — — 31,25 = 1,4948500
One Fifth, is the Log. of the Root 1,990 *&c.* = 0,2989700

Note ; When the *Index* is *Negative*, add to it 10 for the *Square* Root, 20 for the Cube ; 30 for the Biquadrate ; 40 for the Surſolid Root, *&c.* and then divide as before ; as in the following *general Example.*

Example 7. What are the ſeveral Roots of ,27589 ?

The Logarithm of	— ,27589	=	,9,4407132
An Half, is the *Square* Root	,52523 *&c.*	=	,9,7203566
A Third, is the *Cube* Root	,6509 *&c.*	=	,9,8135710
A Fourth, is the *Biquadrate* Root	,7247 *&c.*	=	,9,8601783
A Fifth, is the *Surſolid* Root	,7729 *&c.*	=	,9,8881426

CHAP. XII.

The Uſe of DECIMALS *in* Algebra ; *exemplified in the Reſolution of thirty four Select, Pleaſant, and Uſeful Algebraick Queſtions.*

NOTWITHSTANDING *Algebra* has the Glory and Reputation of being one of the Topmoſt Branches of the Tree of human Arts and Sciences ; yet muſt it be acknowledged that (as ſublime and uſeful a Science as it is) it would anſwer no great Purpoſe of practical Knowledge, were not the Art of *Decimal Arithmetick*, on every Occaſion, called in to its Aſſiſtance. *Vulgar Fractions* and *Algebra* together, may be view'd as the *Blind leading the Blind* ; And Whole Numbers miſerably help the *lame Dog over the Stile.*

'Tis *Decimals* therefore which in all Caſes (not *ſtudied* and *ſtated on purpoſe*, but) which contingently or occaſionally happen, can only ſpeak out plainly and intelligibly the recluſe Meaning of an *Algebraick Equation* or *Theorem.*

This

This I ſhall make appear by the Reſolution of the moſt curious pleaſant and uſeful Queſtions, which I have ſelected from the beſt *Algebraick Authors* extant?

Queſtion 1. The Sum ($= s = 67$) of any two Numbers, and their *Difference* ($= d = 30$) being given to find thoſe Numbers.

Let	1	$a =$ the Greater, and $e =$ the Leſſer Number.
Then	2	$a + e = s = 67$
And	3	$a - e = d = 30$
2 + 3	4	$2a = s + d = 97$
4 ÷ 2	5	$a = \frac{s+d}{2} = 48,5$ the greater Numb. ⎫
2 − 3	6	$2e = s - d - 37$ ⎬ ſought.
6 ÷ 2	7	$e = \frac{s-d}{2} = 18,5$ the leſſer Number ⎭

Queſtion 2. The *Sum* ($= s = 15$) and *Product* ($= p = 15$) of any two Numbers given, to find thoſe Numbers?

Then	1	$a + e = s = 15$ ⎫ Quere *a*, and *e*.
	2	$ae = p = 15$ ⎭
1 ⊙ 2	3	$aa + 2ae + ee = ss = 225$
2 × 4	4	$4ae = 4p = 60$
3 − 4	5	$aa - 2ae + ee = ss - 4p = 165$
5 ɯ 2	6	$a - e = \sqrt{ss - 4p} = 12,845$ &c.
1 + 6	7	$2a = s + \sqrt{ss - 4p} = 27,845$
7 ÷ 2	8	$a = \frac{s + \sqrt{ss-4p}}{2} = 13,921$ &c. Great. ⎫ Numb.
1−6÷2	9	$e - \frac{s - \sqrt{ss-4p}}{2} = 1,038$ &c. Leſſer ⎭ ſought.

Question 3. What two Numbers are thoſe whoſe *Sum* is $= 40 = s$, and the *Greater* divided by the *Leſſer* ſhall quote $50 = q$?

Here	1	$a + e = s = 40$	Quere a, and e?
	2	$\frac{a}{e} = q = 50$	
$2 \times e$	3	$a = qe = 50e$	
$1 - 3$	4	$e = s - qe = 40 - 50e$	
$4 + qe$	5	$qe + e = s = 40$	
$5 \div \overline{q+1}$	6	$e = \frac{s}{q - 1} = 0,7843$ &c. Leſſer	Numb. ſought.
$1 - 6$	7	$a = \frac{qs}{q + 1} = 39,2156$ &c. Greater	

Question 4. What two Numbers are thoſe, whoſe *Sum* is $= 8 = s$, and the *Sum* of their *Squares* $37\frac{5}{9} = z$?

Here	1	$a + e = s = 8$	Quere a, and e?
	2	$aa + ee = z = 37,\dot{5}$	
1 ⊙ 2	3	$aa + 2ae + ee = ss = 64$	
$3 - 2$	4	$2ae = ss - z = 26,\dot{4}$	
$2 - 4$	5	$aa - 2qe + ee = 2z - ss = 11,\dot{1}$	
5 ɯ 2	6	$a - e = \sqrt{2z - ss} = 3,\dot{3}$	
$1 + 6$	7	$2a = s + \sqrt{2z - ss} = 11,\dot{3}$	
$7 \div 2$	8	$a = \frac{s + \sqrt{2z - ss}}{2} = 5,\dot{6}$ Greater	Number ſought.
$\overline{1 - 6} \div 2$	9	$e = \frac{s - \sqrt{2z - ss}}{2} = 2,\dot{3}$ Leſſer	

Question 5. If the *Sum* of any two Numbers be $50 = s$, and the *Difference* of their *Squares* be $273 = x$, What are the ſaid Numbers?

Here	1	$a + e = s = 50$	Quere a, e?
	2	$aa - ee = x = 273$	
$2 \div 1$	3	$a - e = \frac{x}{s} = 5,46$	
$1 + 3$	4	$2a = \frac{ss + x}{2s} = 55,46$	
$\overline{4} \div 2$	5	$a = \frac{ss + x}{2s} = 27,73$ the Greater	Number ſought.
$\overline{1 - 3} \div 2$	6	$e = \frac{ss - x}{2s} = 22,27$ the Leſſer	

Question

Question 6. Suppose the *Difference* of two Numbers be $30 = d$, and the *Product* $512 = p$. *Quere* those Numbers?

Here	1	$a - e = d = 30$	Quere a, e?
	2	$ae = p = 512$	
1 @ 2	3	$aa - 2ae + ee = dd = 900$	
2 × 4	4	$4ae = 4p = 2048$	
3 + 4	5	$aa + 2ae + ee = dd + 4p = 2948$	
5 ω 2	6	$a + e = \sqrt{dd + 4p} = 54,29$	
6+1÷2	7	$a = \frac{d + \sqrt{dd+4p}}{2} = 42,18$ Greater	Numb. sought.
6—÷2	8	$e = \frac{\sqrt{dd+4p} - d}{2} = 12,18$ Lesser	

Question 7. Suppose the *Difference* of two Numbers be the same with the *Quotient* of the *Greater* divided by the *Lesser*, viz. $= 18$. *Quere* those Numbers?

Here	1	$a - e = d = 18$	Quere a, e.
	2	$\frac{a}{e} = q = 18$	
2 × e	3	$a = qe = 18e$	
1 + e	4	$a = d + e = 18 + e$	
3, 4	5	$qe = d + e = 18 + e$	
5 − e	6	$qe - e = d = 18$	
6 ÷ q—1	7	$e = \frac{d}{q-1} = 1,058$ the Lesser	Number sought.
1 − 7	8	$a = \frac{qd}{q-1} = 19,058$ the Greater	

Question 8. The *Difference* of two Numbers $= 5 = d$, and the *Sum* of their *Squares* $= 52 = z$, being given; to find those Numbers.

Here	1	$a - e = d = 5$	Quere a, e?
	2	$aa + ee = z = 55$	
1 @ 2	3	$aa - 2ae + ee = dd = 25$	
2 − 3	4	$2ae = z - dd = 30$	
2 + 4	5	$aa + 2ae + ee = 2z - dd = 85$	

5 ww 2	6	$a+e=\sqrt{2z-dd}=9,219$	
$1+\overline{6}\div 2$	7	$a=\dfrac{d+\sqrt{2z-dd}}{2}=7,109$ Greater	Number sought.
$6-\overline{1}\div 2$	8	$e=\dfrac{\sqrt{2z-dd}-d}{2}=2,109$ Lesser	

Question 9. The *Difference* of any two Numbers $=12=d$, and the Difference of their Squares $=279=x$, being given; to find those Numbers?

Here	1	$a-e=d=12$	Quere a, e?
	2	$aa-ee=x=279$	
$2\div 1$	3	$a+e=\dfrac{x}{d}=23,25$	
$1+\overline{3}$	4	$2a=\dfrac{dd+x}{d}=35,25$	
$4\div\overline{2}$	5	$a=\dfrac{dd+x}{2d}=17,625$ Greater	Number sought.
$3-\overline{5}$	6	$e=\dfrac{x-dd}{2d}=5,625$ Lesser	

Question 10. The *Product* of any two Numbers being $=573=p$, and the *Quotient* of the *Greater* divided by the *Lesser* $=75$; *Quere* those Numbers?

Here	1	$ae=p=573$	Quere a, e?
	2	$\dfrac{a}{e}=q=75$	
1×2	3	$aa=qp=42975$	
3 ww $\overline{2}$	4	$a=\sqrt{qp}=207,304$ &c. Greater	Number sought.
$1\div 2$	5	$ee=\dfrac{p}{q}=7,64$	
5 ww 2	6	$e=\sqrt{\dfrac{p}{q}}=2,764$ &c. Lesser	

Question

Question 11. The *Product* of any two Numbers $=100=p$, and the *Sum* of their *Squares* $=1000=z$; *Quere* those Numbers?

Here	1	$ae = p = 100$	Quere a, e?
	2	$aa + ee = z = 1000$	
1×2	3	$2ae = 2p = 200$	
$2 + 3$	4	$aa + 2ae + ee = z + 2p = 1200$	
$4\ \omega\ 2$	5	$a + e = \sqrt{z + 2p} = 34{,}641$	
$2 - 3$	6	$aa - 2ae + ee = z - 2p = 800$	
$6\ \omega\ 2$	7	$a - e = \sqrt{z - 2p} = 28{,}284$	
$5 + 7 \div 2$	8	$a = \frac{\sqrt{z+2p} + \sqrt{z-2p}}{2} = 31{,}462$ Gr.	Numb. sought.
$5 - 7 \div 2$	9	$e = \frac{\sqrt{z+2p} - \sqrt{z-2p}}{2} = 3{,}178$ Les.	

Question 12. The *Product* of any two Numbers $=10=p$, and the *Difference* of their *Squares* $=20=x$, being given; thence to find those Numbers?

Here	1	$ae = p = 10$	Quere a, e?
	2	$aa - ee = x = 20$	
$1 \odot 2$	3	$aaee = pp = 100$	
$2 \odot 2$	4	$aaaa - 2aaee + eeee = xx = 400$	
3×4	5	$4aaee = 4pp = 400$	
$4 + 5$	6	$aaaa + 2aaee + eeee = xx + 4pp = 800$	
$6\ \omega\ 2$	7	$aa + ee = \sqrt{xx + 4pp} = 28{,}284$	
$2 + 7$	8	$2aa = x + \sqrt{xx + 4pp} = 48{,}284$	
$8 \div 2$	9	$aa = \frac{x + \sqrt{xx+4pp}}{2} = 24{,}142$	
$9\ \omega\ 2$	10	$a = \sqrt{\frac{x + \sqrt{xx+4pp}}{2}} = 4{,}917$ &c. G.	N.
$7 - 2 \div 2\ \omega\ 2$	11	$e = \sqrt{\frac{\sqrt{xx+4pp} - x}{2}} = 2{,}035$ &c. L.	

Question 13. The *Quotient* of any two Numbers $=10=q$, and the *Sum* of their *Squares* $=57=z$, being given; to find those Numbers?

Here	1	$\frac{a}{e}=q=10$	Quere a, e?
	2	$aa+ee=z=57$	
$1\times e$	3	$a=qe=10e$	
3 ⊙ 2	4	$aa=qqee=100ee$	
$2-4$	5	$ee=z-qqee=z-100ee$	
$4+qqe$	6	$qqee+ee=z=57$	
$6\div qq+1$	7	$ee=\frac{z}{qq+1}=,5643$	
$2-7$	8	$aa=\frac{qqz}{qq+1}=56,43$	
8 ⊙ 2	9	$a=\sqrt{\frac{qqz}{qq+1}}=7,512$ &c. Great.	Numb. sought.
7 ⊙ 2	10	$e=\sqrt{\frac{z}{qq+1}}=,7512$ &c. Less.	

Question 14. Suppose the *Quotient* of two Numbers $=20=q$, and the *Difference* of their *Squares* $=100=x$; Thence to find the Numbers.

Here	1	$\frac{a}{e}=q=20$	Quere a, e?
	2	$aa-ee=x=100$	
$1\times e$	3	$a=qe=20e$	
3 ⊙ 2	4	$aa=qqee=400ee$	
$2+ee$	5	$aa=x+ee=100+ee$	
4, 5	6	$qqee=x+ee=100+ee$	
$6-ee$	7	$qqee-ee=x=100$	
$7\div qq-1$	8	$ee=\frac{x}{qq-1}=,2506$ &c.	
$2+8$	9	$aa=\frac{qqx}{qq-1}=100,2506$, &c.	
9 ɯ 2	10	$a=\sqrt{\frac{qqx}{qq-1}}=10,012$ &c. Greater	Numb. sought.
8 ɯ 2	11	$e=\sqrt{\frac{x}{qq-1}}=0,5006$ &c. Lesser	

Question

Question 15. Suppose the *Sum* of the *Squares* of any two Numbers $= 300 = z$, and the *Difference* of the said *Squares* $= 250 = x$, to find the Numbers?

Here	1	$aa + ee = z = 300$	Quere a, e?
	2	$aa - ee = x = 250$	
$1 + 2$	3	$2aa = z + x = 550$	
$3 \div 2$	4	$aa = \frac{z + x}{2} = 275$	
$4 \; \omega \; 2$	5	$a = \sqrt{\frac{z + x}{2}} = 16,583$ &c. Greater	Number sought.
And	6	$e = \sqrt{\frac{z - x}{2}} = 5$ &c. Lesser	

Thus any two of those *Six* Things (viz. *Sum*, *Difference*, *Product*, *Quotient*, *Sum* of the *Squares*, and *Difference* of the *Squares*, of any two Numbers) being given; 'twill be easy to find the Numbers themselves and all the other Particulars.

I have chose to give the *Analytical* Process of the Work of each Question at large, that the Young Student may see the Manner of Investigating *Theorems*; and by viewing the frequent Divisions and Extractions, may the more clearly perceive the great Use, or rather, the absolute Necessity of *Decimals*, in order to express the Equations in Numbers.

But in those Questions which follow, I have only exhibited the *Theorem* or Equation which answers them, and given the Solution of each in *Decimal* Numbers.

Question 16. There are two Numbers a, e. The *Sum* of their *Squares* is $aa + ee = z = 97$ The *Greater* is to the *Less* as $12 = b$ is to $7 = d$ That is $a : e :: b : d$ Quere a, e?

Theorem. $e = \sqrt{\frac{zdd}{dd + bb}}$ } to be resolved.

First Multiply the *Sum* of the *Squares*		$z = 97$	
By the *Square* of d	—	$dd = 49$	
The Product is	—	zdd 4763	Dividend.
Then to the *Square* of d	—	$dd = 49$	
Add the *Square* of b	—	$+ bb = 144$	
The Sum of both is	—	$dd + bb = 193$	Divisor.

By which Divide zdd, The Quotient will be $\Big\}$ — $\dfrac{zdd}{dd+bb} = 24,679$ &c.

The *Square Root* of which is $\sqrt{\dfrac{zdd}{dd+bb}} = e = 4,966$

Then as $\left\{\begin{matrix} d : & e & :: b : & a \\ 7 : & 4,966 & :: 12 : & 8,513 \end{matrix}\right\}$ Thus $\left\{\begin{matrix} a = 8,513 \text{ \&c.} \\ e = 4,966 \text{ \&c.} \end{matrix}\right.$

Question 17. There are three Numbers in *continued* Proportion, *viz.* — — a. m. e.

The *Sum* of the *Extreams* — $a + e = 37\frac{2}{5} = s$

And the *Mean* — $m = 13$. Quere a, e?

Theorem. $a = \dfrac{s + \sqrt{ss - 4mm}}{2}$ $\Big\}$ to be solved.

First, The *Square* of the *Sum* is —	ss =	1398,76
And the *Square* of $m \times 4$ is —	$4mm$ =	676
Which substracted, there remains,	$ss - 4mm$ =	722,76
The *Square Root* whereof is	$\sqrt{ss - 4mm}$ =	26,885
To which add the *Sum* of the *Extreams*	s =	37,4
That Sum is —	$s + \sqrt{ss - 4mm}$ =	64,285
The half of that is the *First Number*	a =	32,142
Then as $\left\{\begin{matrix} a & : m :: m : \\ 32,142 & : 13 :: 13 : \end{matrix}\right\}$	e =	5,257
The Sum of which is the Proof	$a + e$ =	37,4

Question 18. There are three Numbers in *continued* Proportion, *viz.* — $a : m :: m : e$

Their *Sum* is — $a + m + e = s = 31\frac{2}{7}$ Quere a, m, e?

And the *Sum* of their *Sq.* $aa + mm + ee = z = 763$

Theorem. $m = \dfrac{ss - z}{2s}$ $\Big\}$ to be solved.

First,

First, from the Square of their *Sum* $ss = 978{,}7955$ &c.
Substract the *Sum* of their *Squares* $z = 763$

The *Remainder* is — $ss - z = 215{,}7955$
Which divided by *twice* the *Sum* $2s = 62{,}571428$
Gives the *Quotient* — $m = 3{,}446$ &c.
Then $31{,}285714 - 3{,}446 = 27{,}839 = a + e$
Which may be found as in the *Theorem* of the last Question.

Question 19. Suppose *three* Numbers a, b, c in *Musical* Proportion, *viz.* As $a : c :: a - b : b - c$, and any Two of them being given, to find the Third.

Theorem 1. $a = \frac{cb}{2c - b}$ Finds a, if b, c, be given.

Theorem 2. $b = \frac{2ac}{c + a}$ Finds b, if a, c, be given.

Theorem 3. $c = \frac{ba}{2a - b}$ Finds c, if a, b, be given.

Suppose $b = 13$, and $c = 10$ To find a?
Multiply — $b = 13$
By — — $c = 10$

The Product — $cb = 130$
Divide by twice c, less b, $2c - b = 7$) $130 = cb$ ($1{,}857142$.

The Quotient is $a = 18{,}57142$ the Number sought. And so for either of the other.

Question 20. Suppose *four Numbers*, a, b, c, d, in *Musical* Proportion, *viz.* $a : d :: a - b : c - d$; and any *Three* of these given, to find the fourth.

Theorem 1. $a = \frac{db}{2d - c}$ Finds a, if b, c, d, be given.

Theorem 2. $b = \frac{2da - ca}{d}$, Finds b, if a, d, c, be given.

Theorem 3. $c = \frac{2da - db}{a}$, Finds c, if a, b, d, be given.

Theorem 4. $d = \frac{ca}{2a - b}$, Finds d, if a, b, c, be given.

Question 21. Suppose it was required to divide any Number ($20 = s$) into *Extream* and *Mean Proportion*; That is, into two such Parts, a, and e, that $aa = ae + ee = se$. Quere a, e?

Theorem. $a = \sqrt{ss + \frac{1}{4}ss} - \frac{1}{2}s$

Square the given Number	—	$ss = 400$
Add thereto $\frac{1}{4}$ of the said *Square*	—	$\frac{1}{4}ss = 100$
The *Sum* is	—	$ss + \frac{1}{4}ss = 500$
The *square Root* thereof is	—	$\sqrt{ss + \frac{1}{4}ss} = 22{,}3606$ &c.
From which substract $\frac{1}{2}$ the given Numb.		$\frac{1}{2}s = 10$
There Remains the *Greater Part*	—	$a = 12{,}3606$ &c.
Which substracted from the given Numb. There remains the *Lesser Part*		$e = 7{,}6393$ &c.

Note, 'Tis impossible to answer this Question in *Whole Numbers*.

Question 22. What is the *Canon* or *Theorem* for *Extracting* the *square Root*?

Suppose $a + e =$ Root; Then the *Canon* is this, *viz.* $aa + 2ae + ee =$ Square.

Extract the Sq. Root of $\dot{6}5\dot{5},\dot{3}6$	$= aa + 2ae + ee$	
From the Numb. subst. 400	$= aa$ $(a = 20$	
There Remains — $255{,}36$	$= 2ae + ee$	$a = 20$
Divide that by $2a = 40$ $)255{,}36($ $5 = e$		$e = 5$
Then substract — 225	$= 2ae + ee$	$a + e = 25$ $= a$, anew.
There Remains — $30{,}36$	$= 2ae + ee$ anew.	
Which divid by. $2a = 50)30{,}36(,6 = e$ anew		$25 = a$
Then again substract $30{,}36$	$= 2ae + ee$	$,6 = e$
....		$25{,}6 = a + e$

Question 23. What is the *Canon* or *Theorem* for extracting the *Cube Root*?

Suppose $a + e =$ Root, Then $aaa + 3aae + 3aee + eee$ is the *Canon* or Rule for Extracting any *Cube Root*.

Requir'd

Requir'd the *C. Root* of	$\dot{1}95\dot{3},12\dot{5}$	$= aaa + 3aae + 3aee + eee$	
Substract the Cube	1000	$= aaa$	($a = 10$. 1st.
There remains —	$953,125$	$= 3aae + 3aee + eee$	
Div. by $3aa + 3a = 330$)	$953,125$(	$2 = e$	
Then —	600	$= 3aae$	
And — —	120	$= 3aee$	
Lastly —	8	$= eee$	
The Sum of all is	728	$= \left\{ \begin{array}{l} 3aae + 3aee + eee \text{ subst.} \\ \text{from the Remainder.} \end{array} \right.$	
There remains —	$225,125$	$= 3aae + 3aee + eee$, anew.	
Div. by $3aa + 3a = 468$)	$225,125$(	$,5 = e$	Then $a = 10$
Then — —	$216,000$	$= 3aae$	$e = 2$
And — —	$9,000$	$= 3aee$	$a + e = 12$
Lastly —	$0,125$	$= eee$	$= a$, anew.
The Sum of all is	$225,125$	$= 3aae + 3aee + eee$ to be (subst. from the last Rem.	
	......		

Hence the Root is $10 + 2 + ,5 = 12,5$

Note, From hence appears the *Rationale* of the Method of *extracting* the *Cube Root*; for the Precepts there are only the Words expressing the order and *combination* of the *Symbols* of this *Canon*.

Question 24. What is the Manner of *Extraction* by *Converging Series*, or Theorems raised thereby?

There are several Kinds of *Converging Series* for this Purpose, but Mr. *Ward*'s I take to be the best, which is thus.

Let $aa = G$. Quere a?

Let	1	$r + e = a$ The Root sought.
1 ⊙ 2	2	$rr + 2re + ee = aa = G$.
2 — r	3	$2re + ee = G - rr = D$ The Dividend.
Then	4	$\frac{D}{2r + e} = e$, or, $\frac{D}{r + \frac{1}{2}e} = e$ $\left\{ \begin{array}{l} \textit{Theorems} \text{ for} \\ \text{the Sq. Root.} \end{array} \right.$

Let

Let $aaa = G$. Quere a.

Put	1	$r + e = a$ The Root sought
1 © 3	2	$rrr + 3rre + 3ree + eee = aaa = G$
2 — rrr	3	$3rre + 3ree + eee = G - rrr$
$3 \div 3r$	4	$re + ee + \frac{eee}{3r} = \frac{G - rrr}{3r} = D$, the Dividend.
Reject $\frac{eee}{3r}$	5	$re + ee = D$ $\frac{eee}{3r}$ being of small or no Value.
$5 \div$	6	$\frac{D}{r + e} = e$. The *Theorem* for the *Cube*.

If $aaaa = G$ Then $\frac{G - r^4}{2rr} = D$ and $\frac{D}{2r + 3e} = e$
The Theorem for the Biquadrate Root.

If $aaaaa = G$. Then $\frac{G - r^5}{5r^3} = D$. And $\frac{D}{r + 2e}$
The *Theorem* for the *Sursolid Root*.
And in the same Manner proceed for any other Root.

Note; To work by these Theorems, r must be taken less than the Root; otherwise, if r be taken *greater* than the *true* Root, it will be $rr - G$ instead of $G - rr$, and $\frac{D}{2r - e} = e$ and the same in other *Theorems* for the other Roots.

After the same Manner you raise *Theorems* for all kind of *adfected Equations*.

Suppose $aaa + 24a = 587914$ Quere a?

Put	1	$r + e = a$ Put $r = 80$ } The powers of e above ee are all to be rejected.
1 © 3	2	$rrr + 3rre + 3ree = aaa$
1 × 24	3	$24r + 24e = 24a$
2 in Numb.	4	$512000 + 19200e + 240ee = aaa$
3 in Numb.	5	$1920 + 24e = 24a$
4 + 5	6	$513920 + 19224e + 240ee = 587914$
5 − 513920	7	$19224e + 240ee = 73994$
$7 \div 240$	8	$80,1e + ee = 308,31 = D$ the Dividend
$8 \div$	9	$e = \frac{D}{80,1 + e}$ See the Operation.

80,1)

$80,1)\ 308,31 = D\ (3,7 = e$
$+e = 3$ $\quad 2493$
1 Divisor $83,1)\ 59,01$
$+e = ,7$ $\quad 5866$
2 Divisor $83,8)\ \cdot\cdot 35$

$$\left\{\begin{array}{l} r = 80 \\ e = 3,7 \\ r + e = 83,7 = a \end{array}\right.$$

Here 83,7 is a new r for a *second Operation*; but being involved will be found too big, or *greater* than the *true Root*; Therefore it must be made $r - e = a$ the Root.

Thus	1	$r - e = a$ the Root sought.
1 ⊙ 3	2	$rrr - 3rre + 3ree = aaa$
1 × 24	3	$24r - 24e = 24a$
2 in Numb.	4	$586376,253 - 21017,07e + 251,1ee = aaa$
3 in Numb.	5	$2008,8 - 24e = 24a$
4 + 5	6	$588385,053 - 21041,07e + 251,1ee = 587$
6 − 587914	7	$21041,07e - 251,1ee = 471,053$ (914
7 ÷ 251,1	8	$83,7955e - ee = 1,87595778 = D$
8 ÷	9	$e = \dfrac{D}{83,7955 - e}$

Operation 2.

$83,7955)\ 1,87595778(\ ,022392736$
$-e = 02$ $\quad 1,675510$
1 Divisor $83,7755)\ ,2001477$
$-e = ,002$ $\quad 1675470$
2 Divisor $83,7735)\ ,03290078$
$-e = ,0003$ $\quad ,02513196$
3 Divisor $83,7732$ $\quad ,00776882$
753958
22923
16754
6168
5864
304
251
53
49
$\cdot 4$ &c.

From hence 'tis sufficient to work by Contracted Division to find the Remainder of the Root.

Now $r = 83,7$
And $e = 0,022392736$

Then $r - e = 83,677607264 = a$
The Root of 587914 requir'd

And

And if this Root be not exact enough, it may be made a New r, to work as before, in a third Operation.

And thus may the Root of any *Adfected Equation* be found.

Question 25.

Suppose
- Any *Body* — — A
- Pass over a given *Space* — c
- In a given *Time* — f
- And any other Body — B
- Pass over a *Space* — d
- In the *Time* — — g
- Also their *Distance* of *Place* — e
- And the *Interval* of *Time*, in which they begin to move, be h
- Thence to determine — x

The Distance they pass, 'ere the hindmost Body overtakes the foremost, if they both tend the same way; or before they meet, if their Motions be opposite.

First, If they both tend the same Way, and A begin to move first, and is nearest the Place they tend to;

The *Theorem* is $x = \dfrac{hdc + fed}{fd - ge}$ 1.

But if B begin to move first, the *Theorem* is $x = \dfrac{hdc - fed}{fd - gc}$ 2.

If they both begin to move in the same Moment; the *Theorem* will be thus $x = \dfrac{fed}{fd - gc}$ 3.

For if $h = o$, then the Member of the *Equation* that has it, vanisheth.

Secondly. If the *Moveable Bodies meet*, and x, as before, be the Distance of the farthest Body, from the Place of *Rencounter*, or Meeting; then $e - x$ will be the Distance of the other Body. Call the Body at farthest Distance A, and the other B.

Then,

Then, if A moves first, the *Theorem* is — $x = \frac{cdh + cge}{fd + gc}$ 4.

But if B moves first, the *Theorem* is — $x = \frac{cdh + cge}{fd + gc}$ 5.

If they both move at the same time, the *Theorem* is — — $x = \frac{cge}{fd + gc}$ 6.

These Six *Theorems* answer most of the curious (and some of them very useful) Questions that are usually proposed concerning the *Motion* of two Bodies.

Question 26. Suppose the *Sun* (A) in the Beginning of *Virgo*, and eight Days after the *Moon* (B) is in the Beginning of *Gemini*; Quere the Place of the next *New Moon*?

Here are given
$c = 0{,}03285 = 00^{\circ} : 59' : 08''$
$f = 1,$
$d = 0{,}4892 = 13^{\circ} : 10' : 35''$
$g = 1,$
$e = 3,$
$h = 8,$
Quere x?

By *Theorem* 1, work as follows;

Multiply —	$d =$	0,4892	☽'s given Motion.
By —	$c =$	0,03285	☉'s given Motion.
The Product is	$dc =$	0,01607	
Which mult. by	$h =$	8	Difference of Time.
That Prod. is	$dch =$	0,12856	
Again multiply	$d =$	0,4892	☽'s given Motion,
By —	$fe =$	3	Difference of Place multiplied into f.
The Prod. is	$fed =$	1,4676	
To which add	$dch =$	0,12856	
Sum is	$hdc + fed =$	1,59616	The Dividend.
Then from —	$fd =$	0,4892	
Substract —	$gc =$	0,03285	
Remains	$fd - gc =$	0,45635	The Divisor.

By which divide, } Signs ° ′ ″
The Quotient is } $x = 3{,}49769 = 3 : 14 : 55 : 05$
To which add the *Moon*'s pres. Pl. $= 2 : 00 : 00 : 00$

The *Sum* is the Place of the } viz. In ♍ 5 } 14 : 55 : 05
Next New Moon sought

Question 27. If a *Ship B* sail from the *Equator* percisely *North*, at the Rate of $7\frac{1}{2}$ Degrees in 3 Days, and another Ship *A*, 8 Days after set sail on the same Meridian the same Way, in *Latitude* 36° : 30′, and runs 8° : 45′ in five Days; 'tis required to tell in *what Degree of North Latitude B* will overtake *A*?

By *Theorem* 2. Thus;

Multiply the given *Motion* of *A* —	$c =$	8,75
By the given *Motion* of *B* —	$d =$	7,5
The Product is — — —	$dc =$	65,625
Multiply that by the *Interval of Time* —	$b =$	8
The Product is the Substrahend —	$bdc =$	525,000
Then multiply the given *Interval of Places*	$e =$	36,5
By the given *Motion* of *B* —	$d =$	7,5
The Product is — —	$ed =$	273,75
Which multiplied by *A*'s given Time —	$f =$	5
The Product is — —	$fed =$	1368,75
From which substract the Substrahend —	$bdc =$	525,00
Remains the *Dividend* —	$fed - bdc =$	843,75
Then from $7{,}5 \times 5$ —	$fd =$	37,5
Substract $8{,}75 \times 3$ — —	$gc =$	26,25
There remains the *Divisor* —	$fd - gc =$	11,25
By which Divide; the Quotient is $= x$, the *Latitude sought, viz.* —	} $x = 75$th Deg.	

Question 28. *A* challenges *B* to run a Race with him, provided he will give him 30 *Rod* in a 100; now the *Velocity* of *B*'s Running to that of *A*, is as $7\frac{1}{4}$ to $5\frac{1}{2}$. *Quere* which of the two beat?

By

By *Theorem* 3, work thus;

Multiply the *Velocity* of *B* —	$d =$	7,75
By the given *Interval* of *Distance* —	$e =$	30
The Product (because $f = 1$) is the *Dividend*	$fed =$	232,5

Then from — —	$fd =$	7,75
Substract — —	$gc =$	5,5
There Remains the Divisor	$fd - gc =$	2,25

Then — 2,25) 232,5 (103 $= x =$ 103,3 Rods.

Hence *A* beat *B*, since above the 100 Rods were pass'd 'ere *B* came up with, or could overtake him.

Question 29. Suppose the *Hour* and *Minute* Hand of a *Horologium*, or Clock, be now both in *Conjunction* at 12, *Quere* the Place of their next Conjunction?

If you proceed by the same *Theorem* 3, you will find it to be at $\overset{h.}{1} : \overset{,}{09}$ Hours $= \overset{h.}{1} : \overset{\prime}{5} : \overset{\prime\prime}{27} : \overset{\prime\prime\prime}{16} : \overset{\prime\prime\prime\prime}{21} : \overset{\prime\prime\prime\prime\prime}{49}$, &c. the last five Places repeating *ad infinitum*. Hence we may observe, that though there really is *a certain Moment of Time* in which the *Minute-Hand* is precisely in *Conjunction* with the *Hour-Hand*, yet 'tis impossible to determine or represent that *Moment* of *Time* either in *whole Numbers*, or *Decimal Fractions*; But by *Vulgar Fractions* we know it is $1\frac{1}{11}$ Hours, that is just *one Eleventh* Part of an *Hour* after *one a Clock*.

Question 30. From *London* to *Chichester* is 60 Miles; A Post-Boy *(A)* sets out from *London*, and goes $8\frac{3}{4}$ Miles in $2\frac{1}{2}$ Hours; Another Post-Boy *(B)* $1\frac{1}{2}$ Hour after sets out from *Chichester*, and rides 9 Miles in $3\frac{1}{4}$ Hours. I demand how far *A* will have gone before he meets *B*?

This *Question* is answered by *Theorem* 4, thus;

Multiply *A*'s *given Space* — —	$c =$	8,75
By the *Interval* of the *Times* —	$b =$	1,5
Then multiply that Product —	$cb =$	13,125
By the *given Space* of *B* —	$d =$	9
The Product is — —	$cdb =$	118,125

 Again

Again multiply	—	— c =	8,75
By the *given Time* of *B*		— g =	3,25
And that Product •	—	cg =	28,4395
Mult. by the *Interv. of Distance*		— e =	60
To that Product	—	cge =	1706,25
Add that above, *viz.*	—	cdh =	118,125
The Sum is the *Dividend*		$cdh + cge$ =	1824,375
Then to the Product above,		— cg =	28,4375
Add the Prod. of $(f \times d = 2,5 \times 9)$		fd =	22,5
The Sum is the *Divisor*		— $fd + cg$, =	50,9375
By which dividing; the Quotient is the *Dist.* of *A*'s Journey		x =	36,208 } Ans.
Then the *Distance B* will have pass'd is	— —	=	23,791 } Ans.

And thus proceed for answering Questions by *Theorems* the fifth and sixth. By these *Theorems* several other pretty *Problems* proposed, may be resolv'd by any one versed in those Matters.

Question 31. This *present Year of our Lord*, the *Cycle* of the *Sun* is $6 = e$, and the *Cycle* of the *Moon* (call'd the *Prime* or *Golden Number*) is $5 = d$; *Quere* the Year of the *Dionysian Æra* or Period?

Here is given $\begin{cases} e = 6 \\ d = 5 \\ e - d = z = 1 \end{cases}$ Let x be the Year of the *Period* sought.

The *Theorem* is $59,x\, z + 3,x\, d - 2,x\, e = x$.

For, multiply the *Difference* of the *Cycles*		z =	1
By the Number	—	—	59,x
Then that Product is	—	=	59,x
To which add $d \times 3,x$	—	=	15,5
From that *Sum*	—	— =	74,6
Substract $e \times 2,x$	—	— =	12,6
There remains the *Year* of the *Period*		=	62 sought.

This

This *Theorem* I contrived my self; and inserted it here as being a *Decimal* one.

Question 32. Let *A*, and *B*, be two *spherical* Bodies *perfectly elastick*, and let (a) denote the *Velocity* of *A*, and (b) = the *Velocity* of *B*; then the *Motion* of $A = aA$, and the *Motion* of $B = bB$; lastly let x = the *Increase* of *Motion* communicated by the *Impact* or Stroke, to one Body; and the *Decrease* or *Loss* of *Motion* in the *percutient* or *striking* Body.

Let *A* follow *B*, and let it be required to determine the *Celerity* of each Body after the Stroke or *Impulse*.

If *A* and *B* tend both the same Way, the *Theorem* for
$$\begin{cases} A\text{'s } Celerity \text{ is } x = \dfrac{aA - aB + 2bB}{A + B} \\ B\text{'s } Celerity \text{ is } x = \dfrac{2aA - bA + bB}{A + B} \end{cases}$$

But if they meet, the *Theorems* will be altered thus, for
$$\begin{cases} A, \quad x = \dfrac{aA - aB - 2bB}{A + B} \\ B, \quad x = \dfrac{2aA + bA - bB}{A + B} \end{cases}$$

Example. Suppose two Bodies of the same sort, *A* of $5\frac{1}{3}$ Pounds, and 9 Degrees of *Velocity*; and *B* of $6\frac{1}{2}$ Pounds, and 4 Degrees of *Velocity*; tend the same Way; Quere their *Celerities* after the *Impulse*?

Here $A = 5,\dot{3}$. $a = 9$. $B = 6,5$. $b = 4$. Then,

From the *Motion* of *A*	— —	aA	$= 48$
Substract the *Velocity* of *A* into *B*	—	aB	$= 58,5$
There remains *negative*,	—	$aA - aB$	$= -\ 10,5$
To which add twice the *Motion* of *B*	—	$2bB$	$= +\ 52$
There Remains the *Dividend*	—		$= +\ 41,5$

Then $A + B = 11,8) + 41,5\ (3,49 =$ *A*'s Celerity.
In like Manner may be found $8,46 =$ *B*'s Celerity.

Note; If either *Celerity* come out *Negative*, it signifies the *Motion* of that Body, after the *Impulse*, to be *contrary* to what it was before.

I have

I have inserted this Question and *Theorem*, for the Sake of any such Persons as would wish to have always a *Theorem* by them for the ready determining the *Celerity* of *Bodies* in *Motion* by *Calculation*, and the rather because this *Doctrine* of *Motion* is the principal *Basis* of a good Part of the *modern mechanical Philosophy*.

Question 33. Says *A*, I've an *Acre* of Land to inclose; says *B*, I've ten *Thousand* such *Acres* as those, which lie in a *Square*; but the *Form* you design must have the *same Fencing* as goes round all mine. *Quere* the Length and *Breadth* of *A*'s *Acre* of Land?

Let $\begin{cases} a = \;\;\; 1 = \text{The } Area \text{ of } A\text{'s } Plot \text{ of Land.} \\ d = 100 = \text{The } Side \text{ of } B\text{'s } square\ Plot. \\ x = \qquad\;\; \text{The } Side \text{ of } A\text{'s } Plot \text{ to be found.} \end{cases}$

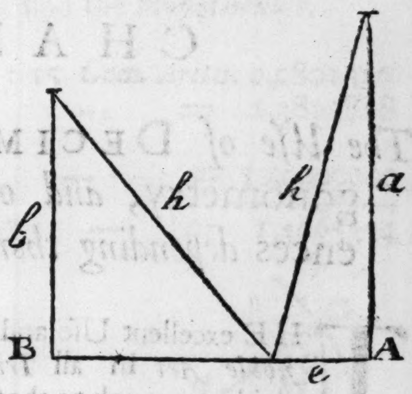
The *Theorem* is $x = \sqrt{dd - a} : + d$.

From the *Square* of d	$dd =$	10000
Substract the given *Area*	$- \; a =$	1
There remains	$- \; dd - a =$	9999
The Sq. Root thereof is	$\sqrt{dd-a} =$	99,995
To which add	$- \; d =$	100
The Sum is one Side of the *Area*	$x =$	199,995 = the Length.
And the other Side is	$- \; \frac{a}{x} =$	0,005 = the Breadth.
Thus *twice* their *Sum* is	$= 4b =$	400 the Perimeter of Both.

Question 34. Suppose the *Tower* *A* 160 Feet high, and another *Tower* *B* 124 Feet high, at the *Distance* *A B* = 150 Feet; 'tis required to set a *Ladder* in some *Point* *(e)* in the Line *A*, of such a *length*, as from thence it may reach the *Tops* of both the *Towers*: *Quere* the Point *e*, and the *length* of such a *Ladder*?

Let

Let $\begin{cases} a = 160 & \text{the Tower } A \\ b = 124 & \text{the Tower } B \\ c = 150 & \text{the Distance } BA \\ e = & \text{the Point's Dist.} \\ h = & \text{the Ladder sought.} \end{cases}$



The *Theorem* for the *Distance* (*e*) $\dfrac{bb + cc - aa}{2c} = e$

Then for the *Length* of the *Ladder* (*h*) $\sqrt{aa + ee} = h$

Thus, to the *Square* of *B*'s *height*	—	$bb =$	15376
Add the *Square* of the *Distance*	—	$cc =$	22500
Then from that *Sum*	—	$bb + cc =$	37876
Substract the *Square* of *A*'s *height*	—	$aa =$	25600
There remains the *Dividend*		$bb + cc - aa =$	12276
Which divided by $2c = 300$, the *Quotient* is the *Distance* (*e*)	—	$e =$	40,92

Then the *Length* of the *Ladder* is $\sqrt{aa + ee} = h = 165,149$ &c.

CHAP.

CHAP. XIII.

The Use of DECIMALS *in Plain* Trigonometry, *and other Mathematical* Sciences *depending thereon.*

THE excellent Use and indispensable Necessity of this *noble Art* in all *Trigonometrical Calculations*, is evident enough to those who are versed therein. Nothing with any *Exactness*, *Ease*, or *Expedition* can be done therein without it; and as *Trigonometry* is the Foundation (yea the very Essence) of *Navigation*, *Fortification*, *Altimetry*, *Longimetry*, and is of Use also in divers Cases of *Astronomy*, *Surveying*, *Dialling*, &c. 'tis manifest the Use and Knowledge of *Decimal Arithmetick* is so requisite in all those *Arts* and *Sciences*, that without its Assistance a Person can make but a gloomy and fruitless Progress in the Study of them.

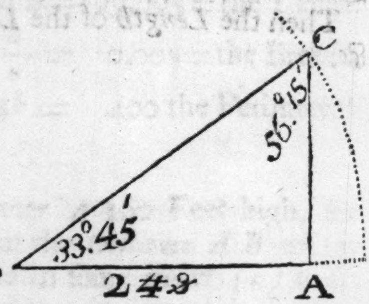
I shall therefore illustrate the Use of *Decimals* in the Resolution of all the Cases of *Right-lined Trigonometry* (for that only is to be understood in this Chapter) both in the Doctrine of *Right-angled*, and *Oblique-angled Triangles*, as follows.

Right-angled Triangles.

Case 1.

Given $\begin{cases} \text{The two } \textit{Angles} \\ \quad B \text{ and } C, \\ \text{The } \textit{Base } B\,A, \end{cases}$

To find the *Cathetus* and Hypothenuse,



The *Analogy* to find the *Cathetus*.

As the *Sine* of the *Angle* C $56^{\circ}:15'$ Com. Arith. 0,0801536
Is to the *Base* B A 24,3 — = 1,3863818

So is the *Sine* of the *Angle* $33^{\circ},45'$ — = 9,7447390

To the *Cathetus* or *Perpendicular* 16,26 = 1,2112744

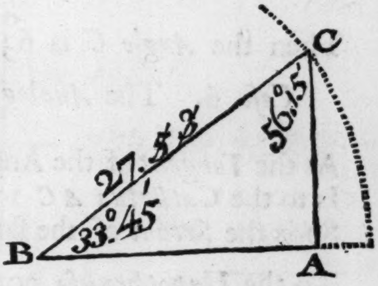
Case

Case 2. The *Analogy* to find the *Hypothenuse*.

As the *Sine* of the Angle *C* $56^{\circ}:15'$	*Com. Arith.*		0,0801536
Is to the *Base* *B A* 24,3	—	=	1,3863818
So is *Radius* 90°	—	— =	10,0000000
To the *Hypothenuse* B *C* 29:27	—	=	1,4665354

Case 3.

Given { The two *Angles* B and *C*, And the *Hypothenuse* B C ;
To find the *Base* and *Cathetus*.



The *Analogy* for finding the *Base*.

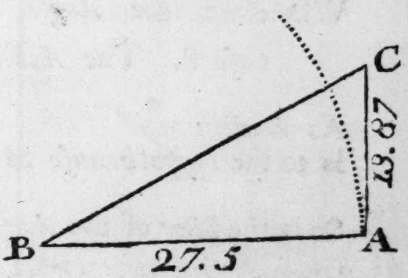
As *Radius* 90°	—	—	10,0000000
Is to the *Hypothenuse* B *C* 27:53	—	=	1,4398906
So is the *Sign* of the Angle *C* $56^{\circ}:15'$	—		9,9198464
To the *Base* B *A*, 22:89	—	=	1,3597370

Case 4. The *Analogy* to find the *Cathetus*.

As the *Radius* 90°	—	—	10,0000000
Is to the *Hypothenuse* BC 27:53		=	1,4398906
So is the *Sine* of the Angle *B* $33^{\circ}:45'$		=	9,7447390
To the *Cathetus*, or *A* C 15:29	—	=	1,1846296

Case 5.

Given { The *Base* B *A*, The *Cathetus* *A C* ;
To find the *Angles*, and the *Hypothenuse*.



S s The

The *Analogy* to find the *Angle* B.

As the *Base* B A 27:5 — = 1,4393327

Is to the *Radius* 90° — — = 10,0000000

So is the *Perpendicular* 13:87 — = 1,1420765

To the *Tangent* of the Angle *B* $26^\circ : 46'$ = 9,7027438

Then the *Angle C* is $63^\circ : 14'$.

Case 6. The *Analogy* to find the *Hypothenuse*.

As the *Tangent* of the Angle B $26^\circ : 46'$ *C. A.* 0,2972562

Is to the *Cathetus A C* 13:87 — = 1,1420765

So is the *Secant* of the same Angle *B* = 10,0492225

To the *Hypothenuse* 30:8 — = 1,4885552

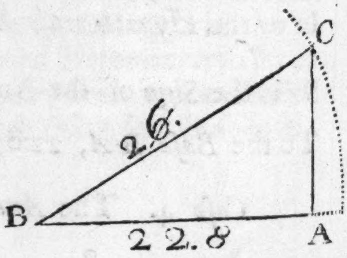
Note. The *Secant* of any Angle is the *Arithmetical Complement* of the *Co sine* of the said Angle, added to *Radius* 10,0000000.

Case 7.

Given { The *Base* B *A*, The *Hypothenuse* BC;

To find the *Angles*, and the *Cathetus A C*,

The *Analogy* for the *Angle C*.



As the *Hypothenuse* B C 26 — = 1,4259687

Is to *Radius* 90° — — = 10,0000000

So is the *Base* B *A* 22:8 — = 1,3579348

To the *Sine* of the *Angle C* $58^\circ:45'$ = 9,9319661

Wherefore the *Angle* B 31:15.

Case 8. The *Analogy* to find the *Cathetus*.

As *Radius* 90° — — 10,0000000

Is to the *Hypothenuse* 26 — = 1,4259687

So is the *Sine* of the *Angle* B $31^\circ:15'$ = 9,7149776

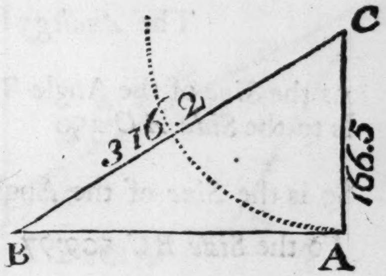
To the *Cathetus A C* 13:83 — = 1,1409463

Case

Case 9.

Given { The *Hypothenuse* B C
The *Cathetus* *A* C,

To find the *Angles*, and the *Base* *B A*.



The *Analogy* to find the *Angle C*.

As the *Cathetus A C* 166:5	— *Arith. Com.*	7,7788467
Is to the *Radius* 90°	— —	10,0000000
So is the *Hypothenuse* B *C*, 316:2	=	2,5000369
To the *Secant* of the *Angle C*, $58^\circ{:}15'$		10,2788836

Then the other *Angle* B will be $31^\circ{:}45'$

Case 10. The *Analogy* for finding the *Base*.

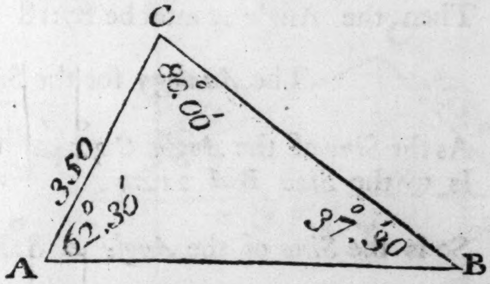
As the *Radius* 90°	— —	10,0000000
Is to the *Cathetus* 166:5	— =	2,2211533
So is the *Tangent* of the Angle *C* $58^\circ{:}15'$	=	10,2084365
To the *Base* B *A* 268:927	— =	2,4295898

Oblique-angled Triangles.

Case 1.

Given { The *Angles* *A*, B, and *C*,
The *Side A C*,

To find the other *two Sides*.



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The *Analogy* for the *Side* B C.

As the *Sine* of the Angle B 37°:30′	*Arith. Com.*		0,2155529
Is to the *Side* A C 350	—	=	2,5440680
So is the *Sine* of the Angle *A* 62°:30′	—		9,9479289
To the *Side* B C 509:97	—	=	2,7075498

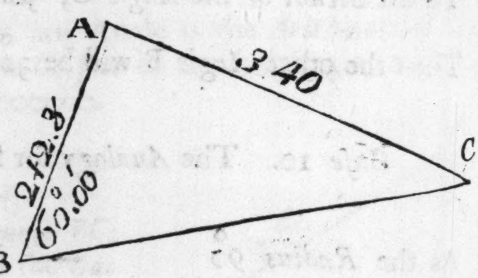
The *Analogy* for the *Side* A B.

As the *Sine* of the *Angle* B 37°:30′	*Arith. Com.*		0,2155529
Is to the *Side* A C 350	—	=	2,5440680
So is the *Sine* of the Angle *C* 80°:00′		=	9,9933515
To the *Side* A B 566:2	—	=	2,7529724

Case 2.

Given { Two *Sides* *A C* and *A* B, And an Angle *opposite* B, }

To find the other Side and Angles.



The *Analogy* for the *Angle* C.

As the *Side* A C 340	—	*Arith. Comp.*	7,4685211
Is to the *Sine* of the *Angle* B 60°:00′		—	9,9375306
So is the *Side* B *A* 242:3	—	=	2,3265407
To the *Sine* of the *Angle* C 32°:42′	—	=	9,7325924

Then the Angle *A* must be 87:18 therefore,

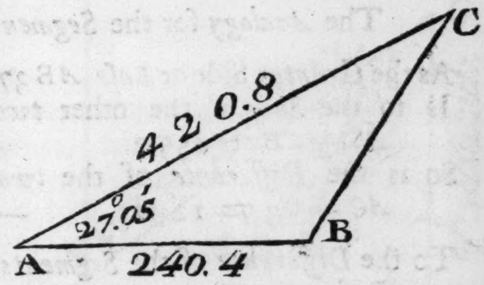
The *Analogy* for the *Side* B C.

As the *Sine* of the *Angle* C 32°:42′	*Com. Arith.*		0,2674076
Is to the *Side* B *A* 242:3	—	=	2,3265407
So is the *Sine* of the *Angle* *A* 87°:18′	—		9,9995170
To the *Side* B C 39:21	—	=	2,5934659

Case

Case 3.

Given { Two Sides *A C* and *A B*, And the Angle *included A*,
To find the other *Side* and *Angles*.



The *Analogy* for the *Angles*.

As the *Sum* of the two Sides 661:2 *Com. A.* = 7,1796672
Is to the *Difference* of the Sides 180:4 = 2,2562365
So is *Tangent* of $\frac{1}{2}$ *Sum* of the unknown Angles *B* and C $76^{\circ}:27'$ — } = 10,6179795

To the *Tangent* of $\frac{1}{2}$ *their Difference* $48^{\circ}:32'$ = 10,0538832

Then, To half the Sum — $76^{\circ}:27'$ of two Ang. *B*, *C*.
Add half the Difference 48:32

The Sum is — 124:59 = Greater Ang. *B*.
Substract the $\frac{1}{2}$ *Differ.* Remains 27:55 = Lesser Angl. C.

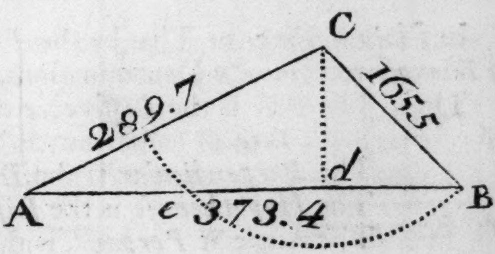
The *Analogy* for the *Side* B C.

As the *Sine* of the *Angle* C $27^{\circ}:55'$ *Arith Com.* 0,3295808
Is to the *Sine* of the *Angle A* $27^{\circ}:05'$ = 9,6582842
So is the *Side A B* 240,4 — = 2,3809345

To the *Side B C* 23,37 — — = 2,3687995

Case 4.

Given { All the *three* Sides, *AB*, *A* C, and C*B*,
To find the *Angles*.



The

The *Analogy* for the *Segments Ad*, and *dB*.

As the *Greater* Side or *Base AB* 373:4 *Ar. Com.* 7,4278258
Is to the *Sum* of the other *two Sides* $AC + CB = 455{\cdot}2$ — $\} = 2{,}6582023$
So is the *Difference* of the *two Sides* $AC - CB = 124{:}2$ — $\} = 2{,}0941216$

To the *Difference* of the *Segments* of the *Base* $Ae = 151{:}4$ — $\} = 2{,}1801497$

Then from the *Grea. Side* or Base = 373:4
Subst. the *Diff.* of the *Segments Ae* = 151:4

There will remain *eB* — = 222
The half of which is *Bd* = *ed* = 111 the Lesser Segm.
Also to *Ae* add *ed*, the Sum is *Ad* = 262:4 the Greater Seg.

The whole *Oblique Triangle ACB* being thus resolved into the two *Right-angled Triangles ACd*, and *BCd*, the Angles *A*, *B*, and *C* are found by the seventh Case of *Right-angled Triangles* foregoing.

Having thus pass'd through all the Cases of *Right* and *Oblique-angled Plain Triangles*, in each of which the absolute Necessity of *Decimal Numbers* to express the Length of the *Sides* sought, is sufficiently evident; I shall next shew, in brief, the Application of the foregoing Doctrine of *Plain Trigonometry* to several *Arts Mathematical*; intending thereby to convince those who purpose to learn them, of the Necessity of their first learning *Decimal Arithmetick*.

The Use of Decimals *in* Navigation *exemplified in all Kinds of Sailing.*

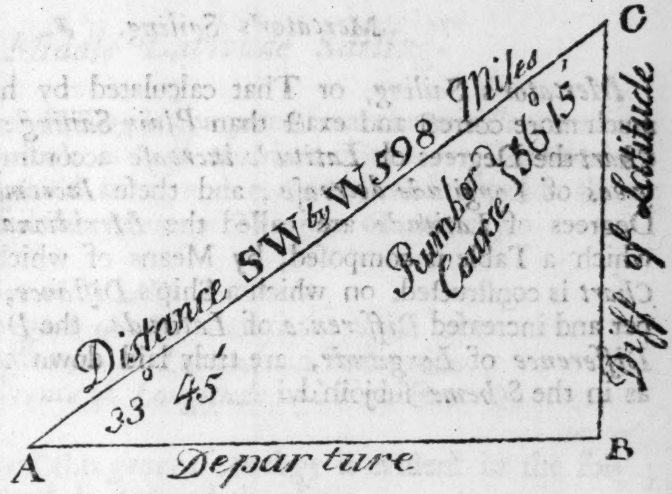
1. *Plain Sailing.*

In *Plain Sailing*, or That by the *Plain Chart*, the Parts of a *Triangle* receive new Denominations.

Thus, The *Base* is the *Difference* of *Longitude* or *Departure*;
The *Perpendicular* is the *Difference* of *Latitude*;
The *Hypothenuse* is the *Distance* the Ship has run;
The Angle at *Perpend.* is the *Course* of the Ship;
And the Angle at *Base* the *Complement* of the *Course*.

Admit

Admit a Ship sails from the *Lat. North* $51^{\circ}:30'$ on the *Rumb.* *A C* 598 *Nautic Miles* whose *Course* is *S W b W*; Quere her *Departure* and *Difference* of *Latitude*?



The *Analogy* for finding the *Departure.*

As *Radius* 90°	—	—	10,0000000
Is to the *Distance* run *A C* 598	—	=	2,7767012
So is the *Sine* of the Course C $56^{\circ}:15'$		—	9,9198464
To the *Departure Westward* from her former *Meridian*	497:2	=	2,6965476

The *Analogy* for the *Difference* of *Latitude.*

As the *Radius* 90°	—	—	10,0000000
Is to the *Distance* run *A C* 598	—	=	2,7767012
So is the *Co-sine* of the *Course* $33^{\circ}:45'$		—	9,7447390
To the *Difference* of *Latitude* 332:2		=	2,5214402

But 332,2 Miles are equal to $5^{\circ}:32',2$, and the Ship's *Course* being *South-westerly.*

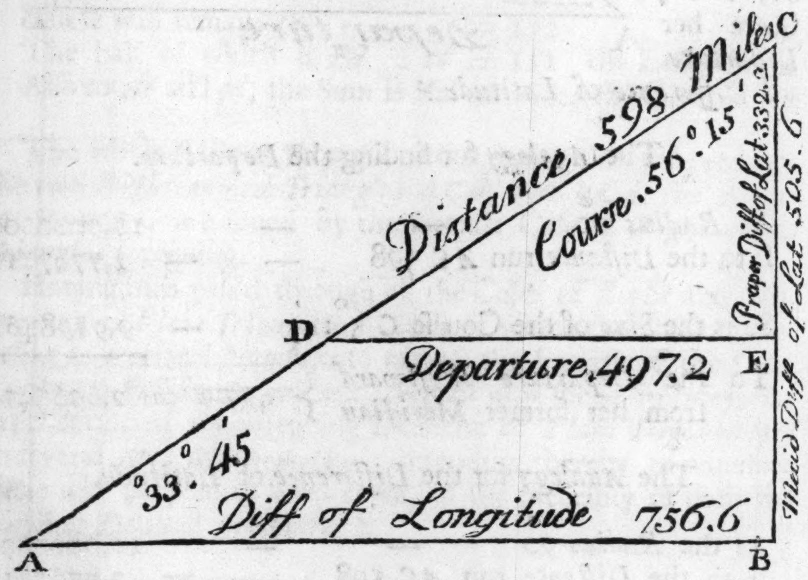
Therefore from the *Latitude* sailed from	—	$51^{\circ}:30'$ *N.*
Substract the *Difference* of *Latitude*	—	5:32,2
Remains the *Latitude* come to	— =	45:57,8 *N.*

The same Case follows in

Mercator's

Mercator's Sailing.

Mercator's Sailing, or That calculated by his *Chart*, is much more correct and exact than *Plain Sailing:* For in this *Chart* the Degrees of *Latitude increase* according as the *Degrees* of *Longitude decrease*; and these *Increments* of the Degrees of *Latitude* are called the *Meridional Parts*; of which a Table is composed, by Means of which *Mercator's Chart* is constructed, on which a Ship's *Distance*, *Course*, proper and increased *Difference* of *Latitude*, the *Departure* and *Difference* of *Longitude*, are truly laid down or delineated, as in the *Scheme* subjoin'd.



The *Analogy* for the Difference of *Longitude*.

As *Radius* 90° — — — 10,0000000
Is to the *Increased Differ.* of *Lat.* 505:6 = 2,7038071
So is the *Tangent* of the *Course* 56°:15' — 10,1751074
To the *Difference* of *Longitude* 756:6 = 2,8789145

The same Case and *Data* follow in

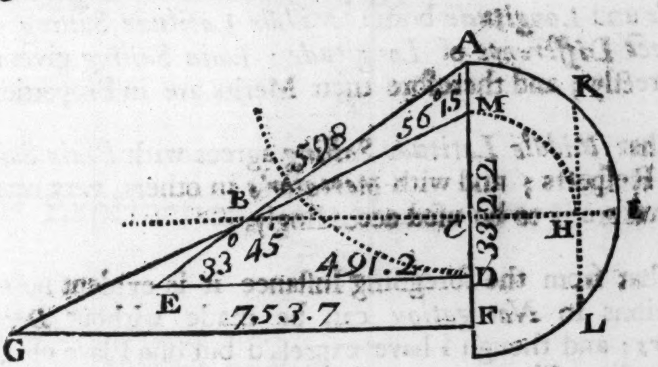
Middle

Middle Latitude Sailing.

This kind of *Sailing* is computed from the *middle Parallel* of *Latitude*, which is *half the Sum* of the *two Latitudes* of the Places, sail'd from, and come to; and depends altogether on the following *general Theorem* or *Analogy.*

As the *Co-ſine* of *Middle Latitude*
Is to the *Tangent* of the *Courſe*,
So is the *Difference* of *Latitude* in Miles, &c.
To the *Difference* of *Longitude* in Miles, &c.

The Reaſon of this *general Analogy* is evident in the following Scheme and is deduced therefrom.



Explanation of the Scheme.

I K = *I L*	Is the *Middle Latitude* 48°.44.
C M = *C H*	The *Co-ſine* of the Middle Latitude.
A D = *M F*	The *Difference* of Latitude.
D E	The *Departure* Weſtward.
F G	The *Difference* of Longitude.
A E	The *Diſtance* ſailed.
E A D	The *Rhumb* or Courſe *S W* by *W*.
B C	The *Tangent* of the Courſe.

Now 'tis manifeſt As *C M* : *C B* :: *M F* : *F G*, which is the ſame as the *Analogy* above in Words at length.

The *Analogy* for the *Difference of Longitude.*

As the *Co-sine* of Middle Lat.	48°.44′	*Arith.* C.	0,1807427
Is to the *Tangent* of the Course	56.15	—	10,1751074
So is the *Difference of Latitude*	332.2	=	2,5213996
To the *Difference* of *Longit.*	753.7	=	2,8772497

Note 1. The Proportions for finding the *Difference* of *Latitude* and *Departure* in *Mecator's* and *Middle Latitude Sailing*, are the same as in *Plain Sailing*, and therefore not repeated.

2. That *Mecator's Sailing* gives the *correct Difference* of *Latitude* and *Longitude* both; *Middle Latitude Sailing*, only the *correct Difference* of *Longitude*; *Plain Sailing* gives neither correctly; and therefore their Merits are in Proportion.

3. That *Middle Latitude Sailing* agrees with *Plain Sailing* in some Respects; and with *Mercator's* in others, very nearly; and therefore is to be used accordingly.

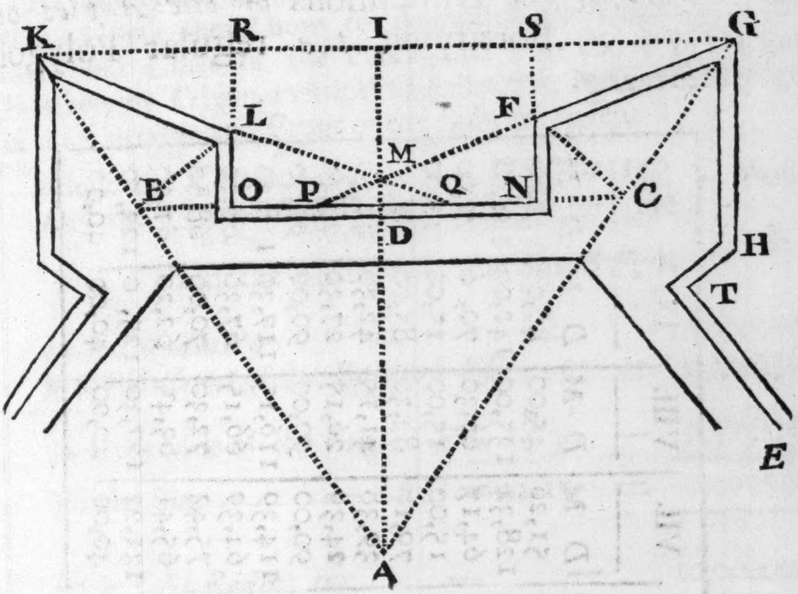
4. That from the foregoing Instance it is evident no exact Calculations in *Navigation* can be made without *Decimal Numbers*; and though I have express'd but one Place of *Decimals*, yet three Places more may be easily found by *Problem* 3. of *Logarithms*.

The Use of Decimals *in Fortification.*

A *Fort* is a Piece of Ground in Form of a *Polygon regular* or *irregular*, environ'd with a *Rampier*, or *Wall*, and a *Ditch* to impede the Assaults of an Enemy.

A Scheme of a *regular Pentagonal Fort*, with its Explanation, is here after subjoin'd.

The



The Explanation, *or Names of the several Parts.*

1. The *Curtaine* — — *O N*
2. The *Bulwark* or *Bastion* — *N F G H T*
3. The *Front* of the *Bulwark* — — *F G*
4. The *Flank* — — *N F*
5. The *Gorge* of a *Bulwark* — — *N T*
6. The *Gorge* Line — — *N C*
7. The *Head* Line — — — *C G*
8. The *Shoulder* — — — *F*
9. The *Flanked* Angle — — *G*
10. The *Inward Flanking* Angle — *S G F*
11. The *Outward Flanking* Angle — *K M G*
12. The *Longest Line of Defence* — *P G*
13. The *Shortest Line of Defence* — *P F*
14. The *False Bray* — — *B C*

A Table *of the* Dimensions *of the Angles observed in* Fortifying *the* regular Polygons *following.*

Number of the Sides of the Polygons.		V. D M	VI. D M	VII. D M	VIII. D M	IX. D M	X. D M
Angle at the Centre —	*BAC*	72,00	60,00	51,26	45,00	40,00	36,00
Angle of the Polygon —	*BCE*	108,00	120,00	128,34	135,00	140,00	144,00
Half thereof —	*BCA*	54,00	60,00	64,17	67,30	70, 0	72,00
To which always add —	—	15,00	15,00	15,00	15,00	15,00	15,00
The Flanked Angle —	*FGH*	69,00	75,00	79,17	82,30	85,00	87,00
The Half thereof —	*FGC*	34,30	37,30	39,38	41,15	42,30	43,30
Inward Flank Angle —	*SGF*	19,30	22,30	24,39	26,15	27,30	28,30
Which add to a Right Angle	—	90,00	90,00	90,00	90,00	90,00	90,00
Angle of the Shoulder —	*NFG*	109, 0	112, 0	114,39	116,15	117,30	118,30
Angle opposite to the Head Line	*GFC*	59,30	62,30	64,39	66,15	67,30	68,30
Angle opposite to the Front	*FCG*	86,00	80,00	75,43	72,30	70,00	68,00
Complement of *SGF*, viz.	*SFG*	70,00	67,30	65,41	63,45	62,30	61,30
Outward Flanking Angle	*KMG*	141, 0	135, 0	131,22	127,30	125, 0	123, 0
Angle fronting the Flank —	*FCN*	40,00	40,00	40,00	40,00	40,00	40,00

Tho'

Tho' 'tis not necessary the Angles in Forts should be precisely such as are before assign'd; yet supposing them to be such, I shall shew how to determine the Quantity of the Sides and Lines of the Pentagonal Fort above in Decimal Numbers by Trigonometrical Calculations, having the Length of the Curtaine and Front of the Bulwark given.

Admit the Curtaine be — $ON = 140$ } Yards,
And the Front of the Bulwark $FG = 93$ }

Then the Analogy for the Sine SF, is

As the *Radius* 90° — — 10,0000000
Is to the *Front* of the *Bulwark* $FG = 93$ = 1,9700368

So is the Sine of the Angle SFG $19^{\circ}.30'$ — 9,5235000

To the Sine — $SF = 31.15$ = 1,4935368

Again; As *Radius* 90° — — 10,0000000
Is to the Front of the *Bulwark* $FG = 93$ = 1,9700368

So is the Co-sine of Angle SFG $70^{\circ}.30'$ 9,9743466

To the Line — $SG = 87.98$ = 1,9443834

Then $SG = 87.98$, and $SI = 70$. Therefore the whole Sine $KG = 315.96$.

Again; as the Sine of the Angle IAG $36^{\circ}.00'$ *A C.* 0,2307813
Is to $\frac{1}{2}$ $KG = IG = 157.98$ — = 2,1986500

So is *Radius* 90° — — 10,0000000

To the *Semidiameter* $AG = 268.8$ = 2,4294313

Again; as the Sine of the Angle IAG $36^{\circ}.00'$ *A. C.* 0,2307813
Is to $\frac{1}{2}$ the Side of the *Pentagon* $IG = 157.98 = 2,1986500$

So is the Sine of the Angle AGI $54^{\circ}.00'$ — 9,9079576

To the *Perpendicular* $AI = 217.4$ = 2,3373889

Again;

Again, as the Sine of the Angle *FCG* $86^{\circ}.00'$ *A.C.* 0,0010592
Is to the *Front* $FG = 93$ — = 1,9700368

So is the Sine of the Angle *FGC* $34^{\circ}.30'$ 9,7531280

To the Line $FC = 52.99$ — = 1,7242240

Also, as the Sine of *FCG* $86^{\circ},00'$ *Arith. C.* 0,0010592
Is to the *Front* $FG = 93$ — = 1,9700368

So is the Sine of the Angle *GFC* $59^{\circ}.30'$ 9,9353204

To the *Head Line* $CG = 80.61$ — = 1,9064164

Then $AG - CG = AC = 188.19$ the Semidiameter of the inner *Pentagon.*

Again, as *Radius* 90° — 10,0000000
Is to the Line $FC = 52.99$ — = 1,7242240

So is the Sine of the Angle *FCN* $40^{\circ}.00'$ 9,8080675

To the *Flank* $FN = 34.06$ — = 1,5322915

Then $FN + SF = ID = 65.21$. And $AI - ID = AD = 152.19$

Again, as *Radius* 90° — — 10,0000000
Is to the Line $FC = 52.99$ — = 1,7242240

So is the Sine of the Angle *NFC* $50^{\circ}.00'$ 9,8842540

To the *Gorge Line* $NC = 40.59$ = 1,6084780

Then $NC + ND = DC = 110.59$ And $2DC = BC = 221.18$

Again, as the Sine of *FPN* $19^{\circ}.30'$ *Arith. C.* 0,4765047
Is to the *Flank* $FN = 34.06$ — = 1,5322915

So is the Sine of the Angle *PFN* $70^{\circ}.30'$ 9,9743466

To the Line $PN = 96.19$ — = 1,9831428

Then $ON - PN = OP = 43.81$ the *second Flank.* And $ON + SG = RG = 227.98$. Then in the Triangle *ROG*

As

As the Line *R O* (= *I D*) = 65.21	*A. C.*	8,1856858
Is to the Line *R G* = 227.98	— =	2,3578967
So is *Radius* 90°	— —	10,0000000
To the *Tangent* of the Angle *R O G* 74°.02′		10,5435825

Then, as the Sine of *R O G* 74°.02′	*Arith.* C.	0,0170850
Is to the Sine *R G* = 227.28	— =	2,3578967
So is *Radius* 90°	— —	10,0000000
To the *Line of Defence* *O G* = 237.13	—	2,3749817

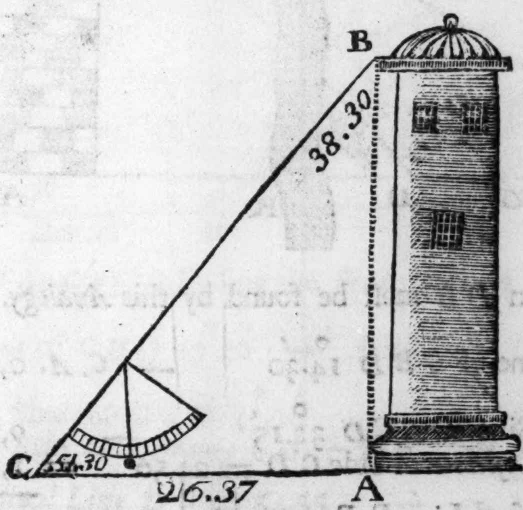
Thus having the *Angles* as in the *Table*, you are here taught the Manner of finding the *Sides* and *Sines* of any *regular Fort*, in any Measure, and *Decimal* Parts thereof.

The Use of Decimals *in* Measuring Heighths, Depths, *and* Distances; *both* accessible *and* inaccessible.

1. *Altimetry*, or the Mensuration of *Altitudes* and *Depths*, is thus performed.

Let *A B* represent a *Tower* whose *Height* is required.

Suppose { The *Distance* *A C* = 26.37 Yards.
The Angle *A C B* (found by a *Quadrant*) 51°.30′.



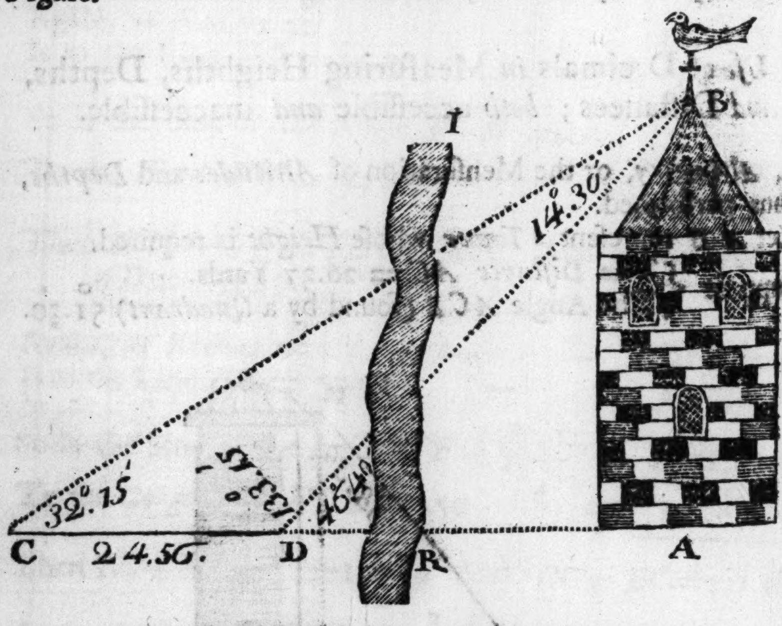
The

The *Analogy* for finding the *Height* *A B*.

As Radius 90°	— — —	10,0000000
Is to the Distance *A C* = 26.37	— =	1,4211101
So is the Tangent of *A C B* $51^{\circ}.30'$	—	10,0993948
To the Height of the *Tower A B* 33.15	=	1,5205049

And thus the *Height* or *Altitude* of any other *accessible* Object may be found.

Suppose the *Steeple A B* be *inaccessible* for the River *R I*; Then with a *Quadrant* at C take the Angle *A C B*, and measure a *Distance* to D, where take again the Angle *A D B*; and let those *Angles*, and the *Distance* be as in the adjoin'd Figure.



Then *D B* must be found by this *Analogy*.

As the Sine of *C B D* $14^{\circ}.30'$	— *C. A.*	0,6014004
Is to the Sine of *B C D* $32^{\circ}.15'$	—	9,7272276
So is the *Distance* or Side *C D* = 24.56	=	1,3902284
To the *Visual Line D B* = 52.34	— =	1,7188564

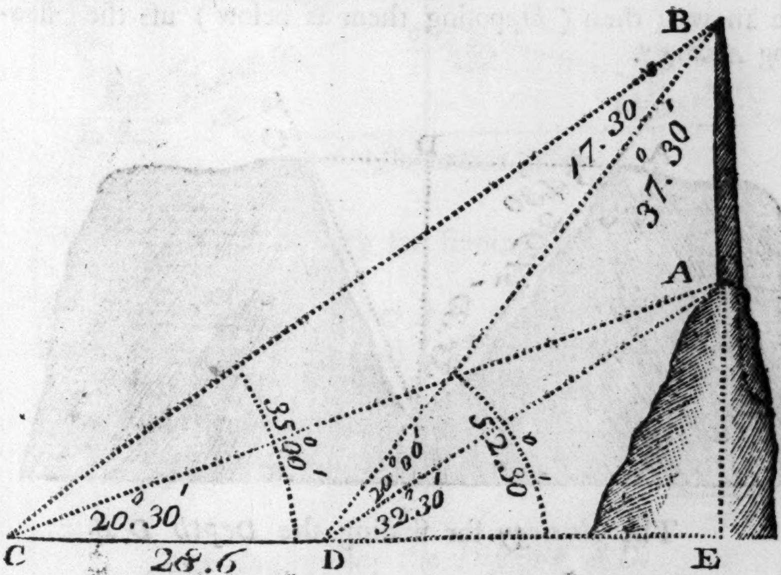
Having

Having found *D B*, you may find *A B*, thus;

As Radius 90°	—	— —	10,0000000
Is to the Line *D B* = 52.34	—	=	1,7188564
So is the Sine of *A D B* $46^{\circ}.45'$		=	9,8623526
To the *Height* required *A B* = 38.12		=	1,5812090

Thus you find the *Height* of any *inaccessible* Objects.

If the Object whose *Height* you would measure standeth aloft, as on a *Hill*, &c. as the *Tower A B*; then take the Angles *B C E* $35^{\circ}.00'$, *A C E* $20^{\circ}.30'$; then from *C* measure the *Distance C D* = 28.6 *Yards*, and at *D*, take the Angles *BDE* $52^{\circ}.30'$, and *A D E* $32^{\circ}.30'$, as below.



Then the *Analogy* for finding the *Side* D B, is

As the *Sine* of C B D $17^{\circ}.30'$	—	*Arith. Com.*	0,5218582
Is to the Sine of B C D $35^{\circ}.00'$		—	9,7585913
So is the *Side* or *Distance* C D = 28.6		=	1,4563660
To the *Visual Line* D B = 54.55	—	=	1,7368155

Then say, as the Sine of Complement of *BAD* (to 180°) $57^{\circ}.30'$ *A. C.* — } 0,0739708

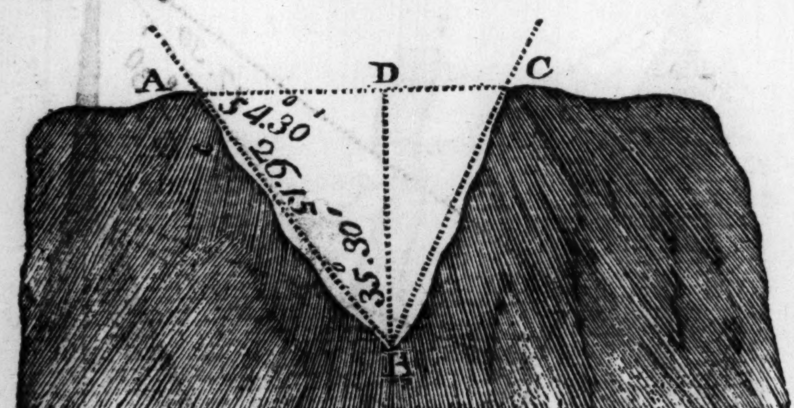
Is to the Sine of *B D A* $20^{\circ}.00'$ — 9,5340517

So is the Side *D B* = 54.55 — = 1,7368155

To the *Height* of the Object *A B* = 22.12 = 1,3448380

Depths, if *Perpendicular*, are most conveniently measured with a *Line* and *Plumet*; but if the *Depth* be *slanting*, such as *Valleys*, &c. and the *perpendicular Profoundity* be required; do as follows.

Let *A B C* be a *Valley*, whose *oblique* Descents or Sides are *A B*, and *B C*; and its perpendicular Depth *B D* required. Then measure the Side *A B*, or *B C*, and take with a *Quadrant* the Angle *A B D*, or *D B C*, whereby the others will be known; then (supposing them as below) use the following *Analogy*.



The *Analogy* for finding the *Depth* *D B*.

As Radius 90° — — 10,0000000

Is to the *slant Descent* *A B* = 26.15 = 1,4174717

So is the Sine of *B A D* $50^{\circ}.30'$ — 9,9106860

To the *Perpendicular Depth* DB = 21.28 = 1,3281577

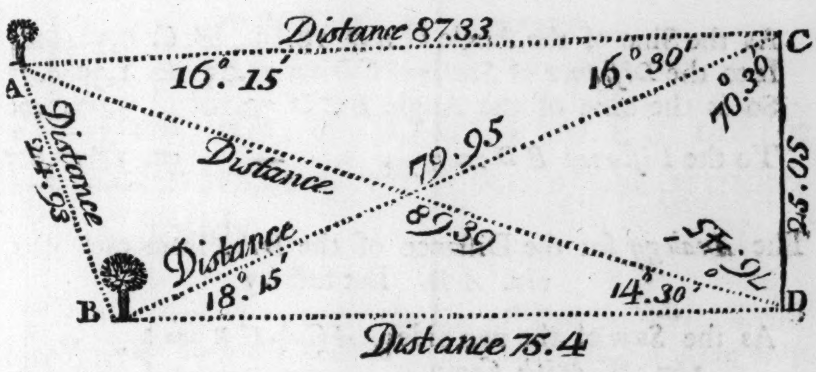
The same might have been equally found by the other *Right-angled* Triangle *C D B*, by the same *Method*.

2. *Longi*

2. *Longimetry*, or the Mensuration of the *Distances* of *Objects*, either from Us, or from one another, is thus performed.

Let *A*, *B*, be two *Trees*; and let it be required to find the *Distance* of *A* from *C* or *D*; as also of *B* from the same two Points; and the *Distance* of *A* from *B*.

Having (by a *Theodolite* or *Semicircle*) at *D*, found the Angles *B D C*, and *A D B*; and at *C* the Angles *A C D* and *A C B*; and measured the *Distance* of the *two Stations C D*, as below: Use the following *Analogies*.



The *Analogy* for finding *A C*.

As the Sine of the Angle *C A D* 16°.15' *A. C.* 0,5531072
Is to the *Distance* of *Stations C D* = 25.05 = 1,3988077

So is the *Sine* of the Angle *A D C* 70°.45' — 9,9882821

To the *Distance A C* = 87.33 — = 1,9411972

The *Analogy* for the *Distance A D*.

As the *Sine* of the Angle *C A D* 16°.15' *A. C.* 0,5531072
Is to the *Distance* of *Stations C D* = 25,05 = 1,3988077

So is the Sine of *A C D* 87°.00' — = 9,9994044

To the *Distance A D* = 89.39 — = 1,9513193

 The

The *Analogy* for the *Distance* BC.

As the Sine of the Angle *CBD* $18^{\circ}.15'$	*A. C.*	0,5042284
Is to the Distance of *Stations* CD 25.05	=	1,3988077
So is the Sine Comp. of the Angle *BDC* (to 180) $88^{\circ}.15'$	—	9,9997974
To the *Distance* BC = 79.95	— =	1,9028335

The *Analogy* for the *Distance* B D.

As the Sine of the Angle *C B D* $18^{\circ}.15'$	*A. C.*	0,5042284
Is to the *Distance* of *Stations* *CD* = 25.05	=	1,3988077
So is the Sine of the Angle *B C D* 70.30		9,9743466
To the *Distance* *B D* = 75.4	— =	1,8773827

The *Analogy* for the Distance of the *Trees* from each other, *viz.* *A B*. But first say

As the *Sum* of the two Sides, *AC* + *CB* = 167.28 *Arith. Comp.*	— —	7,7765050
Is to their *Difference* *AC* — *CB* = 7.38	=	0,8680564
So is the *Tang.* of half the Angles *ABC*+*BAC* $81^{\circ}.45'$	— — —	9,9954822
To the Tangent of half their Difference *ABC* — *BAC* $2^{\circ}.30'$	—	8,6400436

Then the Angle *CBA* $84^{\circ}.15'$; and the Angle *BAC* $79^{\circ}.15'$.

Wherefore the *Analogy* for the *Distance* *A B*, is,

As the Sine of the Angle *ABC* $84^{\circ}.15'$	*A. C.*	0,0021917
Is to the *Distance* of *A C* = 87.33	— =	1,9411972
So is the Sine of the Angle *A C B* $16^{\circ}.30'$	—	9,4533418
To the *Dist.* of the *two Trees* *AB* = 24,93	=	1,3967307

Thus

Thus I have endeavoured to make it appear how abſolutely neceſſary the Uſe and Knowledge of the noble Art of *Decimal Arithmetick* is in thoſe Parts of *Mathematical Science*, which require *Trigonometrical Calculations*, by Examples in the moſt common and uſeful Arts; I might have gone farther, and ſhewn its Uſe in ſeveral Parts of *Aſtronomy*, &c. but I intend only an inſtructive *Specimen* of its excellent Uſe in this Kind of Learning, and ſuch I preſume this Chapter will be found to be.

CHAP. XIV.

The great Uſe of DECIMALS *in the* Menſuration *of all Kinds of* Superficies *and* Solids.

IN this *Tract* (wherein the *Uſe of Decimal Arithmetick* is moſt obvious, neceſſary, and excellent) I have only this to advertiſe the *Reader*, That the Numbers are abſolutely taken; and may repreſent any *Dimenſions*, as *Inches*, *Feet*, *Yards*, *Rods*, *Miles*, *Acres*, &c. in the *Area's*, and *Solid Content* of Bodies. And that after the *Area*, or *Content* is found, I ſhall ſhew the Manner of Reducing it to any of the *Dimenſions* uſed in *Surveying*, *Guaging*, &c. by means of *Decimals*.

Propoſition 1. *To meaſure a* Square.

Rule. Multiply a Side into it ſelf, the Product is the *Area*, or *Square Content*.

Example, Suppoſe the Side *AB* = 12,6
Multiply by it ſelf — 12,6

9) 760

844
2533
12666

The *Square Content* or *Area* is = 150,4

Propo-

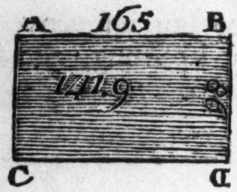
Proposition 2. *To Measure a* Paralellogram.

Rule. Multiply the *Length* by the *Breadth*, the Product is the *Area*, or *Content*.

Example. Mult. the *Length* AB = 16,5
By the *Breadth* BD = 8,6

990
1320

The Product is the *Content* = 141,9



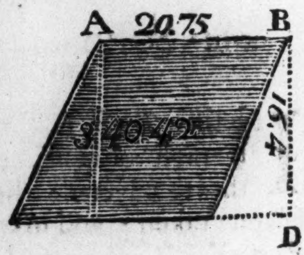
Proposition 3. *To Measure a* Rhombus.

Rule. Multiply one Side into the *perpendicular Height*, the Product is the *Area* or *Content* required.

Exam. Mult. the Side *AB* = 20,75
By the *Height* BD = 16,4

8300
12450
20750

The Product is the *Area* = 340,300



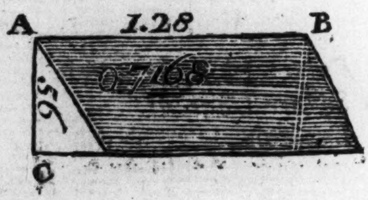
Proposition 4. *To Measure a* Rhomboides.

Rule. Multiply the *Length* by the *perpendicular Height*, or *Breadth*, the Product is the *Content*.

Ex. Mul. the *Length* AB = 1,28
By *Breadth* or *Height* AC = ,56

768
640

The Prod. is the *Area* = 0,7168



Proposition 5. *To Measure a* Plain Triangle.

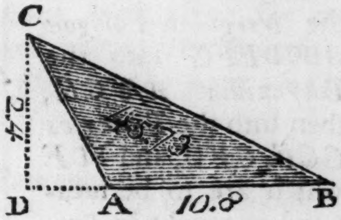
Rule. Multiply the *Base* into half the *Perpendicular Height*; or the *Whole perpendicular Height* into half the *Base*; the Product will give the *Area*.

Ex.

Exam. Mult. the *Base* *AB* = 10,8
By half the *Height* *CD* = 4,2

217
4355

The Product is the *Area* = 45,73



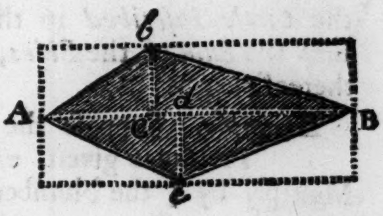
Proposition 6. *To Measure a* Trapezium.

Rule. Multiply the *Diagonal* into the *half Sum* of the two *Perpendiculars*; or the *Contrary*; and the Product will be the *Area* or *Superficial Content.*

Ex. Mul. the *Diagonal* *AB* = 10,5
By *the* ½ *Sum* of *the Perpendiculars be de* } = 3,3

315
315

The Prod. is the *Content* = 34,65



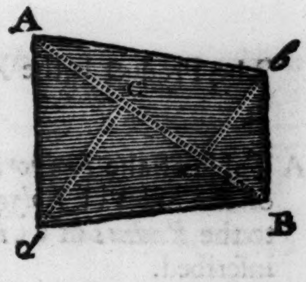
Proposition 7. *To Measure a* Parallelopleuron.

Rule. Multiply the *Diagonal* by the half *Sum* of the two *Perpendiculars*, the Product is the *Area.*

Exam. Mult. the *Diagonal* *AB* = 14,3
By ½ *Sum* of *cd* and *eb* = 6,3

430
860

The Product is the *Content* = 90,3

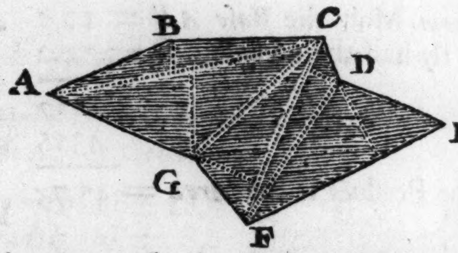


Proposition 8. *To Measure an* irregular Polygon, *or* Polygram.

Rule. Divide all such *multangular* and *irregular* Figures into *Trapeziums* and *Triangles*, then measure them by *Prop.* 5 and 6.

Example.

Example. Divide the *irregular Polygon* ABCDEFG into the *Trapezium* A B C G, then into the *Triangles* GCF, CFD, and FDE which are to be measured as taught in *Proposition* 5, and 6, and the Sum of all added together, will give the *Superficial Content* or *Area* of the given *Polygram.*



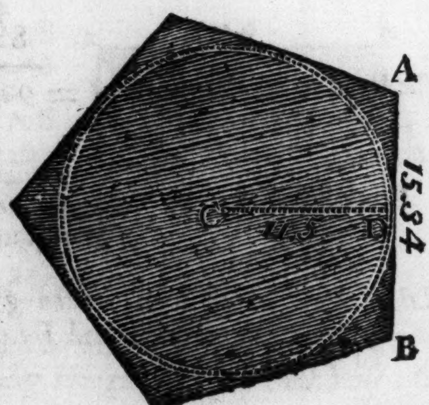
Proposition 9. *To Measure any* regular Polygon, *as a* Pentagon, Hexagon, Heptagon, Octagon, Nonagon, *&c.*

Rule. Multiply half the *Sum* of its *Sides* into the *Radius* of the *Circle inscribed* in the *Figure* ; or *half* that *Radius* into the *Sum* of the *Sides*, the Product will be rhe *Area* thereof.

Example. Suppose the Side of the *Pentagon* given, *viz.* —	} *A* B =	15,34
Multiply by ½ the Number of Sides	—	2,5
		7670
		3068
The Product is the ½ Sum of the Sides	— =	38,35
Multiply that by the *Radius* C D	— =	10,5
		19175
		38350
The Product is the *Area*	— =	402,675

A Table of the *Proportion* of the Sides of *Polygons* to the *Radius* of a *Circle* inscribed.

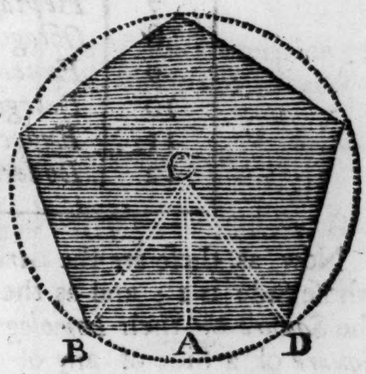
Trigon,	as 1:0,288675 &c.
Pentagon,	as 1:0,68819 &c.
Octagon,	as 1:1,207106 &c.
Decagon,	as 1:1,538841 &c.
Dodecagon,	as 1:1,866320 &c.



Propo-

Proposition 10. *To find such constant Multipliers for any of the* Regular Polygons, *That* multiplying *the* Square *of any* Side *thereby, the* Product *shall be the* Area *of the* Polygon.

Example. In a *Pentagon.*



Div. the whole Circle, *viz.* 360 Deg.
By the Number of Sides; here 5.
The Quotient is the $\angle BCD = 72°$
The $\frac{1}{2}$ thereof is the $\angle ACB = 36°$
Whose Comp. is the $\angle ABC = 54°$

Then make this *Proportion*;

			Arith. C.
As the Sine of the Angle *ACB*	= 36°	=	0,2307813
Is to half the Side (= 1, always)	= ,5	=	,9,6989700
So is the Sine of the Angle *ABC*	= 54	=	9,9079576
To the Perpendicular, or *Radius* of the *inscribed Circle* —	*AC* = ,68819	=	,9,8377089

Then (by the last *Proposition*) *Arith. Comp.* = 0,68819
Multiplied into $\frac{1}{2}$ Sum of the Sides — = 2,5

344095
137638

The Product is the *Area* — — = 1,720475

And thus may the *Area* for any other *Polygons* be found whose Side is 1. And this *Area* will be the *constant Multiplier* for that kind of *Polygon.* A Table of such *Multipliers,* or *Area's,* for the several *Regular Polygons* follow.

Sides.	*Names.*	*Multipliers.*
3	*Trigon*	0,433013
4	*Tetragon*	1,000000
5	*Pentagon*	1,720475
6	*Hexagon*	2,598076
7	*Heptagon*	3,633959
8	*Octagon*	4,828427
9	*Enneagon*	6,181827
10	*Decagon*	7,694209
11	*Endecagon*	8,514250
12	*Dodecagon*	9,330125

Now as these are the *Area's* of each *Polygon* respectively, whose Side is 1; and as the *Area's* of *Like Figures*, are as the *Square* of their *homologous*, or *like* Sides; therefore the *Square* of a Side of any of those *Polygons* multiplied into its respective *Area* in the Table, will produce the true *Area* thereof.

Example. Suppose the Side of a *Heptagon* be 10; the *Square* of which is 100; but $100 \times 3,633959 = 363,3959 =$ *Area* of such a *Heptagon*, and the like for any other.

Proposition 11. *To Measure a* Circle.

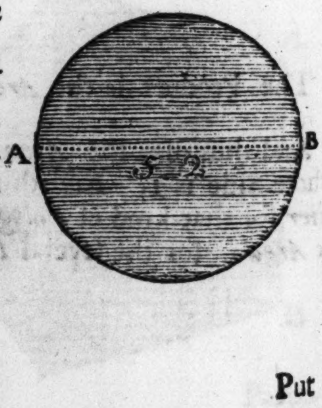
Rule. Multiply the *Square* of the *Diameter* (if that be given) by 0,7854; the Product is the *Area*. Or, (if the *Periphery* be given) Multiply the *Square* of the *Periphery* by 0,07957; the Product is the *Area*, as before.

Exa. Suppose the *Diameter* of a *Circle* be $\Big\}$ $AB = 5,2$

$$\begin{array}{r} 5,2 \\ \underline{5,2} \\ 104 \\ \underline{260} \end{array}$$

The *Square* thereof is $=$ 27,04
Which multiply by — ,7854

$$\begin{array}{r} \overline{10816} \\ 13520 \\ 21632 \\ 18928 \\ \hline \end{array}$$



The *Area* of the *Circle* is $=$ 21,237216

Put

Put $D = Diameter$; $P = Periphery$; and $A = Area$, of any *Circle*.

Then it will be $\left\{\begin{array}{ll} 3{,}1416D = P. & \text{And } 0{,}7854DD = A. \\ 0{,}3183P = D. & \text{And } 0{,}07957PP = A. \\ \sqrt{1{,}2732} = D. & \text{And } \sqrt{12{,}5664A} = P. \end{array}\right.$

Thus by thoſe ſix *Theorems* may all the Varieties relating to the *Diameter*, *Periphery*, and *Area's* of *Circles* be ſolved.

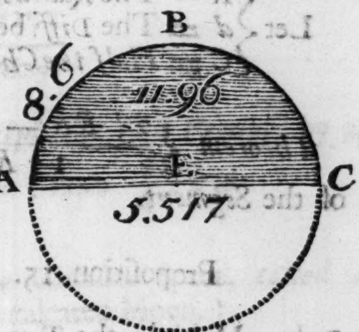
Propoſition 12. *To Meaſure a* Semicircle.

Rule. Multiply half the *Semicircular Arch*, into half the *Diameter*; the Product is the *Area*.

Exam. Multiply $\frac{1}{2}$ $ABC = 4{,}33$
Into $\frac{1}{2}$ $AC = EC = 2{,}76$

```
  2600
 30333
 86666
```

The *Semicircular Area* $= 11{,}96$

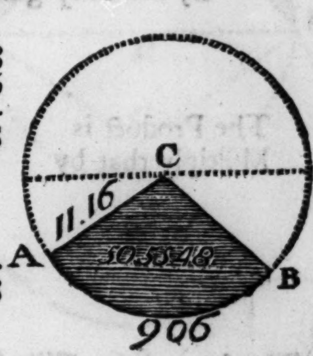


Propoſition 13. *To Meaſure the* Sector *of a* Circle.

Rule. Multiply half the *Arch* into the *Radius*, the Product is the *Area* of the *Sector*.

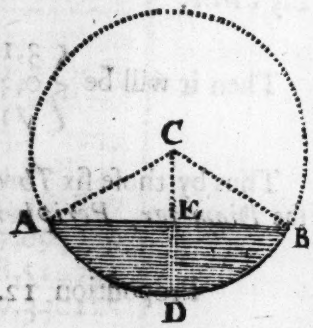
Example. Multiply $\frac{1}{2}$ $AB = 4{,}53$
Into the *Radius* $AC = 11{,}16$

```
   2718
   453
  453
 453
 50,5548
```



Propoſition 14. *To Meaſure the Segment of a* Circle.

Rule. Compleat the *Sector ACBD*, and meaſure it by the laſt *Propoſition*; and then find the *Area* of the *Triangle ABC* by *Propoſition* 5. Then ſubduct the *Area* of the *Triangle* from the *Area* of the *Sector*, the *Remainder* is the *Area* of the *Segment*.



Or thus, (by the Curious *Theorems* of Mr. *Ward.*)

Let $\begin{cases} R = \text{The Radius, or Semidiameter } AC. \\ d = \text{The Diff. between the versed Sine and Radius.} \\ C = \text{Half the Chord or Base of the Seg } AE \text{ (viz. EC.)} \end{cases}$

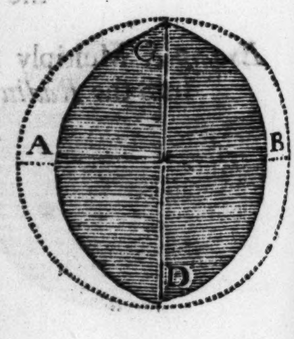
Theorem $\left\{ \dfrac{2\frac{1}{3}\, RR - \frac{1}{3}\, Rd - dd}{1\frac{1}{3}\, R + d} \right. \times C = S,$ The *Area* of the *Segment*.

Propoſition 15. *To Meaſure* an Ellipſis.

Rule. Multiply the *Tranſverſe* and *Conjugate Diameters* into each other; then *multiply* that *Product* by the Number 0,7854, the *Product* is the *Area* required.

Exam. Mult. the *Tranſverſe* CD = 36
By the *Conjugate* AB = 16

		216
		36
The Product is	—	576
Multiply that by	—	0,7854
		2304
		2880
		4608
		4032
The Area of the *Ellipſis*	=	452,3904



Propo-

Propoſition 16. *To Meaſure the* Parabola.

Rule. Multiply the *Greateſt Ordinate*, or *Baſe*, into the perpendicular *Height*, and that *Product* by the ſingle *Repetend* ,6, the *Product* is the *Area.*

Ex. Mult. the *Ordinate* $AB = 53{,}75$
By the *Abſciſſa*, or *Height*, $CD = 43{,}3$

$$\begin{array}{r} 16125 \\ 16125 \\ 21500 \\ \hline \end{array}$$

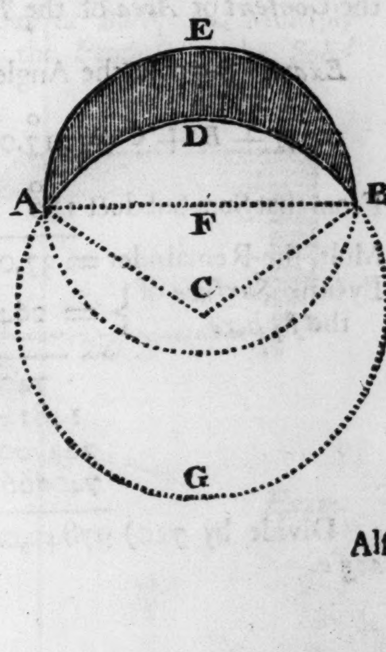
Multiply this Product $= 2327{,}375$
By the *Repetend* $-\ ,6$

$$9)13966250$$

The Area of the *Parab.* $= 1551{,}805$

Note, an eaſier way is to *multiply* the *firſt Product* by 2, and then divide by 3 for the *Area*; ſince $\frac{2}{3} = ,6$

Propoſition 17. *To Meaſure the* Circular Space, *called a* Lune; (*being like the* falcated Moon.)



Rule. In order to find the *Area* of the Lune *A E B D*, ſeek firſt the *Area* of the *Semicircle A E B*, by *Prop.* 12.

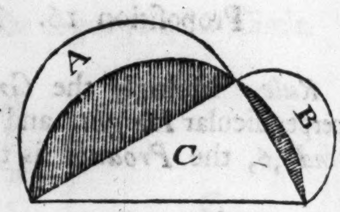
Then find the *Area* of the *Segment A D B F* of the *Circle A D B G*, by *Prop.* 14.

Laſtly; Subſtract the *Area* of the *Segment* from the *Area* of the *Semicirle*; there remains the *Area* of the *Lune* required.

Alſo

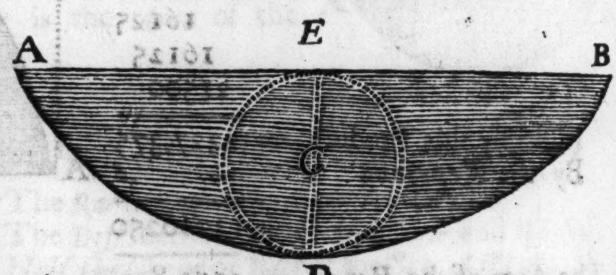
Alſo the *two Lunes A*, B, are together equal to the *Triangle C*.

Note, This is call'd the *Quadrature*, or Squaring the *Lunes* of *Hippocrates*.



Propoſition 18. *To Meaſure the* Cycloid.

Rule. Find the *Area* of the *Circle C*, deſcribed on the *Axis DE*, and multiply that by 3, the *Product* is the *Area* of the *Cycloid* (called alſo the *Trochoid*) *A E B D*.



Propoſition 19. *To Meaſure a* Spherical Triangle.

Rule. From the *Sum* of the *three Angles*, ſubduct 180 Degrees, multiply the *Superficies* of the whole *Sphere* or *Globe* by the Remainder; this Product divide by 720, the *Quotient* is the *Content* or *Area* of the *Triangle*.

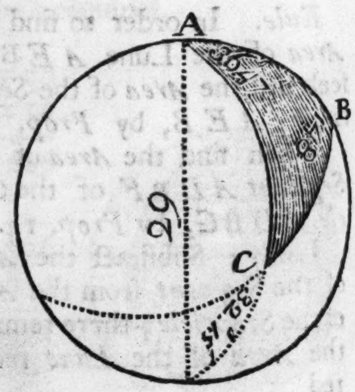
Exam. Suppoſe the Angles $\left. \begin{array}{l} \\ A + B + C = 217^\circ.02' \end{array} \right\}$

From that Sum Subduct 180°

Mult. the Remainder = 37,03

By the *Surface* of the *Sphere* $\}$ = 2642

7406
14813
22220oo
7406666



Divide by 720) 97842,06 (135,891 = *Area* of the *Triangle*.

Note,

Note, This is a very *uncommon*, *curious*, and *uſeful* Propoſition.

Menſuration *of* Solids.

Propoſition 20. *To Meaſure a* Cube.

Rule. Multiply the *Side* of the *Cube* into it ſelf, and that Product again by the *Side* ; this laſt Product will be the *Solid Content*, or *Solidity* of the *Cube*.

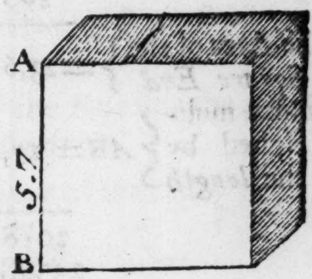
Exam. Mult. the Side *AB* = 5,7
By it ſelf — — 5,7

399
285

This Product — 32,49
Multiply again by — 5,7

22743
16245

The Product is — 185,193 = *Solidity* of the *Cube*.



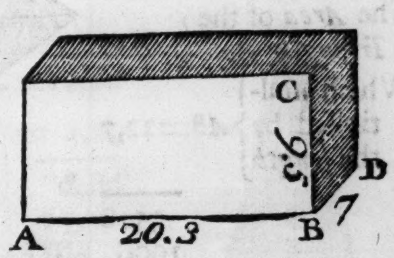
Propoſition 21. *To Meaſure a* Parallelopipedon.

Rule. Find the *Area* of the *End* or *Baſe*, and Multiply that by the *Length* of the *Piece*, the Product is the *Solid Content* thereof.

Exam. Mult.. *C B* = 9,5
By — *B D* = 7

The *Area* of the End — } = 66,5
Mult, that by the *Length A B* } = 20,3

The *Solidity* — = 1349,95



Exam-

Example 2. Of a Square *Prism*, or *Parallelopipedon.*

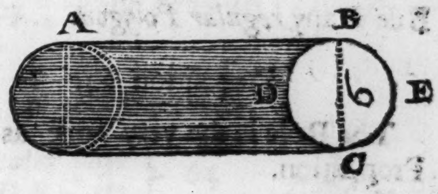
Mult. into it self, or *Square* the Side of the *Square End*	$BC = 5,8$
	5,8
	464
	290
The Area of the *Square End*	$= 23,64$
Which multiplied by the *length*	$AB = 17,6$
	20184
	23548
	3364
The Product is	592,064 = *Solid Content.*

Example 3. Of a *Triangular Prism.*

Mult. the *Base* BC	= 8,4
By the *Height* DE	= 6
The *Area* of the *Trianglar Base*	50,4
Which multiplied by the *length*	$AB = 22,7$
	3528
	1008
	1008
The Prod. is	= 1144,00 = *Solidity* of the *Prism*.

Exam-

Example 4. Of a *Cylinder*.



Square the *Diameter* of the *End BC*	$= 6,6$ $6,6$
The Area of the *circular Base*	$= 43,4$
Which mult. by the *Length AB*	$= 23$
The *Solidity* of the *Cylinder*	$= 1022,2$

Proposition. 22. *To Measure the Convex Superficies* of a *Cylinder*.

Rule. Multiply the *Periphery* of the *Base* into the *Length* of the *Cylinder*; the Product is the *Content*.

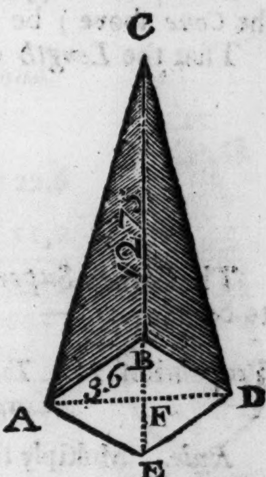
Example. Suppose the *Circumference* of the *Base* (in the last Figure) *BECD* to be — $20,944$
Then multiply that by the *Length AB* $= 23$

62832
41888

The *Superficial Content* of the *Cylinder* $= 481,712$

Proposition 23. *To Measure a Pyramid.*

Rule. Mulriply the *Area* of the *Base* into one Third of its *Altitude* or *Height*; the Product is the *Solid Content*.



Exam. Suppose the *Area* of the *Square Base ABDE* $= 12,96$
Multiply that into $\frac{1}{3}$ of *FC* $= 4,25$

6480
2592
5184

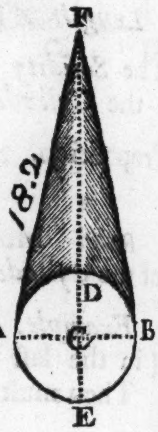
The *Solid Content* of the *Pyramid* — — $= 55,08$

Note

Note, The Rule is general for any kind of *Pyramid* whose Base is any *regular Polygon.*

Proposition 24. *To Measure a* Cone.

The Rule is the very same as for the *Pyramid* in the last Proposition.



Example. Suppose the *Circular Base A D B E* be in *Area* } = 39,5
And ⅓ of the *Height F C*, be = 3,85

1975
3160
1185

The *Solidity* of the *Cone* will be = 152,075 A

Proposition 25. *To Measure the* Curve Superficies *of a* Cone.

Rule. Multiply the *Periphery* of the *Base* into the *Length* of the *Side*; Half that Product is the *Content* or *Area* of the *Curve Surface.*

Example. Suppose the *Periphery A D B E* (of the *Cone* above) be — — — } 23,6
That the *Length* of its *Side A F* — = 18,2

472
1888
236

429,52

The *Convex Superficies* of the *Cone* will be found to be — — — — } 214,76

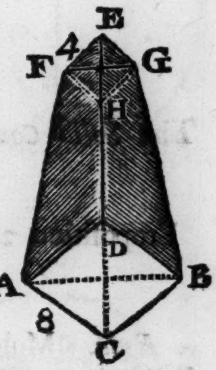
Proposition 26. *To Measure the* Frustum *of a* Pyramid *or* Cone, *cut* parallel *to its* Base.

Rule. Multiply the *Area* of the *Greater Base*, by the *Area* of the *Lesser*, and extract the *Square Root* of the Product;

To

To that *Root*, add the *Sum* of the two *Area's* of the End; then multiply this *last Sum* by $\frac{1}{3}$ of the *Frustum's Height*, the Product is the *Solid Content*.

Example 1. Of any *Pyramid*.



Suppose the *Area* of the *Greater Base* (of a *Square Pyramid* *A D B C* = } 64
And the *Area* of the *Lesser FEGH* = 16

384
64

The Product is — — 1024
The *Square Root* thereof is = 32
To which add the Sum of the 2 *Area's* — = } 80

This Sum — — 112
Multiply by $\frac{1}{3}$ of the *Height* D E = 4,93

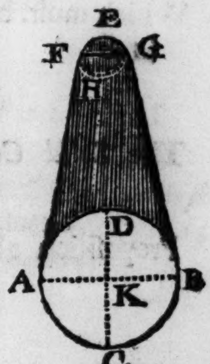
9)336

373
1008
448

The *Solid Content* — = 552,53 of the *Square Frustum*.

And thus the *Frustum* of any other kind of *Pyramid* is to be found.

Example 2. Of a *Cone*.



Suppose the *Area* of the *Greater Base* (of the *Cone* adjoin'd) *A D B C* = } 122,6
And the *Area* of the *Lesser FEGH* = 11,2

2452
1226
1226

The Product is — 1373,12

The

The *Square Root* thereof is — = 37,055 &c.
To which add the *Sum* of the two Area's = 133,8

This *last Sum* — — — 170,855
Multiply by $\frac{1}{3}$ of the *Height* KE = 6,75

854275
1195985
1025130

The *Solid Content* — = 1153,27125

Proposition 27. *To Measure a* Sphere, *or Body* perfectly Round.

Rule. Multiply the *Diameter* into the *Circumference*, the Product is the *Superficial Content*; then multiply that by $\frac{1}{6}$ of the *Diameter*, the Product will be the *Solid Content* of the *Sphere*.

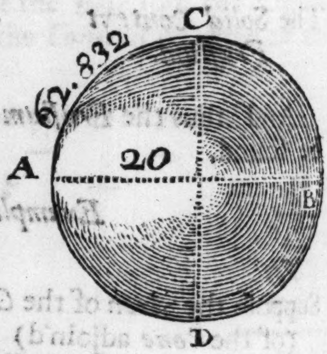
Example. Of the *Superficial* and *Solid Content*.

Mult. the *Periphery* of the *Globe* or Sphere, *ACBD* = } 62,832
By the *Axis* of the *Sphere* AB = } 20

The *Spherical Superficies* = } 1256,640
Which mult. by $\frac{1}{6}$ of *AB* = } 3

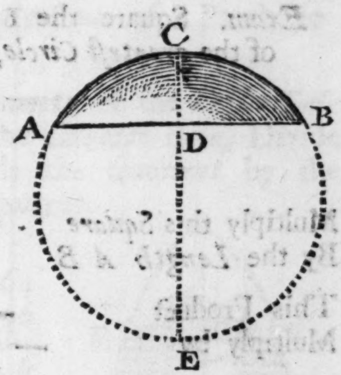
9) 3769,92

The *Solid Content* = 4188,8 of the *Sphere*.



Proposition 28. *To Measure any* Frustum *or* Segment *of a* Sphere *or* Globe.

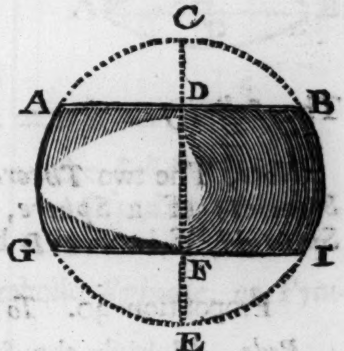
1. Let



1. Let $\left\{\begin{array}{l} D = \textit{Diameter of the} \\ \textit{Sphere } CE \\ H = \textit{Height of the Seg-} \\ \textit{ment } CD \end{array}\right.$

Then $\frac{3DHH - 2H^3}{1,91}$ = The *Content.*

Of the *Polar Segment ACBD.*



2. Let $\left\{\begin{array}{l} D = \textit{Diameter } CE\text{, as} \\ \text{before,} \\ x = \text{the } \textit{Baſe } AB = GI. \\ H = \text{the } \textit{Thickneſs } DF. \end{array}\right.$

Then $\frac{2DD + xx}{3,82} \times H$ = the *Middle Segment ABIG*, called the *Zone.* By theſe two *Theorems*, may the Solidity of any *Segment* of a *Globe* or *Sphere* be found.

To find the *Superficial Content* of any *Segment*, as

Thus $\left\{\begin{array}{l} \text{As the } \textit{Axis}\text{, or } \textit{Diameter} \text{ of the } \textit{Sphere}, \\ \text{Is to the whole } \textit{Superficies} \text{ of the } \textit{Sphere}; \\ \text{So is the } \textit{Height} \text{ of any } \textit{Segment}, \\ \text{To the } \textit{Area} \text{ of its } \textit{Curve Superficies}. \end{array}\right.$

Propoſition 29. *To find the* Content *of a* Spheroid.

Rule. Multiply the *Square* of the *Diameter* of the *Greateſt Circle*, by the *Length*; then multiply that Product by 0,5236; this *laſt Product* will be the *Solidity* of the *Spheroid.*

Exam.

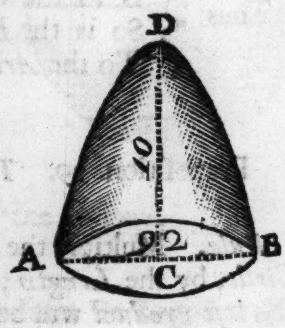
Exam. Square the *Diameter* of the *greatest Circle*, CD = 6,5
6,5
———
325
390
———
Multiply this *Square* — = 42,25
By the *Length A B* — = 10
———
This Product — 422,5
Multiply by — — ,5236
———
25350
12675
8450
21125
———
The *Solidity* — = 221,221 of the *Spheroid.*

Note, The two *Theorems*, which find the *Content* of the *Segments* of a *Sphere*, find those like *Segments* of the *Spheroid*, if in them *D* be made = *C D*, in this *Spheroid.*

Proposition 30. *To Measure a* Parabolic Conoid.

Rule. Multiply the *Square* of the *Diameter* of the Base by the *Height*; and that Product by 0,3927; this last Product is the *solid Content.*

Exam. Mult. into it self, or *square* the *Diameter AB* = 9,2
9,2
———
184
828
———
This *Square* — 84,64
Multiply by the *Height CD* = 10
———
And the Product — 846,4
Multiply by — ,3927
———
59248
16928
76176
25392
———
The *solid Content* = 332,38128 of the *Parabolic* Conoid.

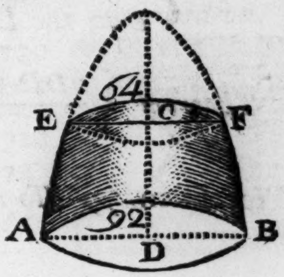


Propo-

Propofition 31. *To Meafure the* Fruftum *of a* Parabolic Conoid.

Rule. Add the *Square* of the *Diameter* of the *Leffer Bafe* to the *Square* of the *Diameter* of the *Greater Bafe*, Divide that *Sum* by 2,5464; then multiply the *Quotient* by the *Height*; the Product is the *Solid Content.*

Exam. The *Square* of $AB = 84,64$
And the *Square* of $EF = 40,96$
Sum of the *Squares* — $= 125,6$

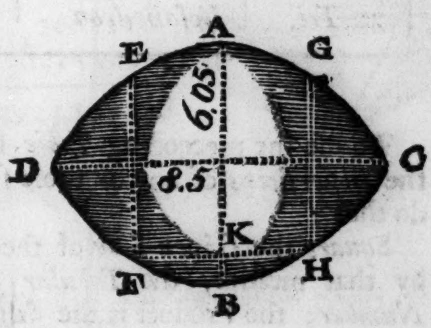


Then 2,5464) 125,600 (49,3244
101856   5 = *CD* the *Height.*
237440   246,622 = the *Solidity* of the
229176   *Fruftum ABFE.*
8264 &c.

Propofition 32. *To Meafure a* Parabolic *Spindle, or* Pyramidoid.

Rule. Multiply the *Square* of the *Diameter* of the *Greateft Circle*, by the *Length*; and that Product by 0,41888; this laft Product is the *Solid Content* of the *Spindle.*

Exam. 1. *Square* the *Diameter* of the *Greateft Circle AB* } 6,05
6,05
3025
36300
Mult. this *Square* 36,6025
By the *Length CD* = 8,5
1830125
2928200
This Product 311,12125
Multiply by ,41888



Solid Cont. = 130,3224692 of the *Parabolic Spindle.*

Example

Example 2. To Measure the Middle *Segment* of the *Spindle*, *EGHF*.

Let $\begin{cases} D = \text{the } \textit{Diameter} \text{ of the } \textit{greatest Circle } AB. \\ C = \text{the } \textit{Diameter} \text{ of either } \textit{Base} \text{ or } \textit{End } EF \text{ or } GH. \\ x = 2BK = \text{the } \textit{Excess} \text{ of } AB \text{ above } EF, \text{ or } GH. \\ L = \text{the } \textit{Length} \text{ of the } \textit{Frustum } FH. \end{cases}$

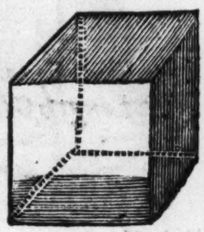
Theorem $\left\{ \dfrac{2DD + CC - 0,4xx}{3,82} \times L = \right\}$ *Solidity* of the *middle Frustum*.

Proposition 33. *To Measure any of the Five* Regular (*or* Platonic) *Bodies*.

Those Bodies being only an *Aggregate* of so many *Pyramids* as they consist of *Sides*, *each Side* being the Base of a *Pyramid*, may, with due Consideration, be measured by *Proposition* 22. However for the more ready and expeditious Practice, I shall here subjoin a Table of the *Solidity* and *Superficies* of each Body whose *Side* is 1. or *Unity*.

Sides.	*Names.*	*Superficies.*	*Solidity.*
4 *Tri.*	*Tetrahedron*	1,732051	0,1178511
6 *Sq.*	*Hexahedron*	6,000000	1,0000000
8 *Tri.*	*Octahedron*	3,464102	0,4714045
12 *Pent.*	*Dodecahedron*	20,645729	7,663119
20 *Tri.*	*Icosahedron*	8,660254	2,181695

Hexahedron.

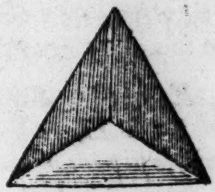


To use the preceeding *Table* for finding the *Superficies* of any of those five Bodies, do thus;

Square the *Given Side* of the Body, and by that multiply the *Tabular Superficial Number*; the Product is the *Superficies* of the Body, which was sought.

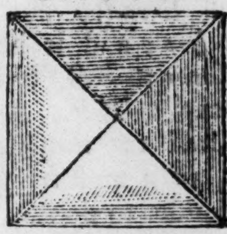
Exam-

Tetrahedron.



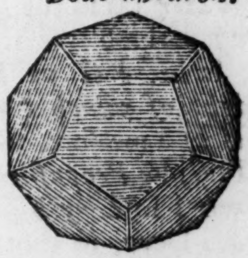
Example. Suppoſe the Side of the *Dodecahedron* be 8, the *Square* of which is 64.

Octahedron.



Then multiply the *Tab. Numb.*	20,645729
By the *Square* of the Side	64
	82582916
	123874374
Superficial Content =	1321,326656

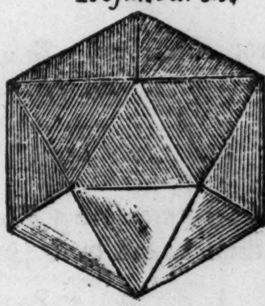
Dodecahedron.



To find the *Solid Content*; Multiply the *Tabular Number* of the *Solidity*, by the *Cube* of the *Side* given, the Product is the *Solid Content.*

Example. Of the aforeſaid *Dodecahedron.*

Icoſahedron.



Multiply the *Tabular Numb.*	7,663119
By the *Cube* of the Side (=8,) *viz.* —	512
	15326238
	7663119
	38315595
Solid Content =	3923,516928

And thus proceed for the *Superficies* and *Solidity* of the other Bodies.

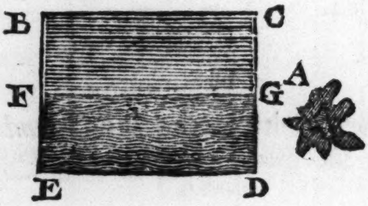
Propoſition 34. *To Meaſure any* ſolid *or* hollow Body *how* irregular ſoever.

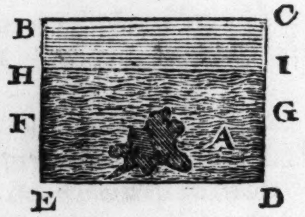
Rule. Take any *Veſſel* in Form of a *Parallelopipedon*, and fill it with *Water* to a certain *Height*, and then immerſe

 the

the *irregular Body* therein, and observe how much the Water is *raised* by the *Side* of the *Vessel*; for that *Water* is equal in *Quantity*, or *Solid Content*, to the *irregular Body*; and may be found by *Proposition* 20.

Example. Suppose the *Vessel* B C D *E*, in Form of a *Parallelopipedon*, whose *Length* is two *Feet*, eight *Inches*; and *Breadth* one *Foot*, ten *Inches*; and it be fill'd with *Water* to *F G*, twelves *Inches* Deep $= EF$. And it is required to measure the Log *A*, of a most *irregular Form*. In order to do this, I take and immerge the *Log* in the *Vessel* of *Water* (as in the *lower Figure*) and observe the Water rise from *F G* to *H I*, the *Height of which* (viz. *F H*, or *G I*) I measure, and find to be 5,5 *Inches*.





Then by *Proposition* 20, I find a Body of *Water*, 2 *Feet*, 8 *Inches* in *Length*; 1 *Foot*, ten *Inches* in *Width*, and 5,5 *Inches* Deep, to contain 3520 *Solid Inches*, or 2,037 *Solid Feet*; which therefore is the *Solid Content* of the *Log A*?

Proposition 35. *To assign the* Dimensions *by which the* several Artificers *measure their Work.*

Masons measure their Work by { *Feet* and *Inches* }:
- *Superficial* { As Pavements, Chimneys, Pieces, Cornishes, &c. }
- *Solid* { Columns, and other *solid* Parts of Buildings. }

Bricklayers measure by the:
- *Foot*; Arches, Quoins, Cornices, Fascia's, &c.
- *Yard*; Pavements, Pieces, &c.
- *Rod*; All manner of Walls, and Chimneys.
- *Square*; All manner of Tyling, and Slating.

Carpenters measure by the { *Square* of 100 Feet } { Roofing, Partitioning, Flooring, &c. }

Join-

Joiners, Painters, Plaisterers measure by the { *Square Yard* for the most part; seldom, by the Foot Square.

Glasiers measure their Work by the { *Decimal Foot* Square; very rarely by Inches and Quarters.

Solid Inches.

Gaugers measure the Area's and Content, of Superficies and Solids, by { 231, For Wine Gallons. 282, For Ale Gallons. 268,8. For Corn Gallons.

Surveyers measure Land by the { *Rod* or *Pole*; but mostly by the *Chain* of an 100 Links = 4 Rod.

Proposition 36. *To assign* Multipliers, *and* Divisors, *whereby the* Gauger *may readily find any* Area *or* Content *in* Gallons *or* Bushels, *whether the* Dimensions *be taken in* Inches, Feet, *or* Yards.

This I shall do by disposing the Numbers in their proper Order in the Table subjoin'd.

Dimensions.	*Multipliers.*		*Divisors.*	
Inches.	0.004329	*W. G.*	231	*W. G.*
	0.003546	*A. G.*	282	*A. G.*
	0.003722	*C. G.*	268.8	*C. G.*
	0.000465	*C. B.*	2150.42	*C. B.*
Feet	7.48052	*W. G.*	0.13368	*W. G.*
	6.12765	*A. G.*	0.16352	*A. G.*
	6.42448	*C. G.*	0.15565	*C. G.*
	0.80356	*C. B.*	1.24446	*C. B.*
Yards	67.32468	*W. G.*	0.014853	*W. G.*
	55.14885	*A. G.*	0.018168	*A. G.*
	57.82[illegible]52	*C. G.*	0.017294	*C. G.*
	7.23204	*C. B.*	0.138273	*C. B.*
Note,	*W. G.* } stands for		*Wine Gallons.*	
	A. G. } stands for		*Ale Gallons.*	
	C. G. } stands for		*Corn Gallons.*	
	C. B. } stands for		*Corn Bushels.*	

The Use of the preceeding Table.

If by the foregoing *Propositions* the *Area* or *Content* of any *Superficies* or *Solid* be found; and you would know how many *Wine*, *Ale*, or *Corn Gallons*, or *Bushels* it contains; Multiply, or Divide, the given *Area* or *Solidity*, by the *Tabular* Number corresponding to the *respective Measure*, and *Dimension*, the Product is the *Area* or *Content* sought in *Gallons*, or *Bushels*.

Example. Suppose the *Parallelopipedon* in *Example* 1. of *Proposition* 20. represent a *Cistern*, and the *Dimensions* there used be *Feet*; then the *solid Content* of the *Cistern* is there found to be 1349,95 *Solid* or *Cubick Feet*.

$$\text{Then}\left\{\begin{array}{l} 1349{,}95\times7{,}48052=\text{Content in } \textit{Wine Gallons.}\\ 1349{,}95\times6{,}12765=\text{Content in } \textit{Ale Gallons.}\\ 1349{,}95\times6{,}42448=\text{Content in } \textit{Corn Gallons.}\\ 1349{,}95\times0{,}80356=\text{Content in } \textit{Corn Bushels.}\end{array}\right.$$

Or by Division.

$$\text{Thus}\left\{\begin{array}{l} {,}13368)\ 1349{,}95\ (=\text{Content in } \textit{Wine Gallons.}\\ {,}16352)\ 1349{,}95\ (=\text{Content in } \textit{Ale Gallons.}\\ {,}15565)\ 1349{,}95\ (=\text{Content in } \textit{Corn Gallons.}\\ 1{,}24446)\ 1349{,}95\ (=\text{Content in } \textit{Corn Bushels.}\end{array}\right.$$

And were *Dimensions* taken by a *Decimal Yard* or *Foot* (which are by far the best *Instruments* for *Mensuration*;) The Business of *Gauging* would be easy, and greatly expedited by a *Table* not before extant, that I know of.

Note; If you would find the *Content* of *Circular Area's* at one *Operation* (without reducing them by the *common Multiplier* 0,785398) do thus,

$$\begin{array}{l}\text{Divide the Square of}\\ \text{the } \textit{Diameter} \text{ by}\end{array}\left\{\begin{array}{ll} 359{,}05 & \text{for } \textit{Ale Gallons.}\\ 294{,}12 & \text{for } \textit{Wine Gallons.}\\ 342{,}24 & \text{for } \textit{Corn Gallons.}\\ 2738, & \text{for } \textit{Corn Bushels.}\end{array}\right.$$

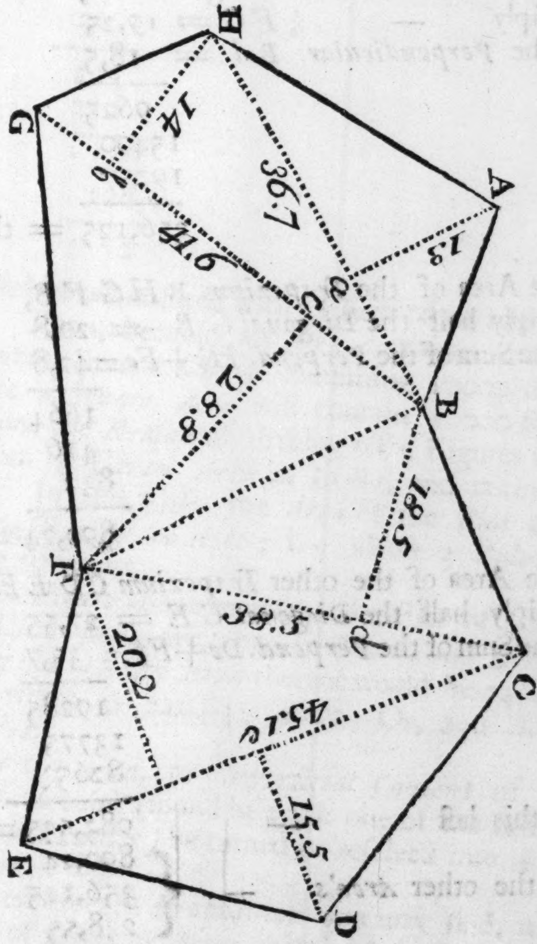
But

But in this Caſe the *Dimenſions* muſt be underſtood of Inches only.

By what I have ſaid in this *Propoſition*, I ſuppoſe the dexterous young *Artiſt* will be eaſily appriſed of the Nature, Manner, and Reaſon of *Gauging*; and how to apply the foregoing *Propoſitions* to that purpoſe.

Propoſition 37. *To apply the foregoing Propoſitions of* Superficial Menſuration *to* Surveying.

Suppoſe a *Field* in Form of an *Irregular Polygon*; as *ABCDEFGH*, below.



The

The Field being measured, *Plotted*, and the *Plot* resolved into *Trapezia*, and *Triangles*, as *per Proposition* 8.

Proceed to find the *Contents* of the *Triangles* by *Proposition* 5. and of the *Trapezia* by *Proposition* 6.

Thus for the *Area* of the *Triangle* *H A B*,
Multiply half the Base *H B* = 18,35
By the *Perpendicular* *A a* = 13

5505
1835

238,55 = the *Area*.

For the *Area* of the *Triangle* *F B C*,
Multiply — $\frac{1}{2}$ *F C* = 19,25
By the *Perpendicular* *B d* = 18,5

9625
15400
1925

356,125 = the *Area*.

For the Area of the *Trapezium* *B H G F B*,
Multiply half the *Diagonal* *G B* = 20,8
By the Sum of the *Perpend.* *Hb+Fc*=42,8

1664
416
832

890,24 = the *Area*.

For the Area of the other *Trapezium* *C D E F C*,
Multiply half the *Diagonal* *C E* = 27,55
By the Sum of the *Perpend.* *De+Ff* = 35,7

19285
13775
8265

To this last — 983,535 = the *Area*.

Add the other *Area's* — { 890,24
356,125
238,55 }

The Sum of all is the *Content* = 2468,45 = the *Field*.

Now

Now if the Numbers are ſuppoſed to be *Poles* or *Rods*, Divide the Whole *Superficial Content* or *Area* of the *Field*, by 160, the Quotient will be the Number of *Statute Acres* the *Field* contains. See the Work.

```
160) 2468,45 (15,4278 Acres ;
     16                  Acres  Rood  Rods  Yards
     ——                 Or, 15  :  1  :  28  :  13
      86
      80
      ——
       68
       64
       ——
       ·44
        32
        ——
        125
        112
        ———
        ·130
         128
         ———
         ···2
```

But if the *Field* was meaſured with a *Chain* of an 100 *Links* (= 4 Rods.) Then, becauſe an *Acre* contains 10 *Square Chains*, and every *Square Chain* containing 10000 *Square Links*, therefore a *Square Acre* will contain 10000 *Square Links*; and hence the *Reaſon* of ſtriking off 5 Figures to the Right-hand from a *Given Area* in *Links*, and taking the *Reſt for Acres*. In this Caſe, the *Area* of the *Plot* above would be but ,0246845 *of an Acre* ; i. e. about 3 *Poles* and *Yards*, *Square Meaſure*.

But ſuppoſe the *Figure* above repreſent the *Plot* of a large *Common* whoſe *Dimenſions* are taken in *Chains* and *Decimal Parts* of a *Chain :* Then the *Area* thereof would be 2468,45 *Square Chains*, which is 246,845 *Acres* ; Or, 246 *Acres*, *Roods*, 14 *Poles*.

And thus may any *Area*, or *Superficial Content* of any *Field* or *Plot* of Ground (found by ſome one of the firſt 19 *Propoſitions* of this *Chapter*) be turned or reſolved into *Acres* by this *Propoſition*.

Note, By the nineteenth *Propoſition* you may find, with Eaſe, the Number of *Miles or Acres* contained in the Whole, or any Part, of the *Superficies* of the *terreſtrial Globe* ; and of any *Province*, *Kingdom*, *Empire*, or *Nation* of the World; and

and it ought therefore to be well underſtood by all who would ſtudy ***Political Arithmetick***, this being the moſt certain, curious, and principal Branch of that Art; Tho' it be not in every Tract on that Subject, nor in any Book of Menſuration that I know of.

Thus I have finiſhed a Tract of *Planometry*, and *Stereometry*, or of the *Menſuration* of *Superficies* and *Solids*; containing a *Greater* Variety than I know to be in *many Books*, wrote wholly on the Subject, of two, three, or four *Shillings Price*; having endeavoured here, as in all other Parts of this *Syſtem*, to oblige the *Reader* with all that could be *uſeful* for him to know, in the moſt plain but *Compendious Manner*, at the eaſieſt Rate.

Vive, Vale; Si quid noviſti rectius iſtis,
Candidus imperti; ſi non, his utere mecum.

FINIS.

A

TABLE

OF THE

LOGARITHMS

TO ALL

NUMBERS,

Not exceeding 10000, or 4 Places, whether they be Intire, Broken, or Mixt Numbers.

Particularly uſeful in Extracting the Square Cube, &c. Roots, and ſolving Queſtions in Compound Intereſt, &c.

Natural Numbers.	0	1	2	3	4
1	0000000	0413927	0791812	1139433	1461280
2	3010300	3222193	3424227	3617278	3802112
3	4771212	4913617	5051500	5185139	5314789
4	6020600	6127838	6232493	6334684	6434527
5	6989700	7075702	7160033	7242759	7323937
6	7781512	7853298	7923917	7993405	8061800
7	8450980	8512583	8573325	8633229	8692317
8	9030900	9084850	9138138	9190781	9242793
9	9542425	9590414	9637878	9684829	9731278
100	0000000	0004341	0008677	0013009	0017337
101	0043214	0047511	0051805	0056094	0060379
102	0086002	0090257	0094509	0098756	0102999
103	0128372	0132587	0136794	0141003	0145205
104	0170333	0174507	0178677	0182843	0187005
105	0211893	0216027	0220157	0224284	0228406
106	0253059	0257154	0261245	0265333	0269416
107	0293838	0297895	0301948	0305997	0310043
108	0334237	0338257	0342273	0346284	0350293
109	0374265	0378247	0382226	0386202	0390173
110	0413927	0417873	0421816	0425755	0429691
111	0453230	0457140	0461048	0464952	0468852
112	0492180	0496056	0499928	0503797	0507663
113	0530784	0534626	0538464	0542299	0546130
114	0569048	0572856	0576661	0580462	0584260
115	0606978	0610753	0614525	0618293	0622058
116	0644580	0648322	0652061	0655797	0659530
117	0681859	0685569	0689276	0692980	0696681
118	0718820	0722499	0726175	0729847	0733517
119	0755470	0759118	0762762	0766404	0770043
120	0791812	0795430	0799045	0802656	0806265
121	0827854	0831441	0835026	0838608	0842187
122	0863598	0867157	0870712	0874264	0877814
123	0899051	0902580	0906107	0909631	0913151
124	0934217	0937718	0941216	0944711	0948204
125	0969100	0972573	0976043	0979511	0982975
126	1003705	1007151	1010593	1014033	1017471
127	1038037	1041455	1044871	1048284	1051694
128	1072100	1075491	1078880	1082266	1085650
129	1105897	1109262	1112625	1115985	1119343
130	1139433	1142773	1146110	1149444	1152776
131	1172713	1176027	1179338	1182647	1185954
132	1205739	1209028	1212314	1215598	1218880
133	1238516	1241780	1245042	1248301	1251558
134	1271048	1274288	1277525	1280760	1283993

Natural Numbers.	5	6	7	8	9
1	1760912	2041200	2304489	2552725	2787536
2	3979400	4149733	4313637	4471580	4623980
3	5440680	5563025	5682017	5797836	5910646
4	6532125	6627578	6720978	6812412	6901961
5	7403627	7481880	7558748	7634280	7708520
6	8129133	8195439	8260748	8325089	8388491
7	8750613	8808136	8864907	8920946	8976271
8	9294189	9344984	9395192	9444827	9493900
9	9777236	9822712	9867717	9912261	9956352
100	0021661	0025980	0030295	0034605	0038912
101	0064660	0068937	0073209	0077478	0081742
102	0107239	0111473	0115704	0119931	0124154
103	0149403	0153597	0157787	0161973	0166155
104	0191163	0195317	0199467	0203613	0207755
105	0232524	0236639	0240750	0244857	0248960
106	0273496	0277572	0281644	0285712	0289777
107	0314085	0318123	0322157	0326188	0330214
108	0354297	0358298	0362295	0366289	0370279
109	0394141	0398105	0402066	0406023	0409977
110	0433623	0437551	0441476	0445398	0449315
111	0472749	0476642	0480532	0484418	0488301
112	0511525	0515384	0519239	0523091	0526939
113	0549958	0553783	0557605	0561423	0565237
114	0588055	0591846	0595634	0599419	0603200
115	0625820	0629578	0633334	0637085	0640834
116	0663259	0666985	0670708	0674428	0678145
117	0700379	0704073	0707765	0711453	0715138
118	0737183	0740847	0744507	0748164	0751818
119	0773679	0777312	0780941	0784568	0788192
120	0809870	0813473	0817073	0820669	0824263
121	0845763	0849336	0852906	0856473	0860037
122	0881361	0884905	0888446	0891984	0895519
123	0916669	0920185	0923696	0927206	0930712
124	0951693	0955180	0958664	0962146	0965624
125	0986437	0989896	0993353	0996806	1000257
126	1020905	1024337	1027766	1031192	1034616
127	1055102	1058506	1061909	1065308	1068705
128	1089031	1092410	1095785	1099159	1102529
129	1122698	1126050	1129400	1132746	1136091
130	1156105	1159432	1162756	1166077	1169396
131	1189257	1192559	1195858	1199 54	1202448
132	1222159	1225435	1228709	1231981	1235250
133	1254813	1258064	1261314	1264561	1267806
134	1287223	1290450	1293676	1296890	1300110

Natural Numbers.	0	1	2	3	4
135	1303338	1306553	1309767	1312978	1316187
136	1335389	1338581	1341771	1344958	2348144
137	1367206	1370374	1373541	1376705	1379867
138	1398791	1401937	1405080	1408222	1411361
139	1430148	1433271	1436392	1439511	1442628
140	1461280	1464381	1467480	1470577	1473671
141	1492191	1495270	1498347	1501422	1504494
142	1522883	1525941	1528996	1532049	1535100
143	1553360	1556396	1559430	1562462	1565491
144	1583625	1586640	1589653	1592663	1595672
145	1613680	1616674	1619666	1622656	1625644
146	1643528	1646502	1649474	1652443	1655411
147	1673173	1676127	1679078	1682027	1684975
148	1702617	1705550	1708482	1711411	1714339
149	1731863	1734776	1737688	1740598	1743506
150	1760913	1763807	1766699	1769590	1772478
151	1789769	1792645	1795518	1798389	1801259
152	1818436	1821292	1824146	1826999	1829850
153	1846941	1849752	1852588	1855421	1858253
154	1875207	1878026	1880844	1883659	1886473
155	1903317	1906118	1908917	1911714	1914510
156	1931246	1934029	1936810	1939590	1942367
157	1958996	1961762	1964525	1967287	1970047
158	1986571	1989319	1992065	1994809	1997552
159	2013971	2016702	2019431	2022158	2024883
160	2041200	2043913	2046625	2049335	2052044
161	2068259	2070955	2073650	2076344	2079035
162	2095150	2097830	2100508	2103185	2105860
163	2121876	2124540	2127201	2129862	2132521
164	2148438	2151086	2153732	2156376	2159018
165	2174839	2177471	2180100	2182727	2185355
166	2201081	2203696	2206310	2208922	2211533
167	2227165	2229764	2232363	2234959	2237554
168	2253093	2259677	2258260	2260841	2263421
169	2278867	2281436	2284003	2286570	2289134
170	2304489	2307043	2309596	2312146	2314696
171	2329961	2332500	2335038	2337574	2340108
172	2355284	2357809	2360331	2362853	2365373
173	2380461	2382971	2385479	2387986	2390491
174	2405492	2407988	2410481	2412974	2415465
175	2430380	2432861	2435341	2437819	2440296
176	2455127	2457593	2460059	2462523	2464986
177	2479733	2482186	2484637	2487085	2489536
178	2504200	2506639	2509077	2511513	2513948

Natural Number.	5	6	7	8	9
135	1319393	1322597	1325798	1328998	1332194
136	1351326	1354507	1357685	1360861	1364034
137	1383027	1386184	1389339	1392492	1395643
138	1414498	1417632	1420765	1423895	1427022
139	1445742	1448854	1451964	1455072	1458177
140	1476763	1479853	1482941	1486026	1489110
141	1507564	1510632	1513698	1516762	1519824
142	1538149	1541195	1544240	1547282	1550322
143	1568519	1571544	1574568	1577589	1580608
144	1598678	1601683	1604685	1607686	1610684
145	1628630	1631614	1634595	1637575	1640553
146	1658376	1661340	1664301	1667260	1670218
147	1687920	1690863	1693805	1696744	1699682
148	1717264	1720188	1723110	1726029	1728947
149	1746412	1749316	1752218	1755118	1758016
150	1875365	1778250	1781132	1784013	1786892
151	1804126	1806992	1809856	1812718	1815578
152	1832698	1835545	1838[illegible]90	1841233	1844075
153	1861084	1863912	1866739	1869563	1872386
154	1889285	1892095	1894903	1897709	1900514
155	1917304	1920096	1922886	1925674	1928461
156	1945143	1947917	1950690	1953460	1956229
157	1972806	1975562	1978317	1981070	1983821
158	2000293	2003032	2005769	2008505	2011239
159	2027607	2030329	2033049	2035768	2038485
160	2054750	2057455	2060159	2062869	2065560
161	2081725	2084413	2087100	2089785	2092468
162	2108534	2111203	2113876	2116544	2119211
163	2135178	2137833	2140487	2143139	2145789
164	2161659	2164298	2166935	2169572	2172206
165	2187980	2190603	2193225	2195845	2198464
166	2214142	2216750	2219356	2221960	2224563
167	2240148	2242740	22453[illegible]1	2247920	2250507
168	2265999	2268576	2271151	2273724	2276296
169	2291697	2294258	2296818	2299377	2301934
170	1317244	2319790	2322335	2324879	2327421
171	2342641	2345173	2347703	2350232	2352759
172	2367891	2370408	2372923	2375437	2377950
173	2392995	2395497	2397998	2400498	2402996
174	2417954	2420442	2422929	2425414	2427898
175	2442771	2445245	2447718	2450189	2452658
176	2467447	2469907	2472365	2474823	2477278
177	2491984	2494430	2496874	2499317	2501759
178	2516382	2518814	2521246	2523675	2526103

Natural Numbers.	0	1	2	3	4
179	2528530	2530956	2533380	2535803	2538224
180	2552725	2555137	2557548	2559957	2562365
181	2576786	2579184	2581582	2583978	2586373
182	2600714	2603099	2605484	2607867	2610248
183	2624511	2626883	2629255	2631625	2633993
184	2648178	2650538	2652896	2655253	2657609
185	2671717	2674064	2676410	2678752	2681097
186	2695129	2697464	2699797	2702128	2704459
187	2718416	2720738	2723058	2725378	2727696
188	2741578	2743888	2746196	2748503	2750809
189	2764618	2766915	2769211	2771506	2773800
190	2787536	2789821	2792105	2794388	2796669
191	2810334	2812607	2814879	2817150	2819419
192	2833012	2835274	2837534	2839793	2842051
193	2855573	2857823	2860071	2862318	2864565
194	2878017	2880255	2882492	2884728	2886963
195	2900346	2902573	2904798	2907022	2909246
196	2922561	2924776	2926990	2929203	2931415
197	2944662	2946866	2949069	2951271	2953471
198	2966652	2968845	2971036	2973227	2975417
199	2988531	2990713	2992893	2995073	2997151
200	3010300	3012471	3014641	3016809	3018977
201	3031961	3034121	3036280	3038438	3040595
202	3053514	3055663	3057811	3059959	3062105
203	3074960	3077099	3079237	3081374	3083509
204	3096302	3098430	3100557	3102684	3104809
205	3117539	3119657	3121774	3123889	3126004
206	3138672	3140780	3142887	3144992	3147097
207	3159703	3161801	3163897	3165993	3168087
208	3180633	3182721	3184807	3186893	3188977
209	3201463	3203540	3205617	3207692	3209767
210	3222193	3224260	3226327	3228393	3230457
211	3242825	3244882	3246939	3248995	3251050
212	3263359	3265407	3267454	3269500	3271545
213	3283796	3285834	3287872	3289909	3291944
214	3304138	3306167	3308195	3310222	3312248
215	3324385	3326404	3328423	3330440	3332457
216	3344537	3346548	3348557	3305565	3352572
217	3364597	3366598	3368598	3370597	3372595
218	3384565	3386557	3388547	3390537	3392526
219	3404441	3406424	3408405	3410386	3412366
720	3424227	3426200	3428173	3430145	3432116
221	3443923	3445887	3447851	3449814	3451776
222	3463530	3465486	3467441	3469395	3471348

Natural Numbers.	5	6	7	8	9
179	2540645	2543063	2545481	2547897	2550312
180	2564772	2567177	2569581	2571984	2574386
181	2588766	2591158	2593549	3595939	2598327
182	2612629	3615008	2617385	2619762	2622137
183	2636361	2638727	2641092	2643455	2645817
184	2659964	2662317	2664669	2667020	2669369
185	2683439	2685780	2688119	2790457	2692794
186	2706788	2709116	2711443	2713769	2716093
187	2730013	2732328	2734643	2736956	2739268
188	2753113	2755417	2757719	2760020	2762320
189	2776092	2778383	2780673	2782962	2785250
190	2798950	2801229	2803507	2805784	2808059
191	2821688	2823955	2826221	2828486	2830750
192	2844307	2846563	2848817	2851070	2853322
193	2866810	2869054	2871296	2873538	2875778
194	2889196	2891428	2893659	2895889	2898118
195	2911468	2913688	2915908	2918127	2920344
196	2933626	2935835	2938044	2940251	2942457
197	2955671	2957869	2960067	2962263	2964458
198	2977605	2979792	2981979	2984164	2986348
199	2999420	3001605	3003781	3005955	3008128
200	3021144	3023309	3025474	3027637	3029799
201	3042751	3044905	3047059	3049212	3051363
202	3064250	3066394	3068537	3070679	3072820
203	3085644	3087778	3089910	3092042	3094172
204	3106933	3109056	3111178	3113299	3115420
205	3128118	3130231	3132343	3134454	3136563
206	3149200	3151303	3153405	3155505	3157605
207	3170181	3172273	3174365	3176455	3178545
208	3191061	3193143	3195224	3197305	3199384
209	3211840	3213913	3215984	3218055	3220124
210	3232521	3234584	3236645	3238706	3240766
211	3253104	3255157	3257209	3259260	3261310
212	3273589	3275633	3277675	3279716	3281757
213	3293979	3296012	3298045	8300077	3302108
214	3314273	3316297	3318320	3320343	3322364
215	3334473	3336488	3338501	3340514	3342526
216	3354579	3356585	3358589	3360593	3362596
217	3374593	3376589	3378584	3380579	3382572
218	3394514	3396501	3398488	3400473	3402458
219	3414345	3416323	3418301	3420277	3422252
220	3434086	3436055	3438023	3439991	3441957
221	3453737	3455698	3457657	3459615	3461573
222	3473300	3475252	3477202	3479152	3481101

Natural. Numbers	0	1	2	3	4
223	3483049	3484996	3486942	3488887	3490832
224	3502480	3504419	3506356	3508253	3510228
225	3521825	3523755	3525684	3527612	3529539
226	3541084	3543006	3544926	3546845	3548764
227	3560259	3562171	3564083	3565994	3567905
228	3579348	3581253	3583156	3585059	3586961
229	3598355	3600251	3602146	3604040	3605934
230	3617278	3619166	3621053	3622939	3624825
231	3636120	3638000	3639878	3641756	3643633
232	3654880	3656751	3658622	3660492	3662361
233	3673559	3675423	3677285	3679147	3681008
234	3692159	3694014	3695869	3697723	3699576
235	3710679	3712526	3714373	3716219	3718065
236	3729120	3730960	3732799	3734637	3736475
237	3747483	3749316	3751147	3752977	3754807
238	3765769	3767594	3769418	3771240	3773062
239	3783979	3785796	3787612	3789427	3791241
240	3802112	3803922	3805730	3807538	3809345
241	3820170	3821972	3823773	3825573	3827373
242	3838154	3839948	3841741	3843534	3845326
243	3856063	3857850	3859636	3861421	3863206
244	3873898	3875678	3877457	3879235	3881012
245	3891661	3893433	3895205	3896975	3898746
246	3909351	3911116	3912880	3914644	3916407
247	3926969	3928727	3930485	3932241	3933997
248	3944517	3946268	3948018	3949767	3951516
249	3961993	3963734	3965480	3967223	3968964
250	3979400	3981137	3982873	3984608	3986343
251	3996737	3998467	4000196	4001925	4003653
252	4014005	4015728	4017451	4019172	4020893
253	4031205	4032921	4034637	4036352	4038066
254	4048337	4050047	4051755	4053464	4055171
255	4065402	4067105	4068807	4070508	4072209
256	4082400	4084096	4085791	4087486	4089180
257	4099331	4101021	4102710	4104398	4106085
258	4116197	4117880	4119562	4121244	4122925
259	4132998	4134674	4136350	4138025	4139700
260	4149733	4151404	4153073	4154742	4156410
261	4166405	4168069	4169732	4171394	4173056
262	4183013	4184670	4186327	4187983	4189638
263	4199557	4201208	4202859	4204509	4206158
264	4216039	4217684	4219328	4220972	4222614
265	4232459	4234097	4225735	4237372	4239009
266	4248816	4250449	4252080	4253712	4255342

Natural Numbers.	5	6	7	8	9
223	3492775	3494718	3496660	3498601	3500541
224	3512163	3514098	3516031	3517963	3519895
225	3531465	3533391	3535316	3537239	3539162
226	3550682	3552599	3554515	3556430	3558345
227	3569813	3571723	3573630	3575537	3577443
228	3588862	3590762	3592662	3594560	3596458
229	3607827	3609719	3611610	3613500	3615390
230	3626709	3628593	3630476	3632358	3634239
231	3645510	3647386	3649260	3651134	3653007
232	3664230	3666097	3667964	3669830	3671695
233	3682869	3684728	3686587	3688445	3690302
234	3701428	3703280	3705131	3706981	3708830
235	3719909	3721753	3723596	3725438	3727279
236	3738311	3740147	3741983	3743817	3745651
237	3756636	3758464	3760292	3762118	3763944
238	3774884	3776704	3778524	3780343	3782161
239	3793055	3794868	3796080	3798492	3800302
240	3811151	3812956	3814761	3816565	3818368
241	3829171	3830969	3832766	3834563	3836359
242	3847117	3848908	3850698	3852487	3854275
243	3864990	3866773	3868555	3870337	3872118
244	3882789	3884565	3886340	3888114	3889888
245	3900515	3902284	3904052	3905819	3907585
246	3918169	3919931	3921691	3923452	3925211
247	3935752	3937506	3939260	3941013	3942765
248	3953264	3955011	3956758	3958504	3960249
249	3970705	3972446	3974185	3975924	3977662
250	3988077	3989811	3991543	3993275	3995007
251	4005380	4007106	4008832	4010557	4012282
252	4022614	4024333	4026052	4027771	4029488
253	4039780	4041492	4043205	4044916	4046627
254	4056878	4058584	4060289	4061994	4063698
255	4073909	4075608	4077307	4079005	4080703
256	4090874	4092567	4094259	4095950	4097641
257	4107772	4109459	4111144	4112829	4114513
258	4124605	4126285	4127964	4129643	4131320
259	4141374	4143047	4144719	4146391	4148063
260	4158977	4196744	4161410	4163076	4164741
261	4174717	4176377	4178037	4179696	4181355
262	4191293	4192947	4194601	4196254	4197906
263	4207806	4209454	4211101	4212748	4214394
264	4224257	4225898	4227539	4229180	4230820
265	4240645	4242281	4243915	4245550	4247183
266	4256972	4258601	4260230	4261858	4263486

Natural. Numbers	0	1	2	3	4
267	4265113	4266739	4268365	4269990	4271614
268	4281348	4282968	4284588	4286207	4287825
269	4297523	4299137	4300751	4302364	4303976
270	4313638	4315246	4316853	4318460	4320067
271	4329693	4331295	4332897	4334498	4336098
272	4345689	4347285	4348881	4350476	4352071
273	4361626	4363217	4364807	4366396	4367985
274	4377506	4379090	4380674	4382258	4383841
275	4393327	4394906	4396484	4398062	4399639
276	4409091	4410664	4412237	4413809	4415380
277	4424798	4426365	4427932	4429499	4431065
278	4442448	4442010	4443571	4445132	4446692
279	4456042	4457598	4459154	4460709	4462264
280	4471580	4473131	4474681	4476231	4477780
281	4487063	4488608	4490153	4491697	4493241
282	4502491	4504031	4505570	4507109	4508647
283	4517864	4519399	4520932	4522466	4523998
284	4533183	4534712	4536241	4537769	4539296
285	4548449	4549972	4551495	4553018	4554540
286	4563660	4565179	4566696	4568213	4569731
287	4578819	4580332	4581844	4583356	4584868
288	4593925	4595433	4596940	4598446	4599953
289	4608978	4610481	4611983	4613484	4614985
290	4623980	4625477	4626974	4628470	4629966
291	4638930	4640422	4641914	4643405	4644895
292	4653828	4655316	4656802	4658288	4559774
293	4668676	4670158	4671640	4673120	4674601
294	4683473	4684950	4686427	4687903	4689378
295	4698220	4699692	4701163	4702634	4704105
296	4712917	4714384	4715850	4717317	4718782
297	4727564	4729027	4730488	4731949	4733410
298	4742163	4743620	4745076	4746533	4747988
299	4756712	4758164	4759616	4761067	4762518
300	4771212	4772660	4774107	4775553	4776999
301	4785665	4787108	4788550	4789991	4791432
302	4800069	4801507	4802945	4804381	4805818
303	4814426	4815859	4817292	4818724	4820156
304	4828736	4830164	4831592	4833019	4834446
305	4842998	4844422	4845845	4847268	4848650
306	4857214	4858633	4860052	4861470	4862888
307	4871384	4872798	4874212	4875626	4877039
308	4885507	4886917	4888326	4889735	4891144
309	4899585	4900990	4902395	4903799	4905203
310	4913617	4915018	4916418	4917818	4919217

Natural Numbers.	5	6	7	8	9
267	4273238	4274861	4276484	4278106	4279727
268	4289443	4291060	4292677	4294293	4295908
269	4305588	4307199	4308809	4310419	4312029
270	4321673	4323278	4324883	4326487	4328090
271	4337698	4339298	4340896	4342494	4344092
272	4353665	4355258	4356851	4358444	4360035
273	4369573	4371161	4372748	4374334	4375920
274	4385423	4387005	4388587	4390167	4391747
275	4401216	4402792	4404368	4405943	4407517
276	4416951	4418522	4420092	4421661	4423229
277	4432630	4434195	4435759	4437322	4438885
278	4448252	4449811	4451370	4452928	4454485
279	4463818	4465372	4466925	4468477	4470029
280	4479329	4480877	4482424	4483971	4485517
281	4494784	4496326	4497868	4499410	4500951
282	4510184	4511721	4513258	4514794	4516329
283	4525531	4527062	4528593	4530124	4531654
284	4540823	4542349	4543875	4545400	4546924
285	4556061	4557582	4559102	4560622	4562142
286	4571246	4572762	4574277	4575791	4577305
287	4586378	4587889	4589399	4590908	4592417
288	4601458	4602963	4604468	4605972	4607475
289	4616486	4617986	4619485	4620984	4622482
290	4631461	4632956	4634450	4635944	4637437
291	4646386	4647875	4649364	4650853	4652341
292	4661259	4662743	4664227	4665711	4667194
293	4676081	4677560	4679039	4680518	4681996
294	4690853	4692327	4693801	4695275	4696748
295	4705575	4707044	4708513	4709982	4711450
296	4720247	4721711	4723175	4724639	4726102
297	4734870	4736329	4737788	4739247	4740705
298	4749443	4750898	4752352	4753806	4755259
299	4763968	4765418	4766867	4768316	4769765
300	4778445	4779890	4781334	4782778	4784222
301	4792873	4794313	4795754	4797192	4798631
302	4807254	4808689	4810124	4811559	4812993
303	4821587	4823018	4824448	4825878	4827307
304	4835873	4837299	4838725	4840150	4841574
305	4850112	4851533	4852954	4854375	4855795
306	4864305	4865721	4867138	4868554	4869969
307	4878451	4879863	4881275	4882686	4884097
308	4892552	4893959	4895366	4896773	4898179
309	4906607	4908009	4909412	4910814	4912216
310	4920616	4922014	4923413	4924810	4926207

Natural Numbers.	0	1	2	3	4
311	4927604	4929000	3930396	4931791	4933186
312	4941546	4942938	4944329	4945720	4947110
313	4955443	4956831	4958218	4959604	4960990
314	4969296	4970679	4972062	4973444	4974825
315	4983106	4984484	4985862	4987240	4988617
316	4996871	4998245	4999619	5000992	5002365
317	5010593	5011962	5013332	5014701	5016069
318	5024271	5025637	5027002	5028366	5029731
319	5037907	5039268	5040629	5041989	5043349
320	5051500	5052857	5054213	5055569	5056925
321	5065050	5066403	5067755	5069107	5070459
322	5078559	5079907	5081255	5082603	5083950
323	5092025	5093370	5094713	5096057	5097400
324	5105450	5106790	5108130	5109469	5110808
325	5118834	5120170	5121505	5122841	5124175
326	5132176	5133508	5134840	5136171	5137501
327	5145478	5146805	5148133	5149460	5150787
328	5158738	5160062	5161386	5162709	5164031
329	5171959	5173279	5174598	5175917	5177236
330	5185139	5186455	5187771	5189086	5190400
331	5198280	5199592	5200903	5202214	5203525
332	5211381	5212689	5213996	5215303	5216610
333	5224442	5225746	5227050	5228353	5229656
334	5237465	5238765	5240064	5241364	5242663
335	5250448	5251744	5253040	5254335	5255631
336	5263393	5264685	5265977	5267269	5268560
337	5276299	5277588	5278876	5280163	5281451
338	5289167	5290452	5291736	5293020	5294303
339	5301997	5303278	5304558	5305839	5307118
340	5314789	5316066	5317343	5318619	5319895
341	5327544	5328817	5330090	5331363	5332635
342	5340261	5341531	5342800	5344069	5345338
343	5352941	5354207	5355473	5356738	5358003
344	5365584	5366847	5368109	5369370	5370631
345	5378191	5379450	5380708	5381966	5383223
346	5390761	5392016	5393271	5394525	5395779
347	5403295	5404546	5405797	5407048	5408298
348	5415792	5417040	5418288	5419535	5420781
349	5428254	5429498	5430742	5431986	5433229
350	5440680	5441921	5443161	5444401	5445641
351	5453071	5454308	5455545	5456781	5458017
352	5465427	5466660	5467894	5469126	5470359
353	5477747	5478977	5480207	5481436	5482665
354	5490033	5491259	5492486	5493712	5494937

Natural Numbers.	5	6	7	8	9
311	4934580	4935974	4937368	4938761	4940154
312	4948500	4949890	4951279	4952667	4954056
313	4962375	4963761	4965145	4966529	4967913
314	4976206	4977587	4978967	4980347	4981727
315	4989994	4)91370	4992746	4994121	4995496
316	5003737	5005109	5006481	5007852	5009222
317	5017437	5018805	5020172	5021539	5022905
318	5031094	5032458	5033821	5035183	5036545
319	5044709	5046068	5047426	5048785	5050142
320	5058280	5059635	5060990	5062344	5063697
321	5071810	5073160	5074511	5075860	5077210
322	5085297	5086644	5087990	5089335	5090680
323	5098743	5100085	5101427	5102768	5104109
324	5112147	5113485	5114823	5116160	5117497
325	5125510	5126844	5128178	5129511	5130844
326	5138832	5140162	5141491	5142820	5144149
327	5152113	5153439	5154764	5156089	5157414
328	5165354	5166676	5167997	5169318	5170639
329	5178554	5179872	5181189	5182506	5183823
330	5191715	5193028	5194342	5195655	5196968
331	5204835	5206145	5207455	5208764	5210073
332	5217916	5219222	5220528	5221833	5223138
333	5230958	5232260	5233562	5234863	5326164
334	5243961	5245259	5246557	5247854	5249151
335	5256925	5258219	5259513	5260807	5262100
336	5269851	5271141	5272431	5273721	5275010
337	5282738	5284024	5285311	5286596	5287882
338	5295587	5296869	5298152	5299434	5300716
339	5308398	5309677	5310955	5312234	5313512
340	5321171	5322446	5323721	5324996	5326270
341	5333907	5335179	5336450	5337721	5338991
342	5346606	5347874	5349141	5350408	5351675
343	5359267	5360532	5361795	5363059	5364322
344	5371892	5373153	5374413	5375672	5376932
345	5384481	5385737	5386994	5388250	5389506
346	5397032	5398286	5399538	5400791	5402043
347	5409548	5410798	5412047	5413296	5414544
348	5422028	5423274	5424519	5425765	5427010
349	5434472	5435714	5436956	5438198	5439439
350	5446880	5448119	5449358	5450596	5451834
351	5459253	5460489	5461724	5462958	5464193
352	5471591	5472823	5474055	5475286	5476517
353	5483894	5485123	5486351	5487578	5488806
354	5406162	5497387	5498612	5499836	5501060

Natural Numbers.	0	1	2	3	4
355	5502283	5503507	5504730	5505952	5507174
356	5514500	5515720	5516939	5518158	5519377
357	5526682	5527898	5529114	5530330	5531545
358	5538830	5540043	5541256	5542468	5543680
359	5550944	5552154	5553362	5554572	5555781
360	5563025	5564231	5565437	5566643	5567848
361	5575072	5576275	5577477	5578680	5579881
362	5587086	5588285	5589484	5590683	5591882
363	5599066	5600262	5601458	5602654	5603849
364	5611014	5612207	5613399	5614592	5615784
365	5622929	5624118	5625308	5626497	5627685
366	5634811	5635997	5637183	5638369	5639555
367	5646661	5647844	5649027	5650209	5651392
368	5658478	5659658	5660838	5662017	5663196
369	5670264	5671440	5672617	5673793	5674969
370	5682017	5683191	5684364	5685537	5686710
371	5693739	5694910	5696080	5697249	5698419
372	5705429	5706597	5707764	5708930	5710097
373	5717088	5718252	5719416	5720580	5721743
374	5728716	5729877	5731038	5732198	5733358
375	5740313	5741471	5742628	5743786	5744943
376	5751878	5753033	5754188	5755342	5756496
377	5763413	5764565	5765717	5766868	5768019
378	5774917	5776067	5777215	5778363	5779511
379	5786392	5787538	5788683	5789828	5790973
380	5797836	5798979	5800121	5801263	5802405
381	5809250	5810389	5811529	5812668	5813807
382	5820634	5821770	5822907	5824043	5825179
383	5831988	5833122	5834255	5835388	5836521
384	5843312	5844443	5845574	5846704	5847834
385	5854617	5855735	5856863	5857990	5859117
386	5865873	5866998	5868123	5869247	5870371
387	5877110	5878232	5879353	5880475	5881596
388	5888317	5889436	5890555	5891674	5892792
389	5899496	5900612	5901728	5902844	5903959
390	5910646	5911759	5912873	5913985	5915098
391	5921768	5922878	5923988	5925098	5926208
392	5932861	5933968	5935076	5936183	5937290
393	5943925	5945030	5946135	5947239	5948344
394	5954962	5956064	5957166	5958268	5959369
395	5965971	5967070	5968169	5969268	5970367
396	5976952	5978048	5979145	5980241	5981336
397	5987905	5988999	5990092	5991186	5992279
398	5998831	5999922	6001012	6002103	6003193

Natural Numbers.	5	6	7	8	9
355	5508396	5509618	5510839	5512059	5513280
356	5520595	5521813	5523031	5524248	5525465
357	5532760	5533975	5535189	5536403	5537617
358	5544892	5546103	5547314	5548524	5549735
359	5556989	5558197	5559404	5560612	5561818
360	5569053	5570257	5571461	5572665	5573869
361	5581083	5582284	5583485	5584686	5585886
362	5593080	5594278	5595476	5596673	5597870
363	5605044	5606239	5607433	5608627	5609820
364	5616975	5618167	5619358	5620548	5621739
365	5628875	5630062	5631250	5632437	5633624
366	5640740	5641925	5643109	5644293	5645477
367	5652573	5653755	5654936	5656117	5657298
368	5664375	5665553	5666731	5667909	5669087
369	5676144	5677320	5678494	5679669	5680843
370	5687882	5689054	5690226	5691397	5692568
371	5699588	5700757	5701926	5703094	5704262
372	5711263	5712428	5713594	5714759	5715924
373	6722906	5724069	5725231	5726393	5727555
374	5734518	5735678	5736837	5737996	5739154
375	5746099	5747256	5748412	5749568	5750723
376	5757650	5758803	5759956	5761109	5762261
377	5769169	5770320	5771470	5772620	5773769
378	5780659	5781806	5782953	5784100	5785246
379	5792118	5793262	5794406	5795550	5796693
380	5803547	5804688	5805829	5806969	5808110
381	5814945	5816084	5817222	5818359	5819497
382	5826314	5827450	5828585	5829719	5830854
383	5837654	5838786	5839918	5841050	5842181
384	5848963	5850093	5851222	5852351	5853479
385	5860244	5861370	5862496	5863622	5864748
386	5871495	5872618	5873742	5874865	5875987
387	5882717	5883838	5884958	5886078	5887198
388	5893910	5895028	5896145	5897262	5898379
389	5905075	5906189	5907304	5908418	5909532
390	5916210	5917322	5918434	5919546	5920657
391	5927318	5928427	5929536	5930644	5931753
392	5938397	5939503	5940609	5941715	5942820
393	5949447	5950551	5951654	5952757	5953860
394	5960470	5961571	5962671	5963771	5964871
395	5971465	5972563	5973660	5974758	5975855
396	5982432	5983527	5984622	5985717	5986811
397	5993371	5994464	5995556	5996648	5997739
398	6004283	6005373	6006462	6007551	6008642

Natural Numbers.	0	1	2	3	4
399	6009729	6010817	6011905	6012993	6014081
400	6020600	6021685	6022771	6023856	6024941
401	6031444	6032527	6033609	6034692	6035774
402	6042261	6043341	6044421	6045500	6046580
403	6053050	6054128	6055205	6056282	6057359
404	6063814	6064888	6065963	6067037	6068111
405	6074550	6075622	6076694	6077766	6078837
406	6085260	6086330	6087399	6088468	6089537
407	6095944	6097011	6098078	6099144	6100210
408	6106602	6107666	6108730	6109794	6110857
409	6117233	6118295	6119356	6120417	6121478
410	6127839	6128898	6129957	6131015	6132073
411	6138418	6139475	6140531	6141587	6142643
412	6148972	6150026	6151080	6152133	6153187
413	6159501	6160552	6161603	6162654	6163705
414	6170003	6171052	6172101	6173149	6174197
415	6180481	6181527	6182573	6183619	6184665
416	6190933	6191977	6193021	6194064	6195107
417	9201360	6202402	6203443	6204484	6205524
418	6211763	6212802	6213840	6214879	6215917
419	6222140	6227177	6224213	6225149	6226284
420	6232493	6233527	6234560	6235594	6236627
421	6242821	6243852	6244884	6245915	6246945
422	6223154	6254153	6255182	6256211	6257239
423	6263404	6264430	6265457	6266483	6267509
424	6273659	6274683	6275707	6276730	6277754
425	6283889	6284911	6285933	6286954	6287975
426	6294096	6295115	6296134	6297153	6298172
427	6304279	6305296	5306312	6307329	6308345
428	6314438	6315452	6316467	6317481	6318495
429	6324573	6325585	6326597	6327609	6328620
430	6334685	6335694	6336704	6337713	6338723
431	6344773	6345780	6346788	6347795	6348801
432	6354837	6355843	6356848	6357852	6358857
433	6364879	6365882	6366884	6367887	6368889
434	6374897	6375898	6376898	6377898	6378898
435	6384893	6385891	6386889	6387887	6388884
436	6394865	6395861	6396857	6397852	6398847
437	6474814	6405808	6406802	6407795	6408788
438	6414741	6415733	6416724	6417715	6418705
439	6424645	6425634	6426623	6427612	6428601
440	6434527	6435514	6436500	6437487	6438473
441	6444386	6445371	6446355	6447339	6448323
442	6454223	6455205	6456187	6457169	6458151

Natural Numbers.	5	6	7	8	9
399	6015168	6016255	6017341	6018428	6019514
400	6026025	6027109	6028193	6029277	6030361
401	6036855	6037937	6039018	6040099	6041180
402	6047659	6048738	6049816	6050895	6051973
403	6058435	6059512	6060587	6061663	6062738
404	6069185	6070259	6071332	6072405	6073478
405	6079909	6080979	6082050	6083120	6084190
406	6090605	6091674	6092742	6093809	6094877
407	6101276	6102342	6103407	6104472	6105537
408	6111921	6112948	6114046	6115109	6116171
409	6122539	6123599	6124660	6125720	6126779
410	6133132	6134189	6135247	6136304	6137361
411	6143698	6144754	6145809	6146863	6147918
412	6154240	6155292	6156345	6157397	6158449
413	6164755	6165805	6166855	6167905	6168954
414	6175245	6176293	6177340	6178387	6179434
415	6185710	6186755	6187800	6188845	6189889
416	6196150	6197193	6198235	6199277	6200319
417	6206565	6207605	6208645	6209684	6210724
418	6216955	6217992	6219030	6220067	6221104
419	6227320	6228355	6229300	6230424	6231459
420	6237660	6238693	6239725	6240757	6241789
421	6247976	6249006	6250036	6251066	6252095
422	6258267	6259295	6260322	6261350	6262377
423	6268534	6269559	6270585	6271610	6272634
424	6278777	6279800	6280823	6281845	6282867
425	6288996	6290016	6291036	6292057	6293076
426	6299190	6300208	6301226	6302244	6303262
427	6309361	6310377	6311392	6312408	6313423
428	6319508	6320522	6321535	6322548	6323560
429	6329632	6330643	6331653	6332664	6333674
430	6339732	6340740	6341749	6342757	6343765
431	6349808	6350814	6351820	6352826	6353832
432	6359861	6360865	6361869	6362872	6363876
433	6369891	6370893	6371894	6372895	6373896
434	6379898	6380897	6381896	6382895	6383894
435	6389882	6390879	6391876	6392872	6393869
436	6399842	6400837	6401832	6402826	6403820
437	6409781	6410773	6411765	6412758	6413749
438	6419696	6420686	6421676	6422666	6423656
439	6429589	6430577	6431565	6432552	6433540
440	6439459	6440445	6441430	6442416	6443401
441	6449307	6450291	6451274	6452257	6453240
442	6459133	6460114	6461095	6462076	6453057

Natural Numbers.	0	1	2	3	4
443	6464037	6465017	6465997	6466977	6467957
444	6473830	6474808	0475785	6476763	6477740
445	6483600	6484576	6485552	6486527	6487502
446	6493349	6494322	6495296	6496269	6497242
447	6503075	6504047	6505018	6505989	6506960
448	6512780	6513749	6514719	6515687	6516656
449	6522463	6523430	6524397	6525364	6526331
450	6532125	6533090	6534055	6535019	6535984
451	6541765	6542728	6543691	6544653	6545616
452	6551384	6552345	6553306	6554266	6555226
453	6560982	6561941	6562899	6563857	6564815
454	6570559	6571515	6572471	6573427	6574383
455	6580114	6581068	6582023	6582976	6583930
456	6589648	6590601	6591553	6592505	6593456
457	6599162	6600112	6601062	6602012	6602962
458	6608655	6609603	6610551	6611499	6612446
459	6618127	6619073	6620019	6620964	6621910
460	6627578	6628522	6629466	6630410	6631353
461	6637009	6637951	6638893	6639835	6640776
462	6646420	6647360	6648299	6649239	6650178
463	6655810	6656748	6657685	6658623	6659560
464	6665180	6666116	6667051	6667987	6668922
465	6674530	6675463	6676397	6677331	6678264
466	6683859	6684791	6685723	6686654	6687585
467	6693169	6694099	6695028	6695958	6696887
468	6702459	6703386	6704314	6705242	6706169
469	6711728	6712654	6713580	6714506	6715431
470	6720979	6721903	6722826	6723750	6724673
471	6730209	6731131	6732053	6732974	6733896
472	6739420	6740340	6741260	6742179	6743099
473	6748611	6749529	6750447	6751365	6752283
474	6757783	6758700	6759615	6760531	6761447
475	0766936	6767850	6768764	6769678	6770592
476	6776069	6776982	6777894	6778806	6779718
477	6785184	6786094	6787004	6787914	6788824
478	6794279	6795187	6796096	6797004	6797912
479	6803355	6804262	6805168	6806074	6806980
480	6812412	6813317	6814222	6815126	6816030
481	6821451	6822354	6823256	6824159	6825061
482	6830470	6831371	6832272	6833173	6834073
483	6839471	6840370	6841269	6842168	6843066
484	6848454	6849351	6850248	6851145	6852041
485	6857417	6858313	6859208	6860103	6860998
486	6866363	6867256	6868149	6869043	6869936

Natural. Numbers	5	6	7	8	9
443	6468936	6469915	6470894	6471873	6472851
444	6478718	6479695	6480671	6481648	6482624
445	6488477	6489452	6490426	6491401	6492375
446	6498215	6499187	6500160	6501132	6502104
447	6507930	6508901	6509871	6510841	6511811
448	6517624	6518593	6519561	6520528	6521496
449	6527297	6528263	6529229	6530195	6531160
450	6536948	6537912	6538876	6539839	6540802
451	6546578	6547539	6548501	6549462	6550423
452	6556186	6557145	6558105	6559064	6560023
453	6565773	6566730	6567688	6568645	6569602
454	6575339	6576294	6577250	6578205	6579159
455	6584884	6585837	6586790	6587743	6588696
456	6594408	6595359	6596310	6597261	6598212
457	6603911	6604860	6605809	6606758	6607706
458	6613393	6614340	6615287	6616234	6617181
459	6622855	6623800	6624745	6625690	6626634
460	6632296	6633239	6634182	6635125	6636067
461	6641717	6642658	6643599	6644539	6645480
462	6651117	6652056	6652995	6653933	6654872
463	6660497	6661434	6662371	6663307	6664244
464	6669857	6670792	6671727	6672661	6673595
465	6679197	6680130	6681062	6681995	6682927
466	6688516	6689447	6690378	6691308	6692239
467	6697816	6698745	6699674	6700602	6701530
468	6707096	6708023	6708950	6709876	6710802
469	6716356	6717281	6718206	6719130	6720054
470	6725596	6726519	6727412	6728365	6729287
471	6734817	6735738	6736659	6737574	6738500
472	6744018	6744937	6745856	6746775	6747693
473	6753200	6754117	6755034	6755951	6756867
474.	6762362	6763277	6764192	6765107	6766022
475	6771505	6772418	6773332	6774244	6775157
476	6780629	6781540	6782452	6783362	6784273
477	6789734	6790643	6791552	6792461	6793370
478	6798819	6799727	6800634	6801541	6802448
479	6807886	6808792	6809697	6810602	6811507
480	6816934	6817838	6818741	6819645	6820548
481	6825963	6826865	6827766	6828668	6829569
482	6834973	6835873	6836773	6837673	6838572
483	6843965	6844863	6845761	6846659	6847556
484	6852938	6853834	6854730	6855626	6856522
485	6861892	6862787	6863681	6864575	6865469
486	6870828	6871721	6872613	6873506	6874398

Natural Numbers.	0	1	2	3	4
487	6875290	6876181	6877073	6877964	6878855
488	6884198	6885088	6885978	6886867	6887757
489	6893089	6893977	6894864	6895752	6896640
490	6901961	6902847	6903733	6904616	6905505
491	6910815	6911699	6912584	6913468	6914352
492	6919651	6920534	6921416	6922298	6923180
493	6928469	6929350	6930231	6931111	6931991
494	6937269	6938148	6939027	6939906	6940785
495	6946052	6946929	6947806	6948683	6949560
496	6954817	6955692	6956568	6957443	6958318
497	6963564	6964438	6965311	6966185	6967058
498	6972293	6973165	6974037	6974909	6975780
499	6981005	6981876	6982746	6983616	6984485
500	6989700	6990569	6991437	6992305	6993173
501	6998377	6999244	7000111	7000977	7001843
502	7007037	7007902	7008767	7009632	7010496
503	7015680	7016543	7017406	7018269	7019132
504	7024305	7025167	7026028	7026890	7027751
505	7032914	7033774	7034633	7035493	7036352
506	7041505	7042363	7043221	7044079	7044937
507	7050080	7050936	7051792	7052649	7053505
508	7058637	7059492	7060347	7061201	7062055
509	7067178	7068031	7068884	7069737	7070589
510	7075702	7076553	7077405	7078256	7079107
511	7084206	7085059	7085908	7086758	7087607
512	7092700	7093548	7094396	7095244	7096091
513	7101174	7102020	7102866	7103713	7104559
514	7109631	7110476	7111321	7112165	7113010
515	7118072	7118915	7119759	7120601	7121444
516	7126497	7127339	7128180	7129021	7129862
517	7134905	7135745	7136585	7137425	7138264
518	7143298	7144136	7144974	7145812	7146650
519	7151674	7152510	7153347	7154183	7155019
520	7160033	7160869	7161703	7162538	7163373
521	7168377	7169211	7170044	7170877	7171710
522	7176705	7177537	7178369	7179200	7180032
523	7185017	7185847	7186677	7187507	7188337
524	7193313	7194142	7194970	7195799	7196627
525	7201593	7202420	7203247	7204074	7204901
526	7209857	7210683	7211508	7212334	7213159
527	7218106	7218930	7219754	7220578	7221401
528	7226339	7227162	7227984	7228806	7229628
529	7234557	7235378	7236198	7237019	7237839
530	7242759	7243578	7244397	3245216	7246035

Natural Numbers.	5	6	7	8	9
487	6879746	6880637	6881528	6882418	6883308
488	6888646	6889535	6890423	6891312	6892200
489	6897527	6898414	6899301	6900188	6901074
490	6906390	6907275	6908161	6909046	6909930
491	6915235	6916119	6917002	6917885	6918768
492	6924062	6924944	6925826	6926707	6927588
493	6932872	6933752	6934631	6935511	6936390
494	6941663	6942541	6943419	6944297	6945174
495	6950437	6951313	6952189	6953065	6953941
496	6959193	6960067	6960942	6961816	6962690
497	6967931	6968804	6969676	6970549	6971421
498	6976652	6977523	6978394	6979264	6980135
499	6985355	6986224	6987093	6987963	6988831
500	6994041	6994908	6995776	6996643	6997510
501	7002709	7003575	7004441	7005307	7006172
502	7011361	7012225	7013089	7013953	7014816
503	7019995	7020857	7021719	7022582	7023444
504	7028612	7029472	7070333	7031193	7032054
505	7037212	7038071	7038929	7039788	7040647
506	7045793	7046652	7047509	7048366	7049223
507	7054360	7055216	7056072	7056927	7057782
508	7062910	7063764	7064617	7065471	7066324
509	7071442	7072294	7073146	7073998	7074850
510	7079957	7080808	7081659	7082509	7083359
511	7088456	7089305	7090154	7091003	7091851
512	7096939	7097786	7098633	7099480	7100327
513	7105404	7106250	7107096	7107941	7108786
514	7113854	7114698	7115542	7116385	7117229
515	7122287	7123129	7123971	7124813	7125655
516	7130703	7131544	7132385	7133225	7134065
517	7139104	7139943	7140782	7141620	7142459
518	7147488	7148325	7149162	7150000	7150837
519	7155856	7156691	7157527	7158363	7159198
520	7164207	7165042	7165876	7166710	7167544
521	7172543	7173376	7174208	7175041	7175873
522	7180863	7181694	7182525	7183356	7184186
523	7189167	7189996	7190826	7191655	7192484
524	7197455	7198283	7199111	7199938	7200766
525	7205727	7206554	7207380	7208206	7209032
526	7213984	7214809	7215633	7216458	7217282
527	7222225	7223048	6223871	7224694	7225517
528	7230450	7231272	7232093	7232914	7233736
529	7238660	7239480	7240300	7241120	7241939
530	7246854	7247672	7248491	7249309	7250127

Natural Numbers.	0	1	2	3	4
531	7250945	7251763	7252581	7253398	7254215
532	7259116	7259933	7260749	7261565	7262380
533	7267272	7268087	7268901	7269716	7270531
534	7275413	7276226	7277039	7277852	7278664
535	7283538	7284349	7285161	7285972	7286784
536	7291648	7292458	7293268	7294078	7294888
537	7299743	7300551	7301360	7302168	7302977
138	7307823	7308630	7309437	7310244	7311051
539	7315888	7316693	7317499	7318304	7319109
540	7323938	7324749	7325546	7326350	7327153
541	3331973	7332775	7333578	7334380	7335182
542	7339993	7340794	7341595	7342396	7343197
543	7347998	7348798	7349598	7350397	7351196
544	7355989	7356787	7357585	7358383	7359181
545	7363965	7364762	7365558	7366355	7367151
546	7371926	7372722	7373517	7374312	7375107
547	7379873	7380667	7381461	7382254	7383048
548	7387806	7388598	7389390	7390182	7390974
549	7395723	7396514	7397305	7398096	7398886
550	7403627	7404416	7405206	7405995	7406784
551	7411516	7412304	7413092	7413880	7414668
552	7419391	7420177	7420964	7421750	7422537
553	7427251	7428037	7428822	7429607	7430392
554	7435098	7435881	7436665	7437449	7438232
555	7442930	7443712	7444495	7445277	7446059
556	7450748	7451529	7452310	7453091	7453871
557	7458552	7459332	7460111	7460890	7461670
558	7466342	7467120	7467898	7468676	7469454
559	7474118	7474895	7475672	7476448	7477225
560	7481880	7482656	7483431	7484206	7484981
561	7489629	7490403	7491177	7491950	7492724
562	7497363	7498136	7498908	7499681	7500453
563	7505087	7505855	7506626	7507398	7508168
564	7512791	7513561	7514331	7515100	7515870
565	7520484	7521253	7522022	7522790	7523558
566	7528164	7528932	7529690	7530466	7531232
567	7535831	7536596	7537362	7538128	7538893
568	7543483	7544248	7545012	7545777	7546541
569	7551123	7551886	7552649	7553412	7554178
570	7558749	7559510	7560279	7561034	7561795
571	7566361	7567122	7567882	7568642	7569402
572	7573960	7574719	7575479	7576237	7576996
573	7581546	7582304	7583062	7583819	7584577
574	7589119	7589875	7590632	7591388	7552144

Natural Numbers.	5	6	7	8	9
531	7255033	7255850	7256667	7257483	7258300
532	7263196	7264012	7264827	7265642	7266457
533	7271344	7272158	7272972	7273786	7274599
534	7279477	7280290	7281101	7281914	7282726
535	7287595	7288406	7289216	7290027	7290838
536	7295697	7296506	7297316	7298125	7298934
537	7303785	7304593	7305400	7306208	7307015
538	7311857	7312663	7313470	7314276	7315082
539	7319914	7320719	7321524	7322329	7323133
540	7327957	7328760	7329564	7330367	7331170
541	7335985	7336787	7337588	7338390	7339191
542	7343997	7344798	7345598	7346398	7347198
543	7351995	7352794	7353593	7354392	7355191
544	7359979	7360776	7361574	7362371	7363168
545	7367948	7368744	7369540	7370335	7371131
546	7375902	7376696	7377491	7378285	7379076
547	7383841	7384634	7385427	7386220	7387013
548	7391766	7392558	7393350	7394141	7394932
549	7399677	7400467	7401257	7402047	7402837
550	7407573	7408362	7409151	7409939	7410728
551	7415455	7416243	7417030	7417817	7418604
552	7423323	7424109	7424895	7425680	7426466
553	7431176	7431961	7432745	7433530	7434314
554	7439015	7439799	7440582	7441365	7442147
555	7446841	7447622	7448404	7449187	7449967
556	7454652	7455432	7456212	7456992	7457772
557	7462449	7463228	7464006	7464785	7465564
558	7470232	7471009	7471787	7472564	7473341
559	7478001	7478777	7479553	7480329	7481105
560	7485756	7486531	7487306	7488080	7488854
561	7493498	7494271	7495044	7495817	7496590
562	7501225	7501997	7502769	7503541	7504312
563	7508939	7509710	7510480	7511251	7512021
564	7516639	7517409	7518178	7518947	7519716
565	7524326	7525094	7525862	7526629	7527397
566	7531999	7532766	7533532	7534298	7535065
567	7539659	7540424	7541189	7541954	7542719
568	7547305	7548069	7548832	7549596	7550359
569	7554937	7555700	7556462	7557224	7557987
570	7562556	7563318	7564079	7564840	7565600
571	7570162	7570922	7571682	7572441	7573201
572	7577755	7578513	7579272	7580030	7580788
573	7585334	7586091	7586848	7587605	7588362
574	7592900	7593656	7594412	7595168	7595923

Natural Numbers.	0	1	2	3	4
575	7596678	7597434	7598189	7598944	7599699
576	7604225	7604979	7605733	7609486	7607240
577	7611758	7612511	7613263	7614016	7614768
578	7619278	7620030	7620781	7621532	7622283
579	7626786	7627536	7628286	7629035	7629785
580	7634280	7635029	7635777	7636526	7637274
581	7641761	7642509	7643256	7644003	7644750
582	7649230	7649976	7650722	7651468	7652214
583	76566[illegible]6	7657430	7658175	7658920	7659664
584	7664128	7664872	7665616	7666359	7667102
585	7671559	7672301	7673043	7673785	7674527
586	7678976	7679717	7680458	7681199	7681940
587	7686381	7687121	7687860	7688600	7689339
588	7693773	7694512	7695250	7695988	7696727
589	7701153	7701890	7702627	7703364	7704101
590	7708520	7709256	7709992	7710728	7711463
591	7715875	7716610	7717344	7718079	7718813
592	7723217	7723951	7724684	7725417	7726150
593	7730547	7731279	7732011	7732743	7733475
594	7737864	7738596	7739326	7740057	7740788
595	7745170	7745899	7746629	7747359	7748088
596	7752463	7753191	7753920	7754648	7755376
597	7759743	7760471	7761198	7761925	7762652
598	7767012	7767738	7768464	7769190	7769916
599	7774268	7774993	7775718	7776443	7777167
600	7781513	7782236	7782960	7783983	7784407
601	7788745	7789467	7790190	7790912	7791634
602	7795965	7796686	7797408	7798129	7798850
603	7803173	7803893	7804613	7805333	7806053
604	7810369	7811088	7811807	7812526	7813245
605	7817554	7818272	7818989	7819707	7820424
606	7824726	7825443	7826159	7826876	7827592
607	7831887	7832602	7833318	7834033	7834748
608	7839036	7839750	7840464	7841178	7841892
609	7846173	7846886	7847599	7848312	7849024
610	7853298	7854010	7854722	7855434	7856145
611	7860412	7861123	7861833	7862544	7863254
612	7867514	7868224	7868933	7869643	7870352
613	7874605	7875313	7876021	7876730	7877438
614	7881684	7882391	7883098	7883805	7884512
615	7888751	7889457	7890163	7890869	7891575
616	7895807	7896512	7897217	7897922	7898626
617	7902852	7903555	7904259	7904963	7905666
618	7909885	7910587	7911290	7911992	791269[illegible]

Natural Numbers.	5	6	7	8	9
575	7600453	7601208	7601962	7602717	7603471
576	7607993	7608746	7609500	7610253	7611005
577	7615520	7616272	7617024	7617775	7618527
578	7623034	7623784	7624535	7625285	7626035
579	7630534	7631284	7632033	7632782	7633531
580	7638022	7638770	7639518	7640266	7641014
581	7645497	7646244	7646991	7647737	7648484
582	7652959	7653705	7654450	7655195	7655941
583	7660409	7661153	7661897	7662641	7663385
584	7667845	7668588	7669331	7670074	7670816
585	7675269	7676011	7676752	7677494	7678235
586	7682680	7683421	7684161	7684901	7685641
587	7690079	7690818	7691557	7692296	7693035
588	7697465	7698203	7698940	7699678	7700416
589	7704838	7705575	7706311	7707048	7707784
590	7712199	7712934	7713670	7714405	7715140
591	7719547	7720282	7721016	7721750	7722483
592	7726884	7727616	7728349	7729082	7729814
593	7734207	7734939	7735670	7736402	7737133
594	7741519	7742249	7742979	7743710	7744440
595	7748818	7749547	7750276	7751005	7751734
596	7756104	7756832	7757560	7758288	7759016
597	7763379	7764106	7764833	7765559	7766286
598	7770642	7771367	7772093	7772818	7773543
599	7777892	7778616	7779340	7780065	7780789
600	7785130	7785853	7786576	7787299	7788022
601	7792356	7793078	7793800	7794522	7795243
602	7799571	7800291	7801012	7801732	7802453
603	7806773	7807492	7808212	7808931	7809650
604	7813963	7814681	7815400	7816118	7816836
605	7821141	7821859	7822576	7823293	7824010
606	7828308	7829024	7829740	7830456	7831171
607	7835463	7836178	7836892	7837607	7838321
608	7842606	7843319	7844033	7844746	7845460
609	7849737	7850450	7851162	7851874	7852586
610	7856857	7857568	7858279	7858990	7859701
611	7863965	7864675	7865385	7866095	7866805
612	7871061	7871770	7872479	7873188	7873896
613	7878146	7878853	7879561	7880269	7880976
614	7885219	7885926	7886632	7887339	7888045
615	7892281	7892986	7893691	7894397	7895102
616	7899331	7900035	7900739	7901444	7902148
617	7906370	7907073	7907776	7908479	7909182
618	7913397	7914099	7914801	7915503	7916205

Natural Numbers.	0	1	2	3	4
619	7916906	7917608	7918309	7919011	7919712
620	7923917	7924617	7925318	7926018	7926718
621	7930916	7931615	7932314	7933014	7933712
622	7937904	7938602	7939300	7939998	7940696
623	7944880	7945578	7946274	7946971	7947668
624	7951846	7952542	7953238	7953933	7954629
625	7958800	7959495	7960190	7950884	7961578
626	7965743	7966437	7967131	7967824	7968517
627	7972675	7973368	7974060	7974753	7975445
628	7979596	7980288	7980979	7981671	7982362
629	7986506	7987197	7987887	7988577	7982267
630	7993405	7994097	7994784	7995473	7996162
631	8000294	8000982	8001670	8002358	8003046
632	8007171	8007858	8008545	8009232	8009919
633	8014037	8014723	8015409	8016095	8016781
634	8020893	8021578	8022262	8022947	8023632
635	8027737	8028421	8029105	8029789	8030472
636	8034571	8035254	8035937	8036619	8037302
637	8041394	8042076	8042758	8043439	8044121
638	8048207	8048887	8049568	8050248	8050929
639	8055009	8055688	8056368	8057047	8057726
640	8061800	8062478	8063157	8063835	8064513
641	8068580	8069258	8069935	8070612	8071290
642	8075350	8076027	8076703	8077379	8078055
643	8082110	8082785	8083460	8084136	8084811
644	8088859	8089533	8090207	8090881	8091555
645	8095597	8096270	8096944	8097617	8098290
646	8102325	8102997	8103670	8104342	8105013
647	8109043	8109714	8110385	8111056	8111727
648	8115750	8116420	8117090	8117760	8118430
649	8122447	8123116	8123785	8124454	8125123
650	8129134	8129802	8130470	8131138	8131805
651	8135810	8136477	8137144	8137811	8138478
652	8142476	8143142	8143808	8144474	8145140
653	8149132	8149797	8150462	8151127	8151791
654	8155777	8156441	8157105	8157769	8158433
655	8162413	8163076	8163739	8164402	8165064
656	8169038	8169700	8170362	8171024	8171686
657	8175654	8176315	8176976	8177636	8178297
658	8182259	8182919	8183579	8184239	8184898
659	8188854	8189513	8190172	8190831	8191489
660	8195439	8196097	8196755	8197413	8198071
661	8202015	8202672	8203328	8203987	8204642
662	8208580	8209236	8209892	8210548	8211203

Natural Numbers.	5	6	7	8	9
619	7920413	7921114	7921815	7922516	7923216
620	7927418	7928118	7928817	7929517	7930217
621	7934411	7935110	7935809	7936507	7937206
622	7941394	7942091	7942789	7943486	7944183
623	7948365	7949061	7949757	7950454	7951150
624	7955324	7956020	7956715	7957410	7958105
625	7962273	7962967	7963662	7964356	7965050
626	7969211	7969904	7970597	7971290	7971983
627	7976137	7976829	7977521	7978213	7978905
628	7983053	7983744	7984435	7985125	7985816
629	7989957	7990647	7991337	7992027	7992716
630	7996851	7997540	7998228	7998917	7999605
631	8003734	8004421	8005109	8005796	8006484
632	8010605	8011292	8011978	8012665	8013351
633	8017466	8018152	8018837	8019522	8020208
634	8024316	8025001	8025685	8026369	8027053
635	8031156	8031839	8032522	8033205	8033888
636	8037984	8038665	8039348	8040031	8040712
637	8044802	8045483	8046164	8046845	8047526
638	8051609	8052289	8052969	8053649	8054329
639	8058405	8059085	8059763	8060442	8061121
640	8065191	8065869	8066547	8067225	8067903
641	8071967	8072643	8073320	8073997	8074874
642	8078731	8079407	8080083	8080759	8081434
643	8085485	8086160	8086835	8087510	8088184
644	8092229	8092903	8093577	8094250	8094924
645	8098962	8099635	8100308	8100980	8101653
646	8105685	8106357	8107029	8107700	8108371
647	8112398	8113068	8113739	8114409	8115080
648	8119100	8119769	8120439	8121108	8121778
649	8125792	8126460	8127129	8127797	8128465
650	8132473	8133141	8133808	8134475	8135143
651	8139144	8139811	8140477	8141144	8141810
652	8145805	8146471	8147136	8147801	8148467
653	8152456	8153120	8153785	8154449	8155113
654	8159096	8159760	8160423	8161087	8161750
655	8165727	8166389	8167052	8167714	8168376
656	8172347	8173009	8173670	8174331	8174993
657	8178958	8179618	8180278	8180939	8181599
658	8185558	8186217	8186877	8187536	8188195
659	8192146	8192806	8193455	8194123	8194781
660	8198728	8199386	8200043	8200700	8201358
661	8205298	8205955	8206611	8207268	8207924
662	8211859	8212514	8213170	8213825	8214480

Natural Numbers.	0	1	2	3	4
663	8215135	8215790	8216445	8217100	8217755
664	8221681	8222335	8222989	8223643	8224296
665	8228216	8228869	8229522	8230175	8230828
666	8234742	8235394	8236046	8236698	8237350
667	8241258	8241909	8242560	8243211	8243862
668	8247765	8248415	8249065	8249715	8250364
669	8254261	8254910	8255559	8256208	8256857
670	8260748	8261396	8262044	8262692	8263340
671	8267225	8267872	8268519	8269166	8269813
672	8273693	8274339	8274985	8275631	8276277
673	8280151	8280796	8281141	8282086	8282731
674	8286599	8287243	8287887	8288532	8289176
675	8293038	8293681	8294324	8294967	8295611
676	8299467	8300109	8300752	8301394	8302036
677	8305887	8306528	8307169	8307811	8308452
678	8312297	8312937	8313578	8314218	8314858
679	8318698	8319337	8319977	8320616	8321255
680	8325089	8325728	8326366	8327005	8327643
681	8331471	8332109	8332746	8333384	8334021
682	8337844	8338480	8339117	8339754	8340390
683	8344207	8344843	8345479	8346114	8346750
684	8350561	8351196	8351831	8352465	8353100
685	8356906	8357540	8358174	8358807	8359441
686	8363241	8363874	8364507	8365140	8365773
687	8369567	8370199	8370832	8371463	8372095
688	8375884	8376516	8377147	8377778	8378409
689	8382192	8382822	8383453	8384083	8384713
690	8388491	8389120	8389750	8390379	8391008
691	8394780	8395409	8396037	8396666	8397294
692	8401061	8401688	8402316	8402943	8403571
693	8407332	8407959	8408586	8409212	8409838
694	8413595	8414220	8414846	8415472	8416097
695	8419848	8420473	8421098	8421722	8422347
696	8426092	8426716	8427340	8427964	8428588
697	8432328	8432951	8433574	8434197	8434819
698	8438554	8439176	8439798	8440420	8441042
699	8444772	8445393	8446014	8446635	8447256
700	8450980	8451601	8452221	8452841	8453461
701	8457180	8457800	8458419	8459038	8459658
702	8463371	8463990	8464608	8465227	8465845
703	8469553	8470171	8470789	8471406	8472024
704	8475727	8476343	8476960	8477577	8478193
705	8481891	8482507	8483123	8483739	8484355
706	8488047	8488662	8489277	8489892	8490507

Natural Numbers.	5	6	7	8	9
663	8218409	8219064	8219718	8220372	8221027
664	8224950	8225603	8226257	8226910	8227563
665	8231481	8232133	8232786	8233438	8234090
666	8238002	823 653	8239305	8239956	8240607
667	8244513	8245163	8245814	8246464	8247114
668	8251014	8251664	8252313	8252963	8253612
669	8257506	8258154	8258803	8259451	8260100
670	8263988	8264635	8265283	8265931	8266578
671	8270460	8271107	8271753	8272400	8273046
672	8276923	8277569	8278214	8278860	8279505
673	8233376	8284021	8284665	8285310	8285955
674	8289820	8290463	8291107	8291751	8292394
675	8296254	8296896	8297539	8298182	8298824
676	8302678	8303320	8303962	8304603	8305245
677	8309093	8309734	8310375	8311016	8311656
678	8315499	8316139	8316778	8317418	8318058
679	8321895	8322534	8323173	8323812	8324450
680	8328281	8328919	8329558	8330195	8330833
681	8334659	8335296	8335933	8336570	8337207
682	8341027	8341663	8342299	8342937	8343571
683	8347385	8348021	8348656	8349291	8349926
684	8353735	8354369	8355003	8355638	8356272
685	8360075	8360708	8391341	8361975	8362608
686	8366405	8367038	8367670	8368303	8368935
687	8372727	8373359	8373990	8374622	8375253
688	8379039	8379670	8380301	8380931	8381562
689	8385343	8385973	8386602	8387232	8387861
690	8391637	8392266	8392895	8393523	8394152
691	8397922	8398550	8399178	8399806	8400433
692	8404198	404825	8405452	8406079	8406706
693	8410465	8411091	8411717	8412343	8412969
694	8416722	8417348	8417973	8418598	8419223
695	8422971	8423596	8424220	8424844	8425468
696	8429211	8429735	8430458	8431081	8431705
697	8435442	8436065	8436687	8437310	8437932
698	8441664	8442286	8442907	8443529	8444150
699	8447877	8448498	8449119	8449739	8450360
700	8454081	8454701	8455321	8455941	8456561
701	8460277	8460896	8461515	8462134	8462752
702	8466463	8467081	8467700	8468318	8468935
703	8472641	8473258	8473876	8474493	8475110
704	8478810	8479426	8480043	8480659	8481275
705	8484970	8485586	8486201	8486817	8487432
706	8491122	8491736	8492351	8492965	8493580

Natural Numbers.	0	1	2	3	4
707	8494194	8494808	8495423	8496037	8496651
708	8500333	8500946	8501559	8502172	8502786
709	8506462	8507075	8507687	8508300	8508912
710	8512583	8513195	8513807	8514418	8515030
711	8518696	8519307	8519917	8520528	8521139
712	8524800	8525410	8526020	8526629	8527239
713	8530895	8531504	8532113	8532722	8533331
714	8536982	8537590	8538198	8538806	8539414
715	8543060	8543668	8544275	8544882	8545489
716	8549130	8549737	8550343	8550949	8551556
717	8555192	8555797	8556403	8557008	8557614
718	8561244	8561849	8562454	8563559	8563663
719	8567289	8567893	8568497	8569101	8569704
720	8573325	8573928	8574531	8575134	8575737
721	8579353	8579955	8580557	8581159	8581761
722	8585372	8585973	8586575	8587176	8587777
723	8591383	8591984	8592584	8593185	8593785
724	8597386	8597985	8598585	8599185	8599784
725	8603380	8603979	8604578	8605177	8605776
726	8609366	8609964	8610562	8611160	8611758
727	8615344	8615941	8616539	8617136	8617733
728	8621314	8621910	8622507	8623103	8623699
729	8627275	8627871	8628467	8629062	8629658
730	8633229	8633823	8634418	8635013	8635608
731	8639174	8639768	8640362	8640956	8641550
732	8645111	8645704	8646297	8646890	8647483
733	8651040	8651632	8652225	8652817	8653409
734	8656961	8657552	8658144	8658735	8659327
735	8662873	8663464	8664055	8664646	8665236
736	8668778	8669368	8669958	8670548	8671138
737	8674675	8675264	8675853	8676442	8677031
738	8680564	8681152	8681740	8682329	8682917
739	8686444	8687032	8687620	8688207	8688794
740	8692317	8692904	8693491	8694077	8694664
741	8698182	8698768	8699354	8699940	8700526
742	8704039	8704624	8705209	8705795	8706380
743	8709888	8710473	8711057	8711641	8712226
744	8715729	8716313	8716897	8717480	8718064
745	8721563	8722146	8722728	8723311	8723894
746	8727388	8727970	8728552	8729134	8729716
747	8733206	8733788	8734369	8734950	8735531
748	8739016	8739597	8740177	8740757	8741338
749	8744818	8745398	8745978	8746557	8747137
750	8750613	8751192	8751771	8752349	8752928

Natural Numbers.	5	6	7	8	9
707	8497264	8497878	8498492	8499106	8499719
708	8503399	8504011	8504624	8505237	8505850
709	8509524	8510136	8510748	8511360	8511972
710	8515641	8516252	8516863	8517474	8518085
711	8521749	8522359	8522970	8523580	8524190
712	8527849	8528458	8529068	8529677	8530286
713	8533940	8534548	8535157	8535765	8536374
714	8540022	8540630	8541238	8541845	8542453
715	8546096	8546703	8547310	8547917	8548524
716	8552162	8552768	8553374	8553980	8554586
717	8558219	8558824	8569429	8560035	8560640
718	8564268	8564872	8555476	8566081	8566685
719	8570308	8570912	8571515	8572118	8572722
720	8576340	8576943	8577545	8578148	8578750
721	8582363	8582965	8583567	8584169	8584770
722	8588379	8588980	8589581	8590181	8590782
723	8594385	8594986	8595586	8596186	8596786
724	8600384	8600983	8601583	8602182	8602781
725	8606374	8606973	8607571	8608170	8608768
726	8612356	8612954	8613552	8614149	8614747
727	8618330	8618927	8619524	8620120	8620717
728	8624296	8624892	8625488	8626084	8626679
729	8630253	8630848	8631443	8632039	8632634
730	8636202	8636797	8637391	8637985	8638580
731	8642143	8642737	8643331	8643924	8644517
732	8648076	8648669	8649262	8649855	8650447
733	8654001	8654593	8655185	8655777	8656369
734	8659918	8660509	8661100	8661691	8662282
735	8665827	8666417	8667008	8667598	8668188
736	8671728	8672317	8672907	8673496	8674086
737	8677620	8678209	8678798	8679387	8679975
738	8683505	8684093	8684681	8685269	8685857
739	8689382	8689969	8690556	8691143	8691730
740	8695251	8695837	8696423	8697010	8697596
741	8701112	8701697	8702283	8720868	8703454
742	8706965	8707549	8708134	8708719	8709304
743	8712810	8713394	8713978	8714562	8715146
744	8718647	8719230	8719814	8720397	8720980
745	8724476	8725059	8725641	8726224	8726806
746	8730298	8730880	8731461	8732043	8732625
747	8736112	8736693	8737274	8737855	8738435
748	8741918	8742498	8743078	8743658	8744238
749	8747716	8748296	8748875	8749454	8750034
750	8753507	8754086	8754664	8755243	8755821

Natural Numbers.	0	1	2	3	4
751	8756399	8756978	8757556	8758134	8758712
752	8762178	8762756	8763333	8763911	8764488
753	8767950	8768526	8769103	8769680	8770256
754	8773713	8774289	8774865	8775441	8776017
755	8779469	8780045	8780620	8781195	8781770
756	8785218	8785792	8786367	8786941	8787515
757	8790959	8791532	8792106	8792680	8793253
758	8796692	8797265	8797838	8798411	8798983
759	8802418	8802990	8803562	8804134	8804706
760	8808136	8808707	8809279	8809850	8810421
761	8813847	8814417	8814988	8815558	8816129
762	8819550	8820120	8820689	8821259	8821829
763	8825245	8825815	8826384	8826953	8827522
764	8830934	8831502	8832070	8832639	8833207
765	8836614	8837182	8837750	8838317	8838885
766	8842288	8842855	8843421	8843988	8844555
767	8847954	8848520	8849086	8849652	8850218
768	8853612	8854178	8854743	8855308	8855874
769	8859263	8859828	8860393	8860957	8861522
770	8864907	8865471	8866035	8866599	8867163
771	8870544	8871107	8871670	8872233	8872796
772	8876173	8876736	8877298	8877860	8878423
773	8881795	8882357	8882918	8883480	8884042
774	8887410	8887971	8888532	8889093	8889653
775	8893017	8893577	8894138	8894698	8895258
776	8898617	8899177	8899736	8900296	8900855
777	8904210	8904765	8905328	8905887	8906445
778	8909796	8910354	8910912	8911470	8912028
779	8915375	8915932	8916489	8917047	8917604
780	8920946	8921503	8922059	8922616	8923173
781	8926510	8927066	8937622	8928178	8928734
782	8932068	8932623	8933178	8933733	8934288
783	8937618	8938172	8938727	8939281	8939836
784	8943161	8943715	8944268	8944822	8945376
785	8948697	8949250	8949803	8950356	8950909
786	8954225	8954778	8955330	8955883	8956435
787	8959747	8960299	8960851	8961403	8961954
788	8965262	8965813	8966364	8966915	8967466
789	8978770	8971320	8971871	8972421	8972971
790	8976271	8976821	8977370	8977920	8978469
791	8981765	8982314	8982863	8983412	8983960
792	8987252	8987800	8988348	8988897	8989445
793	8992732	8993279	8993827	8994375	8994922
794	8998205	8998752	8999299	8999846	9000392

Natural Numbers.	5	6	7	8	9
751	8759290	8759868	8760445	8761023	8761601
752	8765065	8765642	8766219	8766796	8767373
753	8770833	8771409	8771985	8772561	8773137
754	8776592	8777168	8777743	8778319	8778894
755	8782345	8782919	8783494	8784069	8784643
756	8788089	8788663	8789237	8789811	8790385
757	8793826	8794400	8794973	8795546	8796119
758	8799556	8800128	8800701	8801273	8801846
759	8805278	8805850	8806421	8806993	8807564
760	8810992	8811563	8812134	8812705	8813276
761	8816699	8817269	8817840	8818410	8818980
762	8822398	8822968	8823537	8824107	8824676
763	8828090	8828659	8829228	8829797	8830365
764	8833775	8834343	8834911	8835479	8836047
765	8839452	8840019	8840586	8841154	8841721
756	8845122	8845688	8846255	8846821	8847387
767	8850784	8851350	8851915	8852481	8853047
768	8856439	8857004	8857569	8858134	8858699
769	8866086	8862651	8863215	8863779	8863343
770	8867726	8868290	8868854	8869417	8869980
771	8873359	8873922	8874485	8875048	8875610
772	8878985	8879547	8880109	8880671	8881233
773	8884603	8885165	8885726	8886287	8886848
774	8890214	8890775	8891336	8891896	8892457
775	8895818	8896378	8896935	8897498	8898058
776	8901415	8901974	8902533	8903092	8903651
777	8907004	8907562	8908121	8908679	8909238
778	8912586	8913144	8913702	8914259	8914817
779	8918161	8918718	8919275	8919832	8920389
780	8923729	8924285	8924842	8925398	8925954
781	8929290	8929846	8930401	8930957	8931512
782	8934843	8935398	8935953	8936508	8937063
783	8940390	8940944	8941498	8942053	8942607
784	8945929	8946483	8947037	8947590	8948143
785	8951462	8952015	8952567	8953120	8953673
786	8956987	8957539	8958092	8958644	8959195
787	8962506	8963057	8963608	8964160	8964711
788	8968017	8968568	8969118	8969669	8970219
789	8973521	8974071	8974621	8975171	8975721
790	8979019	8979568	8980117	8980667	8981216
791	8984509	8985058	8985606	8986155	8986703
792	8989993	8990541	8991089	8991636	8992184
793	8995469	8996017	8996564	8997111	8997658
794	9000939	9001486	9002032	9002579	9003125

Natural Numbers.	0	1	2	3	4
795	9003671	9004218	9004764	9005310	9005856
796	9009131	9009676	9010222	9010767	9011313
797	9014583	9015128	9015673	9016218	9016762
798	9020029	9020573	9021117	9021661	9022205
799	9025468	9026011	9026555	9027098	9027641
800	9030900	9031443	9031985	9032528	9033071
801	9036325	9036867	9037409	9037951	9038493
802	9041744	9042285	9042827	9043368	9043909
803	9047155	9047696	9048237	9048778	9049318
804	9052560	9053101	9053641	9054181	9054721
805	9057960	9058498	9059038	9059577	9060116
806	9066351	9063889	9064428	9064967	9065505
807	9068735	9069273	9069812	9070350	9070887
808	9074114	9074651	9075188	9075726	9076263
809	9079485	9080022	9080559	9081095	9081632
810	9084850	9085386	9085922	9086458	9086994
811	9090209	9090744	9091279	9091815	9092350
812	9095560	9096095	9096630	9097165	9097699
813	9100905	9101440	9101974	9102508	9103042
814	9106244	9106778	9107311	9107844	9108378
815	9111576	9112109	9112642	9113174	9113707
816	9116902	9117434	9117966	9118498	9119030
817	9122220	9122752	9123234	9123815	9124346
818	9127533	9128064	9128595	9129126	9129656
819	9132839	9133369	9133899	9134430	9134960
820	9138139	9138668	9139198	9139727	9140257
821	9143432	9143961	9144489	9145018	9145547
822	9148718	9149246	9149775	9150303	9150831
823	9153998	9154526	9155054	9155581	9156109
824	9159272	9159799	9160326	9160853	9161380
825	9164539	9165066	9165592	9166118	9166645
826	9169800	9170326	9170852	9171378	9171903
827	9175055	9175580	9176105	9176630	9177155
828	9180303	9180828	9181352	9181877	9182401
829	9185545	9186069	9186593	9187117	9187640
830	9190781	9191304	9191827	9192350	9192873
831	9196010	9196533	9197055	9197578	9198100
832	9201233	9201755	9202277	9202799	9203321
833	9206450	9206971	9207493	9208014	9208535
834	9211661	9212181	9212702	9213222	9213743
835	9216865	9217385	9217905	9218425	9218945
836	9222063	9222582	9223102	9223621	9224140
837	9227255	9227773	9228292	9228811	9229330
838	9232440	9232958	9233477	9233995	9234513

Natural Numbers.	5	6	7	8	9
795	9006402	9006948	9007494	9008039	9008585
796	9011858	9012403	9012984	9013448	9014038
797	9017307	9017851	9018396	9018940	9019485
798	9022749	9023293	9023837	9024381	9024924
799	9028185	9028728	9029271	9029814	9030357
800	9033613	9034156	9034698	9035241	9035783
801	9039035	9039577	9040119	9040661	9041202
802	9044450	9044992	9045533	9046073	9046615
803	9049859	9050399	9050940	9051480	9052020
804	9055261	9055800	9056340	9056880	9057419
805	9060655	9061195	9061734	9062274	9062812
806	9066044	9066582	9067121	9067659	9068197
807	9071425	9071963	9072501	9073038	9073576
808	9076800	9077337	9077874	9078411	2078948
809	9082169	9082705	9083241	9083778	9084314
810	9087530	9088066	9088602	9089137	9089673
811	9092885	9093420	9093955	9094490	9095025
812	9098234	9098768	9099303	9099837	9100371
813	9103576	9104109	9104643	9105177	9105710
814	9108911	9109444	9109977	9110510	9111043
815	9114240	9114772	9115305	9115837	9116369
816	9119562	9120094	9120626	9121157	9121689
817	9124878	9125409	9125940	9126471	9127002
818	9130187	9130717	9131248	9131778	9132309
819	9135490	9136019	9136549	9137079	9137609
820	9140786	9141315	9141844	9142373	9142905
821	9146076	9146604	9147133	9147661	9148190
822	9151359	9151887	9152415	9152943	9153471
823	9156636	9157163	9157691	9158218	9158745
824	9161907	9162433	9162960	9163487	9164013
825	9167171	9167697	9168223	9168749	9161275
826	9172429	9172954	9173479	9174005	9174530
827	9177680	9178205	9178730	9179254	9179779
828	9182925	9183449	9183973	9184497	9185021
829	9188164	9188687	9189211	9189734	9190258
830	9193396	9193919	9194442	9194965	9195488
831	9198623	9199145	9199667	9200189	9200711
832	9203842	9204364	9204886	9205407	9205929
833	9209056	9209577	9210098	9210619	9211140
834	9214263	9214784	9215304	9215824	9216345
835	9219465	9219984	9220504	9221024	9221543
836	9224659	9225179	9225698	9226217	9226736
837	9229848	9230367	9230885	9231404	9231922
838	9235031	9235549	9236066	9236584	9237192

Natural Numbers.	0	1	2	3	4
839	9237620	9238137	9238655	9239172	9239690
840	9242793	9243310	9243827	9244344	9244860
841	9247960	9248476	9248993	9249509	9250025
842	9253121	9253637	9254152	9254668	9255184
843	9258276	9258791	9259306	9259821	9260336
844	9263424	9263939	9264453	9264968	9265482
845	9268567	9269081	9269595	9270109	9270622
846	9273704	9274217	9274730	9275243	9275757
847	9278834	9279347	9279859	9280372	9280885
848	9283959	9284471	9284983	9285495	9286007
849	9289077	9289588	9290100	9290611	9291123
850	9294189	9294700	9295211	9295722	9296233
851	9299296	9299806	9300316	9300826	9301336
852	9304396	9304906	9305415	9305925	9306434
853	9309490	9309999	9310508	9311017	9311526
854	9314579	9315087	9315596	9316104	9316612
855	9319661	9320169	9320677	9321185	6321692
856	9324738	9325245	9325752	9326259	9326767
857	9329808	9330315	9330822	9331328	9331835
858	9334873	9335379	9335885	9336391	9336897
859	9339932	9340437	9340943	9341448	9341953
860	9344984	9345489	9345994	9346499	9347004
861	9350032	9350536	9351040	9351544	9352049
862	9355073	9355576	9356080	9356584	9357087
863	9360108	9360611	9361114	9361617	9362120
864	9365137	9365640	9366143	9366645	9367148
865	9370161	9370663	9371165	9371667	9372169
866	9375179	9375680	9376182	9376683	9377184
867	9380191	9380692	9381193	9381693	9382194
868	9385197	9385697	9386198	9386698	9397198
869	9390198	9390697	9391197	9391697	9392196
870	9395193	9395692	9396191	9396690	9397189
871	9400182	9400680	9401179	9401677	9402176
872	9405165	9405663	9406161	9406659	9407157
873	9410142	9410640	9411137	9411635	9412132
874	9415114	9415611	9416108	9416605	9417101
875	9420081	9420577	9421073	9421569	9422065
876	9425041	9425537	9426032	9426528	9427024
877	9429996	9430491	9430986	9431481	9431976
878	9434945	9435440	9435934	9436429	9436923
879	9439889	9440383	9440877	9441371	9441865
880	9444827	9445320	9445814	9446307	9446800
881	9449759	9450252	9450745	9451238	9451730
882	9454686	9455178	9455671	9456163	9456655

Natural Numbers.	5	6	7	8	9
839	9240208	9240724	9241246	9241759	9242276
840	9245377	9245894	9246410	9246927	9247444
841	9250541	9251057	9251573	9252689	9252605
842	9255699	9256215	9256730	9257245	9257761
843	9260851	9261366	9261880	9262395	9262910
844	9265995	9266511	9267025	9267539	9268053
845	9271136	9271650	9272163	9272677	9273190
846	9276270	9276783	9277296	9277808	9278321
847	9281397	9281909	9282422	9282934	9283446
848	9286518	9287030	9287542	9288054	9288565
849	9291634	9292145	9292656	9293167	9293678
850	9296743	9297254	9297764	9298275	9298785
851	9301847	9302357	9302866	9303376	9303886
852	9306944	9307453	9307963	9308472	9308981
853	9312035	9312544	9313053	9313561	9314070
854	9317121	9317629	9318137	9318645	9319153
855	9322200	9322708	9323215	9323723	9324230
856	9327274	9327781	9328288	9328795	9329301
857	9332341	9332848	9333354	9333860	9334367
858	9337403	9337909	9338415	9338920	9339426
859	9342459	9342964	9343469	9343974	9344479
860	9347509	9348013	9348518	9349022	9349527
861	9352553	9353057	9353561	9354065	9354569
862	6357591	9358095	9358598	9359101	9359605
863	9362623	9363126	9363629	9364132	9364635
864	9367650	9368152	9368655	9369157	9369659
865	9372671	9373172	9373674	9374176	9374677
866	9377686	9378187	9378688	9379189	9379690
867	9382695	9383195	9383696	9384196	9384697
868	9387698	9388198	9388698	9389198	9389698
869	9392696	9393195	9393695	9394194	9 94693
870	9397688	9398187	9398685	9399184	9399683
871	9402674	9403172	9403670	9404169	9404667
872	9407654	9408152	9408650	9409147	9409645
873	9412629	9413126	9413623	9414120	9414617
874	9417598	9418095	9418591	9419088	9419584
875	9422561	9423058	9423554	9424049	9424545
876	9427519	9428015	9428510	9429005	9429501
877	9432471	9432966	9433461	9433956	9434450
878	9437418	9437912	9438406	9438900	9439395
879	9442358	9442852	9443346	9443840	9444333
880	9447294	9447787	9448280	9448773	9449266
881	9452223	9452716	9453208	9453701	9454193
882	9457147	9457639	9458131	9458623	9459115

Natural Numbers.	0	1	2	3	4
883	9459607	9460099	9460591	9461082	9461574
884	9464523	9465014	9465505	9465996	9466487
885	9469433	9469923	9470414	9470905	9471395
886	9474337	9474827	9475317	9475807	9476297
887	9479236	9479726	9480215	9480705	9481194
888	9484130	9484619	9485108	9485597	9486085
889	9489018	9489506	9489994	9490483	9490971
890	9493900	9494388	9494876	9495364	9495852
891	9498777	9499264	9499872	9500239	9500726
892	9503649	9504135	9504622	9505109	9505596
893	9508515	9509001	9509487	9509973	9510459
894	9513375	9513861	9514347	9514832	9515318
895	9518230	9518716	9519201	9519686	9520171
896	9523080	9523565	9524049	9524534	9525018
897	9527924	9528409	9528893	9529377	9529861
898	9532763	9533247	9533730	9534214	9534697
899	9537597	9538080	9538563	9539046	9539529
900	9542425	9542908	9543390	9543872	9544355
901	9547248	9547730	9548212	9548694	9549176
902	9552065	9552547	9553028	9553510	9553991
903	9556877	9557358	9557839	9558320	9558801
904	9561684	9562165	9562645	9563125	9563605
905	9566486	9566966	9567445	9567925	9568405
906	9571282	9571761	9572241	9572720	9573199
907	9576073	9576552	9577030	9577509	9577988
908	9580858	9581337	9581815	9582293	9582771
909	9585639	9586117	9586594	9587072	9587549
910	9590414	9590891	9591368	9591845	9592322
911	9595184	9595660	9596137	9596614	9597090
912	9599948	9600425	9600901	9601377	9601853
913	9604708	9605183	9605659	9606135	9606610
914	9609462	9609937	9610412	9610887	9611362
915	9614211	9614686	9615160	9615635	9616109
916	9618955	9619429	9619903	9620377	9620851
917	9623693	9624167	9624640	9625114	9625587
918	9628427	9628900	9629373	9629846	9630319
919	9633155	9633628	9634100	9634573	9635045
920	9637878	9638350	9638822	9639294	9639766
921	9642596	9643068	9643539	9644011	9644482
922	9547309	9647780	9648251	9648722	9547193
923	9652017	9652488	9652958	9653428	9653899
924	9656720	9657190	9657660	9658130	9658599
925	9661417	9661887	9662356	9662826	9663295
926	9666110	9667579	9667048	9667517	9667985

Natural Nnmbers.	5	6	7	8	9
883	9462066	9462557	9463048	9463540	9464031
884	9466978	9467469	9467960	9468451	9468942
885	9471886	9472376	9472866	9473357	9473847
886	9476787	9477277	9477767	9478257	9748747
887	9481684	9482173	9482662	9483151	9483641
888	9486574	9487063	9487552	9488040	9488529
889	9491460	9491948	9492436	9492924	9493412
890	9495330	9496827	9497314	9497802	9498290
891	9501213	9501701	9502188	9502675	9503162
892	9506082	9506569	9507055	9507542	9508028
893	9510946	9511432	9511918	9512404	9512889
894	9515803	9516289	9516774	9517260	9517745
895	9520656	9521141	9521626	9522111	9522595
896	9525503	9525987	9526472	9526956	9527440
897	9530345	9530828	9531312	9531796	9532280
898	9535181	9535664	9536147	9536631	9537114
899	9540012	9540494	9540977	9541460	9541943
900	9544837	9545319	9545802	9546284	9546766
901	9549657	9550139	9550621	9551102	9551584
902	9554472	9554953	9555434	9555915	9556397
903	9559282	9559762	9560243	9560723	9561204
904	9564086	9564566	9565046	9565526	9566006
905	9568885	9569364	9569844	9570323	9570803
906	9573678	9574157	9574636	9575115	9575594
907	9578466	9578945	9579423	9579902	9580380
908	9583249	9583727	9584205	9584683	9585161
909	9588027	9588505	9588982	9589459	9589937
910	9592799	9593276	9593754	9594230	9594707
911	9597567	9598043	9598520	9598996	9599472
912	9602329	9602805	9503280	9603756	9604232
913	9607086	9607561	9608036	9608511	9608987
914	9611837	9612312	9612787	9613261	9613736
915	9616583	9617058	9617532	9618006	9618481
916	9621325	9621799	9622272	9622746	9623220
917	9626061	9626534	9627007	9627481	9627954
918	9630792	9631264	9631737	9632210	9632683
919	9635517	9635990	9636462	9636934	9637406
920	9640238	9640710	9641181	9641653	9642125
921	9644953	9645425	9645896	9646367	9646838
922	9649664	9650134	9650605	9651076	9651546
923	9654369	9654839	9655309	9655780	9656250
924	9659069	9559539	9660009	9660478	9660948
925	9663764	9664233	9664703	9665172	9665641
926	9668454	9668923	9669392	9669860	9670329

Natural Numbers.	0	1	2	3	4
927	9670797	9671266	9671734	9672203	9672671
928	9675480	9675948	9676416	9676883	9677351
929	9680157	9680625	9681092	9681559	9682027
930	9684829	9685296	9685763	9686230	9686697
931	9689497	9689963	9690430	9690896	9691362
932	9694159	9694625	9695091	9695557	9696023
933	9698816	9699282	9699747	9700213	9700678
934	9703469	9703934	9704399	9704863	9705328
935	9708116	9708581	9709045	9709509	9709974
936	9712758	9713222	9713686	9714150	9714614
937	9717396	9717859	9718323	9718786	9719249
938	9722028	9722491	9722954	9723417	9723880
939	9726656	9727118	9727581	9728043	9728506
940	9731278	9731741	9732202	9732664	9733126
941	9735896	9736358	9736819	9737281	9737742
942	9740509	9740970	9741431	9741892	9742353
943	9745117	9745577	9746038	9746498	9746959
944	9749720	9750180	9750640	9751100	9751560
945	9754318	9754778	9755237	9755695	9756156
946	9758911	9759370	9759829	9760288	9760747
947	9763500	9763958	9764417	9764875	9765334
948	9768083	9768541	9768999	9769457	9769915
949	9772662	9773120	9773577	9774035	9774492
950	9777236	9777693	9778150	9778607	9779064
951	9781805	9782262	9782718	9783175	9783631
952	9786369	9786826	9787282	9787738	9788194
953	9790929	9791385	9791840	9792296	9792751
954	9795484	9795939	9796394	9796849	9797304
955	9800034	9800488	9800943	9801398	9801852
956	9804579	9805033	9805487	9805942	9806396
957	9809119	9809573	9810027	9810481	9810934
958	9813655	9814108	9814562	9815015	9815468
959	9818186	9818639	9819092	9819544	9819997
960	9822712	9823165	9823617	9824069	9824522
961	9827234	9827686	9828138	9828589	9829041
962	9831751	9832202	9832654	9833105	9833556
963	9836263	9836714	9837165	9837616	9838066
964	9840770	9841221	9841671	9842122	9842572
965	9845273	9845723	9846173	9846623	9847073
966	9849771	9850221	9850670	9851120	9851569
967	9854265	9854714	9855163	9855612	9856061
968	9858754	9859202	9859651	9860099	9860548
969	9863238	9863686	9864134	9864582	9865030
970	9867717	9868165	9868613	9869060	9869508

Natural Numbers.	5	6	7	8	9
927	9673139	9673607	9674076	9674544	9675012
928	9677819	9678287	9678754	9679222	9679690
929	9682494	9682961	9683428	9683895	9684362
930	9687164	9687630	9688097	9688564	9689030
931	9691829	9692295	9692761	9693227	9693693
932	9696488	9696954	9697420	9697885	9698351
933	9701143	9701608	9702074	9702539	9703004
934	9705793	9706258	9706722	9707187	9707652
935	9710438	9710902	9711366	9711830	9712294
936	9715078	9715542	9716005	9716469	9716932
937	9719713	9720176	9720639	9721102	9721565
938	9724343	9724805	9725208	9725731	9726193
939	9728968	9729430	9729892	9730354	9730816
940	9733588	9734050	9734511	9734973	9735435
941	9738203	9738664	9739126	9739587	9740048
942	9742814	9743274	9743735	9744196	9744656
943	9747419	9747879	9748340	9748800	9749260
944	9752020	9752479	9752939	9753399	9753858
945	9756615	9757075	9757534	9757993	9758452
946	9761206	9761665	9762124	9762582	9763041
947	9765792	9766251	9766709	9767167	9767625
948	9770373	9770831	9771289	9771747	9772204
949	9774950	9775407	9775864	9776322	9776779
950	9779521	9779978	9780435	9780892	9781348
951	9784088	9784544	9785001	9785457	9785913
952	9788650	9789106	9789562	9790017	9790473
953	9793207	9793662	9794118	9794573	9795028
954	9797759	9798214	9798669	9799124	9799579
955	9802307	9802761	9803216	9803670	9804125
956	9806850	9807304	9807758	9808212	9808666
957	9811388	9811841	9812295	9812748	9813202
958	9815921	9816374	9816827	9817280	9817733
959	9820450	9820902	9821355	9821807	9822260
960	9824974	9825426	9825878	9826330	9826782
961	9829493	9839945	9830396	9830848	9831299
962	9834007	9834459	9834910	9835361	9835812
963	9838517	9838968	9839419	9839869	9840320
964	9843022	9843473	9843923	9844373	9844823
965	9847523	9847973	9848422	9848872	9849322
966	9852019	9852468	9852917	9853366	9853816
967	9856510	9856959	9857407	9857856	9858305
968	9860956	9861445	9861893	9862341	9862790
969	9865478	9865926	9866374	9866822	9867270
970	9869955	9870403	9870850	9871298	6871745

Natural Numbers.	0	1	2	3	4
971	9872192	9872640	9873087	9873534	9873981
972	9876663	9877109	9877556	9878003	9878449
973	9881128	9881575	9882021	9882467	9882913
974	9885590	9886035	9886481	9886927	9887373
975	9890046	9890492	9890937	9891382	9891828
976	9894498	9894943	9895388	9895833	9896278
977	9898946	9899390	9899835	9900279	9900723
978	9903389	9903833	9904277	9904721	9905164
979	9907827	9908270	9908714	9909158	9909601
980	9912261	9912704	9913147	9913590	9914033
981	9916690	9917133	9917575	9918018	9918461
982	9921115	9921557	9921999	9922441	9922884
983	9925535	9925977	9926419	9926860	9927302
984	9929951	9930392	9930834	9931275	9931716
985	9934362	9934803	9935244	9935685	9936126
986	9938769	9939210	9939650	9940090	9940531
987	9943172	9943612	9944051	9944491	9944931
988	9947569	9948009	9948448	9948888	9949327
989	9951963	9952402	9952841	9953280	9953719
990	9956352	9956791	9757229	9957668	9958106
991	9960737	9961175	9951613	9962051	9962489
992	9965117	9965554	9965992	9966430	9966868
993	9969492	9969930	9970367	9970804	9971242
994	9973864	9974301	9974738	9975174	9975611
995	9978231	9978667	9979104	9979540	9979976
996	9982593	9983029	9983465	9983901	9984337
997	9986952	9987387	9987823	9988258	9988694
998	9991305	9991740	9992176	9992611	9993046
999	9995655	9996090	9996524	9996959	9997393

Natural Numbers.	5	6	7	8	9
971	9874428	9874875	9875322	9875769	9876216
972	9878896	9879343	9879789	9880236	9880682
973	9883360	9883806	9884252	9884698	9885144
974	9887818	9888264	9888710	9889155	9889601
975	9892273	9892718	9893163	9893608	9894050
976	9896722	9897167	9897612	9898056	9898501
977	9901168	9901612	9902056	9902500	9902944
978	9905608	9906052	9906496	9906940	9907383
979	9910044	9910488	9910931	9911374	9911818
980	9914476	9914919	9915362	9915805	9916247
981	9918903	9919345	9919788	9920230	9920673
982	9923726	9923758	9924210	9924651	9925093
983	9927744	9928185	9928627	9929068	9929510
984	9932157	9932598	9933039	9933480	9933921
985	9936566	9937007	9937448	9937888	9938329
986	9940971	9941411	9941851	9942291	9942731
987	9945371	9945811	9946251	9946690	9947130
988	9949767	9959206	9950645	9951085	9951524
989	9954158	9954597	9955036	9955474	9955913
990	9958545	9958983	9959422	9959860	9960298
991	9962927	9963365	9963803	9964241	9964679
992	9967305	9967743	9968180	9968618	9969055
993	9971679	9972116	9972553	9972990	9973427
994	9976048	9976485	9976921	9977358	9977794
995	9980413	9980849	9981285	9981721	9982157
996	9984773	9985209	9985645	9986080	9986516
997	9989129	9989564	9990000	9990435	9990870
998	9993481	9993916	9994350	9994785	9995220
999	9997828	9998292	9998697	9999131	9999566

10000 it's Log. = 4.0000000

The End of the Table of the Logarithms.

THE

CONTENTS.

Deci-

CHAP.

The CONTENTS.

CHAP. IX.

CHAP. X.

CHAP. XI.

CHAP.

The CONTENTS.

CHAP. XII.

CHAP. XIII.

CHAP. XIV.

F I N I S.